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Effect of a Magnetic Field on Tunneling Conductance in Normal Metal d-Wave Superconductor Interfaces

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Through the analytic solutions of the Bogoliubov de Gennes (BdG) equations the effect of a static and homogeneous magnetic field applied parallel to the interface of an NIS (N: Normal metal, S: superconductor and I: Insulator) junction on the differential conductance is calculated. For a d_{xy} - symmetry we obtain zero bias conductance peak that can be split by a magnetic field. The shift of the zero bias conductance peak depends on the spread (β) of the tunneling electrons in k space, on the magnitude of the applied field H and on the ratio between the Fermi energy of the superconductor and the normal region, E_{FS}/E_{FN}. Finally we estimate the minimum value of the magnetic field, H_{min}, that splits the zero bias conductance peak. In general H_{min} depends on β, E_{FS}/E_{FN}, the strength of the insulating barrier Z and on the temperature T.

I. INTRODUCTION

In high temperature superconductivity different experiments have been interpreted assuming a d-wave symmetry of the pair potential, see for example [1]. A feature of d symmetry is the zero-bias conductance peak (ZBCP) observed in tunneling conductance in NIS junctions [1]-[6]. The ZBCP appears when the angle between the a axis of the superconductor and the vector normal to the interface is nonzero; it is a maximum when this angle is $\pm\pi/4$, (110) orientation. This peak is due to the formation of the zero energy states (ZES) that are originated by the Andreev reflection at the interface; it undergoes a difference in phase of π due to the anisotropy of the pair potential [7]-[10]. Different experiments have shown that the ZBCP can be split due to application of a magnetic field [2]-[5]. From numerical solutions of the Eilenberger's equations, it has been shown that the effect of the magnetic field is to produce a shift of the ZES proportional to the applied magnetic field [11]-[12], other experiments show [13]-[14] that the effect of the magnetic field is to decrease of the heigth of the ZBCP. In this work we solve the BdG equations for this system and show that the splitting depends on Z, β, χ and T, The above mentioned experimental characteristics are explained in this work.

II. THEORY

The quasiparticles in a superconductor are described by the BdG equations. For steady states and anisotropic superconductors these equations are [15]

$$\int d\mathbf{r}_2 \hat{H}(\mathbf{r}_1,\mathbf{r}_2)\psi(\mathbf{r}_2) = E\psi(\mathbf{r}_1), \ \psi(\mathbf{r}_1) = \begin{pmatrix} u(\mathbf{r}_1) \\ v(\mathbf{r}_1) \end{pmatrix}$$

$$\hat{H}(\mathbf{r}_1,\mathbf{r}_2) = \begin{pmatrix} H_e(\mathbf{r}_1)\delta(\mathbf{r}_1-\mathbf{r}_2) & \tilde{\Delta}(\mathbf{r}_1,\mathbf{r}_2) \\ \tilde{\Delta}^*(\mathbf{r}_1,\mathbf{r}_2) & -H_e^*(\mathbf{r}_1)\delta(\mathbf{r}_1-\mathbf{r}_2) \end{pmatrix}, \quad (1)$$

where $H_e(\mathbf{r}_1) = (-i\hbar\nabla - e\mathbf{A}(\mathbf{r_1}))^2/2m + V(\mathbf{r}_1) - \mu$ is an electronic hamiltonian, with $\mathbf{A}(\mathbf{r_1})$ the vector potential associated with the magnetic fields present in the system, $V(\mathbf{r_1})$ the scalar potential and μ the chemical potential, $\tilde{\Delta}(\mathbf{r}_1,\mathbf{r}_2)$ is the pair potential, $u(\mathbf{r}_1)$and $v(\mathbf{r}_1)$ are the wave function for the electron- and hole-like components of a quasiparticle. The insulating

barrier of height V_0 and thickness d is located in $x > 0$, this barrier can be modeled by a delta function, $V(x) = U_0\delta(x)$, where $U_0 = V_0 d$. We concentrate on cuprate superconductor junctions. It is supposed that the quasiparticle moves on the CuO$_2$ plane with the a and b axes in the x-y plane, the interfaces are normal to the x-axis, see Fig. 1. The Fourier transform of the pair potential is modeled by

$$\Delta_{FT}(\mathbf{k},\mathbf{r}) = \int d\mathbf{R}\, e^{i\mathbf{k}\cdot\mathbf{R}}\tilde{\Delta}(\mathbf{r}_1,\mathbf{r}_2) \simeq \Delta(\hat{\mathbf{k}})\Theta(x), \quad (2)$$

where $\mathbf{R} = \mathbf{r}_1 - \mathbf{r}_2, \mathbf{r} = (\mathbf{r}_1+\mathbf{r}_2)/2$, $\Theta(x)$ the Heaviside function and $\Delta(\hat{\mathbf{k}})$ is the pair potential that undergoes a quasiparticle with momentum $\mathbf{k}$, for s-symmetry $\Delta(\hat{\mathbf{k}}) = \Delta_0$ and for d_{x-y}-symmetry $\Delta(\hat{\mathbf{k}}) = \Delta_0\sin 2\theta_s$,where θ_s is the quasiparticle angle in the superconductor region $\theta_s = \sin^{-1}(k_y/|\mathbf{k}|)$. The magnetic field H, is applied parallel to the z-axis, therefore the vector potential can be written as $\mathbf{A}(\mathbf{r}) = A_y(x)\hat{\mathbf{j}}$. We consider the situation of a high Tc superconductor where the coherence length of the pair potential ξ is much smaller than the penetration length λ of the magnetic field and approximate $A_y = -H\lambda$. As the potentials depends only on x, the solutions of the BdG equations can be written as $\psi(\mathbf{r}) = e^{ik_y y}\tilde{\psi}(\mathbf{r})$.
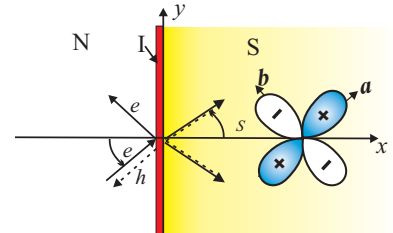

FIG. 1: NIS junction, The insulating barrier is located in $x = 0$ and is modeled by a delta function. For d$_{xy}$-symmetry, the pair potential is modeled as $\Delta(\theta_S) = \Delta_0\sin(2\theta_S)$. The solid and dashed lines represent the electron and the hole-like components of a quasiparticle, respectively.

Considering an incoming electron from the normal region,

the wave functions $\tilde{u}(x)$ and $\tilde{v}(x)$ are given taking into account the in the Andreev approximation [15] by

$$\begin{pmatrix} u(x) \\ v(x) \end{pmatrix} = \left(e^{i\tilde{k}_+ x} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + a e^{i\tilde{k}_- x} \begin{pmatrix} 0 \\ 1 \end{pmatrix} + b e^{-i\tilde{k}_+ x} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right) \Theta(-x)$$
$$+ \left(C e^{i\tilde{k}_+^+ x} \begin{pmatrix} u_0^+ \\ v_0^+ e^{-i\varphi_+} \end{pmatrix} + D e^{-i\tilde{k}_-^- x} \begin{pmatrix} v_0^- \\ u_0^- e^{-i\varphi_-} \end{pmatrix} \right) \Theta(x), \quad (3)$$

where

$$\tilde{k}_\pm = \sqrt{k_{FNx}^2 \pm 2m\tilde{E}/\hbar^2}, \ k_{FNx} = \sqrt{k_{FN}^2 - k_y^2},$$

$$\tilde{k}_\pm^\pm = \sqrt{k_{FSx}^2 \pm 2m\tilde{\Omega}_\pm/\hbar^2}, \ k_{FSx} = \sqrt{k_{FS}^2 - k_y^2},$$

$$\tilde{u}_0^\pm = \sqrt{\frac{1 + \tilde{\Omega}_\pm/\tilde{E}}{2}}, \ \tilde{v}_0^\pm = \sqrt{\frac{1 - \tilde{\Omega}_\pm/\tilde{E}}{2}}, \ \text{if} \ E > V_A$$

$$\tilde{u}_0^\pm = -\sqrt{\frac{1 - \tilde{\Omega}_\pm^*/\tilde{E}}{2}}, \ \tilde{v}_0^\pm = \sqrt{\frac{1 + \tilde{\Omega}_\pm^*/\tilde{E}}{2}}, \ \text{if} \ E < V_A,$$

$$\tilde{\Omega}_\pm = \sqrt{\tilde{E} - |\Delta_\pm|^2}, \tilde{E} = E - V_A, V_A = \frac{e\hbar k_y \lambda H}{m}. \quad (4)$$

The wavenumbers k_{FN} and k_{FS} are determined by the Fermi energy in the normal and superconducting regions respectively, $k_{FS}^2/k_{FN}^2 = E_{FS}/E_{FN} = \chi^2$. The quasiparticles with k_+^+ and k_-^- wavenumber move in the pair potential Δ_+ and Δ_- respectively

$$\Delta_\pm(\theta_s) = \Delta(\pm k_\pm^\pm \hat{\imath} + k_y \hat{\mathbf{j}}) \equiv \Delta_\pm e^{i\varphi_\pm}. \quad (5)$$

The effect of the magnetic field is an energy shift that depends on k_y and H. One finds a, b, C and D using the boundary conditions in $x = 0$. The electron-electron and electron-hole reflection coefficients are respectively, $R_e = |b|^2$, $R_h = |a|^2$.

III. DIFFERENTIAL CONDUCTANCE

Using the model developed by Blonder et al.[16] the differential conductance for an angle θ_S and for $T = 0$K is calculated from R_e and R_h coefficients as

$$\sigma_S(eV, \theta_S) = \sigma_N(\theta_S)(1 + Z^2)(1 + R_h(eV, \theta_S) - R_e(eV, \theta_S))$$
$$= \sigma_N \frac{1 + T_N \tilde{\Gamma}_+^2 + (T_N - 1)|\tilde{\Gamma}_+^2 \tilde{\Gamma}_-^2|}{|1 - (T_N - 1)\tilde{\Gamma}_+ \tilde{\Gamma}_- e^{-i(\varphi_+ - \varphi_-)}|^2}, \quad (6)$$

with

$$\tilde{\Gamma}_\pm = \frac{\tilde{v}_0^\pm}{\tilde{u}_0^\pm}, \chi = \frac{k_{FS}}{k_{FN}}, \ Z = \frac{mU_0}{\hbar^2 k_{FN}},$$

$$T_N = \frac{4\chi \cos\theta_N \cos\theta_S}{(\cos\theta_N + \chi\cos\theta_S)^2 + 4Z^2}, \quad (7)$$

σ_N is the differential conductance when $\Delta = 0$ (NIN junction), Z is the strength of the barrier and θ_N is the quasiparticle angle in the normal region and is determined by the momentum conservation condition in y direction

$$\sin\theta_N = \chi \sin\theta_S. \quad (8)$$

Firstly the case of $\chi = 1$ ($\theta_N = \theta_S \equiv \theta$) is analyzed. The relative total differential conductance is found by integration in the k-space as

$$\sigma_R(eV) = \frac{\langle \sigma_S(eV, \theta)\rangle}{\langle \sigma_N(\theta)\rangle}, \quad (9)$$

where

$$\langle...\rangle = \frac{\int_{-\pi/2}^{\pi/2} d\theta \cos\theta \, e^{-\beta\sin^2\theta} ...}{\int_{-\pi/2}^{\pi/2} d\theta \cos\theta \, e^{-\beta\sin^2\theta}}, \quad (10)$$

and β is related to the spread of tunneling electron in k- space, it is given by

$$\beta = \hbar d k_F^2 / \sqrt{2mV_0}. \quad (11)$$

It is important to note that the insulating barrier is characterized by the parameter Z and β. If in the equation (6) $H = 0$, our results agree with [7]-[8]. When $H \neq 0$ the differential conductance $\sigma_S(eV, \theta)$ has a shift given by $V_A(\theta)$. The Gaussian distribution, $e^{-\beta\sin^2\theta}$, diminishes 99% for a angle given by $\theta_c = \sqrt{\ln(100)/\beta}$. The maximum peak energy shift is determined by $V_A(\theta_{\max})$, where $\theta_{\max}$ is the maximum angle for which an electron tunnels the insulating barrier. This angle is $\pi/2 - \varepsilon$ ($\varepsilon \to 0$) if $0 < \beta \le 4.2$ or approximately θ_c if $\beta > 4.2$.

$$V_{A,\max} = e\hbar k_F \sin\theta_{\max} \lambda H / m. \quad (12)$$

The maximum value of V_A is $\max\{\Delta(\theta)\} = \Delta_0$, therefore the maximum magnetic field is

$$H_{\max} = H_c / \sin\theta_{\max}, \quad (13)$$

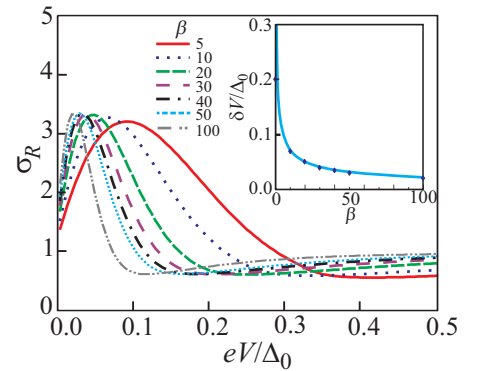with H_c the bulk critic magnetic field. The average shift is

FIG. 2: Relative differential conductance σ_R for different values of β with $Z = 3$, the inset shows how the maximum of σ_R changes with β.

$\delta V = \langle |V_A| \rangle = e\hbar\lambda H \langle |\sin\theta|\rangle / m$. For $\beta >> 1$, δV is approximately

$$\delta V = \langle |V_A|\rangle = \Delta_0 \frac{H}{H_c} \frac{1}{\sqrt{\pi\beta}}. \quad (14)$$
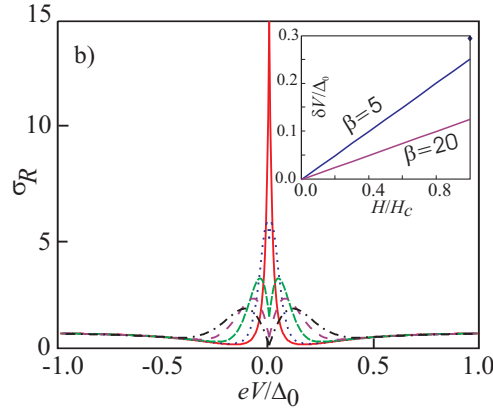
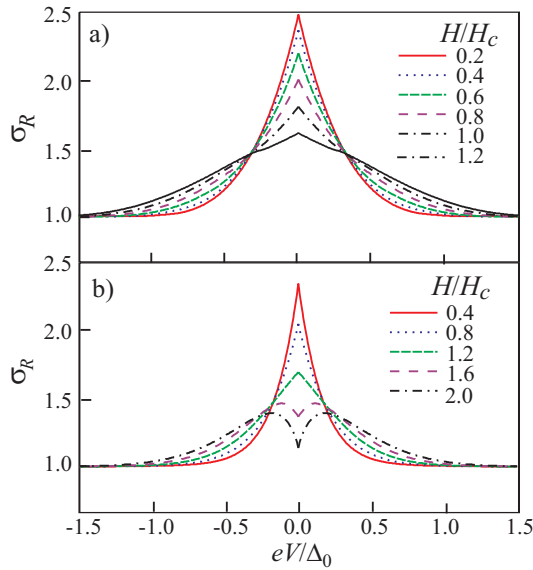FIG. 3: σ_R for different values of H with $Z = 3$, the inset shows how the peak of σ_R changes with H.

FIG. 4: σ_R for different values of H with $Z = 0.5$, (a) $\beta = 5$, (b)$\beta = 10$.

The average shift δV is strongly affected by β, this behavior is due to the fact that as β increases, the tunneling cone diminishes and therefore the shift energy decreases. Fig. 2 shows σ_R at different values of β, the splitting of ZBCP increases as β decreases. In the inset of the Fig. 2 it is contrated the average shift obtained in Eq. (14) with the numerically value. Fig. 3 shows σ_R for different values of H with $Z = 3$, ZBCP is split and the shift depends on the magnetic field and β, as shown the inset of Fig. 3. Fig. 4(a) shows σ_R for different values of H with $Z = 0.5$. Is observed that beyond some value the magnetic field ($H_{\min}$) the ZBCP is split. This value depends on Z and β as shown the figures 4(a) and 4(b). For an estimation of $H_{\min}$ one can compare δV with the width Γ_0 of the ZBCP. For an angle θ, the width is given by

$$\Gamma_0(\theta) = \frac{T_N |\Delta(\theta)|}{2(1-T_N)^{1/2}}. \tag{15}$$

Averaging over θ the width is approximately

$$\bar{\Gamma}_0 = \langle \Gamma_0(\theta)\rangle \simeq \frac{2\Delta_0}{Z(1+Z^2)^{1/2}\sqrt{\pi\beta}}. \tag{16}$$

As β increases, $\bar{\Gamma}$ decreases, this is because the tunneling cone decreases and therefore average gap diminishes. The splitting appears when $\delta V \sim \bar{\Gamma}/2$, from this relation we obtained $H_{\min}$

$$H_{\min} = \frac{H_c p}{Z(1+Z^2)^{1/2}}, \tag{17}$$

with $p \sim 1$. $H_{\min}$ is independent of β, this is due to the fact that as β increases, the width of ZBCP decreases but also δV decreases and these effects compensate each other. It is important to stand out that although in this case $H_{\min}$ is independent of β, $H_{\max}$ depends on β. If $H > H_{\min}$ the magnetic field induces a splitting of the ZBCP, but if $H_{\min} > H_{\max}$ it is not possible to observe this splitting, as is shown in figures 4(a) and 4(b) for $\beta = 5$ and $\beta = 20$ respectively, and $H_{\min} \simeq 1.4 H_c$ with $p = 0.8$. If $\beta = 5$, $H_{\max} = 1.2 H_c < H_{\min}$ and the splitting does not take place. If $\beta = 20$, $H_{max} \simeq 2.2 H_c$ and ZBCP is split. In both cases if $H < H_{\min}$ the effect of the magnetic field is to decrease the height of the ZBCP.

Now we study the effect of the Fermi energy difference between the normal and superconductor regions. This difference is quantified by the parameter $\chi = k_{FS}/k_{FN}$, the average angle in Eq. (10) is modified by

$$\langle ...\rangle = \frac{\int_{-\theta_m}^{\theta_m} d\theta_S e^{-\beta\chi^2 \sin^2 \theta_S}\cos\theta_S}{\int_{-\theta_m}^{\theta_m} d\theta_S e^{-\beta\chi^2 \sin^2 \theta_S}\cos\theta_S...}. \tag{18}$$

where θ_m is the maximum angle in the superconductor, for $\chi < 1$, $\theta_m = \pi/2$ and for $\chi > 1$, $\theta_m = \sin^{-1}(1/\chi)$. If $\beta\chi^2 >> 1$ from Eq´s. (10), (14), (15), δV and $\bar{\Gamma}_0$ are

$$\delta V = \Delta_0 \frac{H}{H_c}\frac{1}{\sqrt{\pi\beta\chi}} \tag{19}$$

$$\bar{\Gamma}_0 \simeq \frac{4\Delta_0}{\sqrt{\pi\beta\left(1+\chi^2+4Z^2\right)\left((1-\chi)^2+4Z^2\right)}}. \tag{20}$$

In this case $H_{\min}$ and $H_{\max}$ are

$$H_{\min} \simeq \frac{4\chi H_c p}{\sqrt{\pi\beta\left(1+\chi^2+4Z^2\right)\left((1-\chi)^2+4Z^2\right)}}, \tag{21}$$

$$H_{\max} \simeq \frac{\chi H_c\sqrt{\beta}}{2.15}. \tag{22}$$

In Figs. 5(a) and 5 (b) σ_R is plotted against V for $\chi = 0.5$ and $\chi = 1.5$ respectively. From Eq. (22) $H_{\min} = 0.8 H_c$ and $H_{\min} = 1.6 H_c$ in agreement with the numerical results shown in the figures.

Finally we study the differential conductance at finite temperature, in this case σ_R is calculated from

$$\sigma_R(eV) = \int_{-\infty}^{\infty} dE \sigma_R(E)\frac{d\left(f(E-eV)-f(E)\right)}{d(eV)}, \tag{23}$$

with $f(E)$ the Fermi-Dirac distribution function at temperature T: Fig. 6 shows σ_R for $T = 0.1 T_c$ and for different values

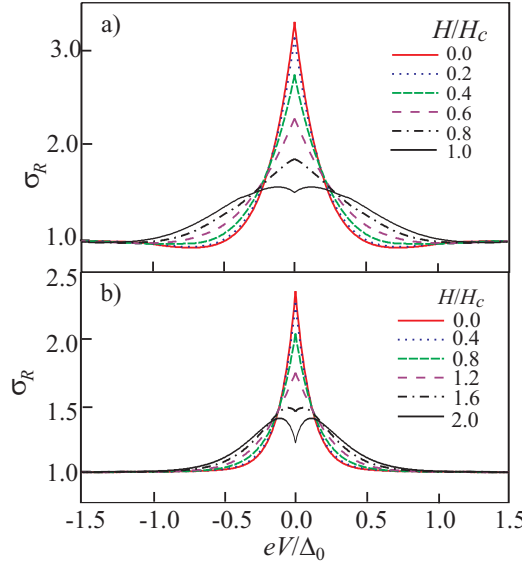FIG. 5: σ_R when $E_{FN} \neq E_{FS}$ for different values of H. a) $\chi = 0.5$, (b)$\chi = 1.5$. In both cases $Z = 0.5$ and $\beta = 20$.
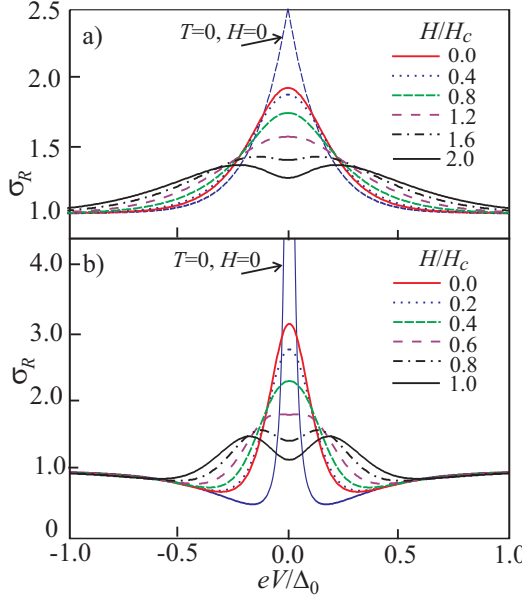

FIG. 6: σ_R for different values of H with $T = 0.1 T_c$, (a)$Z = 0.5$, (b)$Z = 3$. In both cases $\beta = 20$ and $\chi = 1$.

of Z. The main effect of T is to increase the width of the peak to $H = 0$ and therefore to increase $H_{\min}$. In order to analyze the width of the ZBCP we considerer first the case $k_B T << \bar{\Gamma}_0$, we find that width of the ZBCP is

$$\bar{\Gamma}(T) = \bar{\Gamma}_0 \left(1 + \frac{3}{2} \left(\frac{K_B T}{\bar{\Gamma}_0 / 2} \right)^2 + O \left(\frac{K_B T}{\bar{\Gamma}_0 / 2} \right)^4 \right), \quad (24)$$

and $H_{\min}$ is given by

$$H_{\min}(T) = H_{\min}(0) \left(1 + \frac{3}{2} \left(\frac{K_B T}{\bar{\Gamma}_0 / 2} \right)^2 \right). \quad (25)$$

Therefore the value $H_{\min}$ at finite temperature is larger than $H_{\min}$ for $T = 0$. The relative difference$\Delta H / H_{\min}(0) = (H_{\min}(T) - H_{\min}(0)) / H_{\min}(0)$ is proportional to $K_B^2 T^2 / \bar{\Gamma}_0^2$, and therefore in this case the relative increase of $H_{\min}$ is always less than one, as shown in Fig, 6(a) where $H_{\min} \simeq 1.6 H_c$; from Eq's. (17) and (22) $H_{\min}(0) = 1.4 H_c$ and $H_{\min}(T) = 1.57 H_c$ as shown in the figures. For $k_B T >> \bar{\Gamma}_0$

$$\bar{\Gamma}(T) = 1.76 k_B T \left(1 + O(\bar{\Gamma}_0 / (k_B T)^2) \right),$$

$$H_{\min} \simeq 1.76 k_B T \sqrt{\pi \beta} p H_c / (2 \Delta_0). \quad (26)$$

In this case the width is determined main by the temperature $\bar{\Gamma} \sim K_B T$, $H_{\min}(T)$ is proportional to T and $\beta^{1/2}$, and can be larger than $H_{\min}(0)$, in contrast to the previous case , see Fig. 6(b).

IV. CONCLUSIONS

Starting from the solutions of the Bogoliubov of Gennes equations in an NIS junctions we have determined the effect of a magnetic field on the differential conductance. For d_{xy} symmetries the differential conductance presents a ZBCP. The magnetic field induces a splitting of the ZBCP when $H > H_{\min}$, where $H_{\min}$ depends on Z, χ,and T, in this case the shift of the peak is proportional to $H / (\beta^{1/2} \chi)$. When $H < H_{\min}$ the effect of the magnetic field is to decrease of the height of the ZBCP. If $H_{\min} > H_{\max}$, the splitting does not take place when the magnetic field is applied; the value of $H_{\max}$ depends on χ,and β. This results can be used for the interpretation of the tunneling characteristic in high Tc superconducting junctions in applied magnetic fields.

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