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Quark Matter in a QCD Coulomb Gauge Quark Model

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In this communication we present results of a study of chiral symmetry in quark matter using an effective Coulomb gauge QCD Hamiltonian. QCD in Coulomb gauge is convenient for a variational approach based on a quasiparticle picture for the transverse gluons, in which a confining Coulomb potential arises naturally. We show that such an effective Hamiltonian predicts chiral restoration at too low quark densities. Possible reasons for such deficiency are discussed.

I. INTRODUCTION

The study of the temperature and baryon chemical potential dependence of chiral parameters such as the quark condensate and the pion decay constant is of importance for the understanding of several aspects of the nonperturbative phase of quantum chromodynamics (QCD), in particular to the phenomenon of chiral restoration. Questions related to the nature of the quark-gluon deconfinement transition and possible changes of hadron properties at high temperatures and baryon densities might be connected to chiral restoration. Experiments of high-energy heavy ions collisions are designed to produce highly excited hadronic matter with the hope that it will shed light on these questions. Theoretically, these questions have been discussed since long time in the context of the Nambu–Jona-Lasinio (NJL) model [1]. The reason for the great popularity of the NJL model is essentially its mathematical tractability, giving a simple picture for the phenomenon of dynamical chiral symmetry breaking (DχSB). However, the use of models closer to QCD, which can be derived by means of some truncation scheme from the fundamental theory is clearly preferable to purely phenomenological models. In the present communication we present the results of a study of DχSB in quark matter using a model derived directly from QCD, based on a truncation scheme of QCD in Coulomb gauge [2, 3]. We investigate DχSB by means of the self-consistent solution of the Schwinger-Dyson equation for the quark propagator in the Hartree-Fock approximation.

II. EFFECTIVE HAMILTONIAN DERIVED FROM COULOMB GAUGE QCD

One of the main motivations for employing the Coulomb gauge in QCD is that an *instantaneous* long-range confining potential appears in the lattice Coulomb-gauge Hamiltonian [4]. In the continuum, the formulation of QCD in this gauge has shown to be very convenient for a variational approach based on a quasiparticle picture for the transverse gluons [2, 3], in which a confining Coulomb V_{coul} potential arises naturally. In addition, since in the Coulomb gauge Hamiltonian all degrees of freedom are physical, it provides immediate contact with quantum mechanical formulations of quark models, in which hadron bound states can be constructed as Fock space states built on the top of a vacuum state.

The variational ansatz for the gluonic vacuum functional is written in terms of the Fourier transform of the transverse gluon vector fields $\mathbf{A}^a(\mathbf{x})$, $a = 1,2,\cdots,8$, which satisfy the transverse gauge condition, $\nabla\cdot\mathbf{A}^a(\mathbf{x}) = 0$, in the form

$$\langle A|\omega\rangle = \Phi[A,\omega] = \exp\left[-\frac{1}{2}\int\frac{d^3k}{(2\pi)^3}\,\omega(k)\,\mathbf{A}^a(\mathbf{k})\cdot\mathbf{A}^a(\mathbf{k})\right], \quad (1)$$

in which the variational quasi-particle energy $\omega(k)$ is determined through the minimization condition

$$\frac{\delta}{\delta\omega}\langle\Phi|H|\Phi\rangle = 0. \quad (2)$$

For our purposes here, only the quark sector of the effective Hamiltonian is relevant. This part is given by

$$\begin{aligned} H^q_{eff} &= \int d\mathbf{x}\,\Psi^\dagger(\mathbf{x})\,(-i\boldsymbol{\alpha}\cdot\nabla + m_q\beta)\,\Psi(\mathbf{x}) \\ &+ g\int d\mathbf{x}\,\Psi^\dagger(\mathbf{x})\,\boldsymbol{\alpha}\cdot\mathbf{A}^a(\mathbf{x})T^a\,\Psi(\mathbf{x}) \\ &- \frac{1}{2}\int d\mathbf{x}\,d\mathbf{y}\,\rho^a(\mathbf{x})\,V_C(|\mathbf{x}-\mathbf{y}|)\,\rho^a(\mathbf{y}), \end{aligned} \quad (3)$$

where $\rho^a(\mathbf{x})$ are the color densities, given in terms of the quark field operators (the gluonic part is not written) as

$$\rho^a(\mathbf{x}) = \Psi^\dagger(\mathbf{x})T^a\Psi(\mathbf{x}). \quad (4)$$

In these, m_q is the current quark mass, $\boldsymbol{\alpha}^a$ and β are Dirac matrices, $T^a = \lambda^a/2$ where the λ^a are the generators of color SU(3), g is the quark-gluon coupling constant, and V_C is the Coulomb potential.

The numerical solution of Schwinger-Dyson equations in the mean field approximation of the Coulomb kernel leads to an expression that can be well fitted by the analytical formula [2]

$$k^2 V(k) = \begin{cases} 12.25\,(m_g/k)^{1.93} & \text{for } k \le m_g \\ 8.07\,\log^{-0.62}\left(k^2/m_g^2 + 0.82\right)\,\log^{-0.8}\left(k^2/m_g^2 + 1.41\right) & \text{for } k \ge m_g\,, \end{cases} \quad (5)$$

where m_g is a renormalization scale. The interaction between the quarks and the transverse gluons although calculable within the same approach, has not yet been computed. The effect of this interaction on DχSB is investigated here by means of a parametrized form, motivated by a lattice QCD calculation in Coulomb gauge. Its explicit form in Euclidean space is [4]

$$V_T(k_4, \mathbf{k}) = \frac{1}{\mathbf{k}_4^2 + \mathbf{k}^2 + M^4/\mathbf{k}^2}, \quad (6)$$

with $M \approx 768$ MeV. This form in Euclidean space is particularly suitable for finite-temperature calculations, where the time component of four vectors are imaginary.

The effective Hamiltonian is very similar to the phenomenological model employed previously to study mesons, baryons and chiral loops [5].

III. DYNAMICAL CHIRAL SYMMETRY BREAKING AT FINITE BARYON DENSITY

One way to investigate DχSB is by means of the self-consistent solution of the Schwinger-Dyson equation for the quark propagator in the Hartree-Fock (HF) approximation. In the following we present the equations for the general case of finite temperature and finite baryon chemical potential [6], although we shall report results for zero temperatures only. Writing the inverse of the quark propagator $S(k)$ in momentum space in terms of the quark self-energy $\Sigma(k)$ as $S^{-1}(k) = \gamma^\mu k_\mu - \Sigma(k)$, one has that in the HF approximation at finite temperature T and quark baryon chemical potential μ_B, the self-energy $\Sigma(k)$ is given by

$$\Sigma(k) = m_q + \frac{4}{3}\sum_n \int \frac{d^3q}{(2\pi)^3} D^{\mu\nu}(k-q)\,\gamma_\mu S(q) \gamma_\nu, \quad (7)$$

where the time component of the four vector q is given in terms of the Matsubara frequencies $\omega_n = (2n+1)\pi T$ as $q^0 = i\omega_n + \mu_B$. The gluon "propagator" is given in terms of the Coulomb potential $V_C(\mathbf{k})$ and the quark-transverse gluon interaction $V_T(\mathbf{k})$ as

$$D^{00}(\mathbf{k}) = V_C(\mathbf{k}), \qquad D^{ij}(\mathbf{k}) = (\delta^{ij} - \hat{k}^i \hat{k}^j) V_T(\mathbf{k}). \quad (8)$$

Substituting these into Eq. (7), one obtains that the general form of the quark self-energy $\Sigma(k)$ is given as $\Sigma(k) = a_k + b_k \gamma \cdot \mathbf{k} + \gamma_0 c_k$, where a_k, b_k, c_k are functions of $k = |\mathbf{k}|$ given by

$$a_k = m_q + \int \frac{d^3q}{(2\pi)^3} F_q \frac{a_q}{\omega_q}\Big[V_C(\mathbf{k}-\mathbf{q}) - 2V_T(\mathbf{k}-\mathbf{q})\Big], \quad (9)$$

$$b_k = -\frac{3}{8}\int \frac{d^3q}{(2\pi)^3} F_q \frac{1+b_q}{\omega_q}\frac{q}{k}\left[V_C(\mathbf{k}-\mathbf{q})\,\hat{\mathbf{k}}\cdot\hat{\mathbf{q}} + 2\,V_T(\mathbf{k}-\mathbf{q})\frac{(\mathbf{k}\cdot\mathbf{q} - k^2)(\mathbf{k}\cdot\mathbf{q} - q^2)}{kq\,(\mathbf{k}-\mathbf{q})^2}\right], \quad (10)$$

$$c_k = -\frac{3}{8}\int \frac{d^3q}{(2\pi)^3}\Big[V_C(\mathbf{k}-\mathbf{q}) + 2\,V_T(\mathbf{k}-\mathbf{q})\Big](n_q - \bar{n}_q), \quad (11)$$

where $\omega_q = [a_q^2 + (1+b_q)^2]^{1/2}$, $F_q = 1 - n_q - \bar{n}_q$, with n_q and $\bar{n}_q$ being the Fermi-Dirac distributions

$$n_q = \left\{\exp[\beta(\omega_q - \nu_q)] + 1\right\}^{-1}, \quad \bar{n}_q = \left\{\exp[\beta(\omega_q + \nu_q)] + 1\right\}^{-1}, \quad \nu_q = \mu_B - c_q. \quad (12)$$

IV. NUMERICAL RESULTS AND CONCLUSIONS

In this section we present results for $T = 0$ only. In this case, $n_k = \theta(k_F - |\mathbf{k}|)$, where k_F is the Fermi momentum, given in terms of the baryon density $\rho_B = N_f/\pi^2 k_F^3$, where N_f is the number of flavors. Sometimes it is more convenient to solve the equations for the chiral angle $\phi(k)$ or the gap function $\psi(k)$, related to the functions a_k, b_k and c_k by

$$\frac{a_k}{\omega_k} = \sin\phi(k) = \frac{2\,\psi(k)}{1+\psi^2(k)},$$

$$(1+b_k)\frac{k}{\omega_k} = \cos\phi(k) = \frac{1-\psi^2(k)}{1+\psi^2(k)}. \quad (13)$$

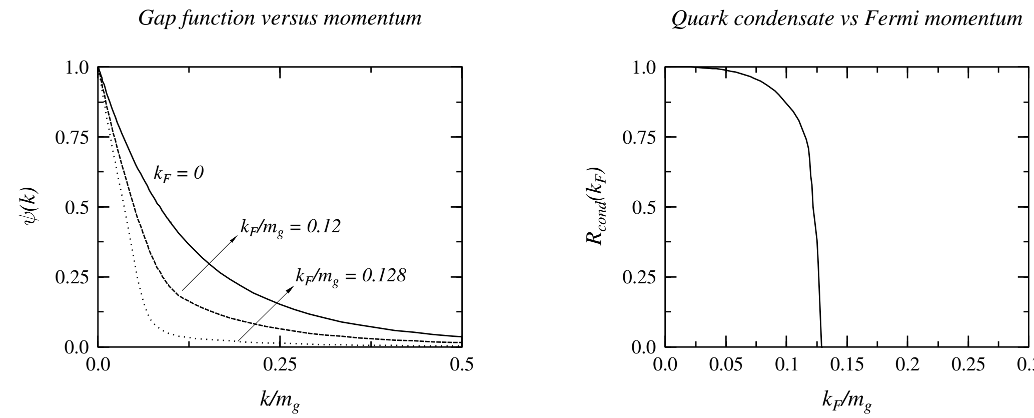

FIG. 1: Left panel: The gap function $\psi(k)$ as a function of momentum for different values of Fermi momenta . Right panel: The ratio of the in-medium to vacuum quark condensates as a function of the Fermi momentum. All dimensionful quantities are scaled by m_g.

In terms of the gap function $\psi(k)$, the quark condensate of a given flavor f is given by

$$\langle\bar{\Psi}_f\Psi_f\rangle = -12\int_{k_F}^{\infty}\frac{d^3k}{(2\pi)^3}\frac{\psi(k)}{1+\psi^2(k)}. \tag{14}$$

Initially, in order to see the effects of the Coulomb sector on DχSB we consider $V_T = 0$. In Fig. 1, on the left panel we present the the gap function $\psi(k)$ as a function of momentum for different values of Fermi momenta. For $k_F = 0$, corresponding to the vacuum, one obtains $\langle\bar{\Psi}_f\Psi_f\rangle^{1/3} = -0.183\,m_g$. Using the value $m_g = 600$ MeV, the value that fits the lattice heavy quark potential within the model [2], one sees that the condensate comes out too small.

On the right panel, we present the ratio of the in-medium to vacuum quark condensates as a function of the Fermi momentum

$$R_{cond}(k_F) = \frac{\langle\bar{\Psi}_f\Psi_f\rangle_{k_F}}{\langle\Psi_f\Psi_f\rangle_{k_F=0}}. \tag{15}$$

The chiral restoration is seen at the Figure to occur at $k_F \approx 0.13\,m_g$, precisely the value obtained in Ref. [6]. This number is clearly unacceptable, since it would indicate chiral restoration for densities much below the normal nuclear matter density, whose Fermi momentum is $k_F = 1.36$ fm^{-1}.

Next, adding the transverse piece V_T as given by Eq. (6) with $m_g = M$, the condensate increases substantially to $\langle\bar{\Psi}_f\Psi_f\rangle^{1/3} = -200$ MeV. We do not show the corresponding changes on $\psi(k)$ - the changes are in the direction of increasing the area under the curves of the left panel of Fig. 1. In the same way, the transverse potential improves substantially the value of k_F for the restoration of chiral symmetry, but the new value is still too low. This might indicate that the mean field treatment of the quark vacuum is not a good approximation for this model.

One should also keep in mind that at low densities, quark matter in the form of a Fermi sea of constituent quarks does not exist. At such low densities, one should take into account the fact that quarks appear confined into nucleons and mesons [7]. Therefore, the study of chiral symmetry at densities around normal nuclear matter density should take into account this fact. Work in this direction is in progress.

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