Brazilian Journal of Physics
ISSN: 0103-9733
luizno.bjp@gmail.com
Sociedade Brasileira de Física
Brasil

Grating, D. R.; Peres, O. L. G.
Effects of Solar Neutrinos Scale on Atmospheric Neutrino Flux
Brazilian Journal of Physics, vol. 37, núm. 2B, june, 2007, pp. 626-629
Sociedade Brasileira de Física
Sâo Paulo, Brasil

Effects of Solar Neutrinos Scale on Atmospheric Neutrino Flux

D. R. Grating and O. L. G. Peres
Universidade Estadual de Campinas, Instituto de Física Gleb Wataghin,
Departamento de Raios Cósmicos, C.P. 6165, Campinas, 13083-970, SP, Brazil

Received on 13 December, 2006

In this work we try to understand the phenomena of neutrino oscillations, and use this to obtain a more precise description of the atmospheric neutrino data. The two neutrino oscillation mechanism solves the problem of the up-down muon neutrino asymmetry successfully. Our main motivation is to describe the excess of events of electron-neutrino type found in the SuperKamiokande results at low energies when compared with the predictions of the two-generation neutrino oscillation. To do this we generalize the oscillation model from two to three neutrino flavors, opening the possibility of oscillation between electron neutrino type and the others. Then we obtain a semi-analytic solution of the three flavors problem using the neutrino phenomenological limits on oscillation parameters, squared masses differences and mixing angles. For this we must take into account matter effects on the electronic neutrino when it cross the Earth and has its oscillation pattern changed.

Keywords: Neutrino oscillations; Atmospheric neutrinos

I. INTRODUCTION

Atmospheric neutrinos have their origin in the cosmic ray collisions with Earth's atmosphere. In this process are formed initially hadrons whose decay mainly into pions and kaons that have neutrinos as product of it's decay. As kaons are produced in more energetic and rare showers, the principal decay mode for neutrino production is $\pi \to \mu + \nu_\mu$ followed by $\mu \to e + \nu_e + \bar{\nu}_\mu$, that implies that the ratio between muon-type and electron-type neutrino fluxes is equal two. As shown in Fig .(1), the atmospheric neutrino problem is related to the fact the SuperKamiokande (SK)[1] results of measurement of muon events induced by atmospheric neutrinos, is smaller than the theoretical previsions [2, 3]. This deficit in muon-like neutrino events depends on the neutrino zenith angle and may be even 50% when $cos\theta_z \to -1$. The fact that this deficit depends on the zenith angle, θ_z, excludes the possibility to describe by a renormalization in the SK data. To explain this phenomena is used the flavor oscillation model in two generations of neutrinos induced by mass, whose describes the experimental data.

II. NEUTRINO FLAVOR OSCILLATIONS

A. Vacuum oscillation in two generations

This is the most simple case of the flavor oscillation model [4], which assumes that the flavor neutrino eigenstates, $|\nu_\alpha\rangle$, are not the eigenstates of Hamiltonian, but a linear combination of these, which are the mass eigenstates, $|\nu_i\rangle$. For example,the mixing between the masses eigenstates $|\nu_2\rangle$, $|\nu_3\rangle$, to form the flavor eigenstates $|\nu_\mu\rangle$ and $|\nu_\tau\rangle$ is relevant for the atmospheric neutrino case, and can be written as

$$|\nu_\alpha\rangle = \sum_{i=2}^{3} U^*_{\alpha i}|\nu_i\rangle \ , \tag{1}$$

where U is the neutrino mixing matrix [5, 6], for two neutrino generations. Explicitly, the Eq .(1) may be written as a function of the atmospheric mixing angle $\theta_{atm} = \theta_{23}$ as,

$$\begin{pmatrix} \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} cos\,\theta_{23} & sen\,\theta_{23} \\ -sen\,\theta_{23} & cos\,\theta_{23} \end{pmatrix} \begin{pmatrix} \nu_2 \\ \nu_3 \end{pmatrix}. \tag{2}$$

So, the temporal evolution of flavor neutrinos must be given by the evolution of the mass eigenstates,

$$i\frac{d}{dt}\begin{pmatrix} \nu_\mu \\ \nu_\tau \end{pmatrix} = UHU^\dagger \begin{pmatrix} \nu_\mu \\ \nu_\tau \end{pmatrix}, \tag{3}$$

where $H = diag(E_2, E_3)$, and E_i is the eigenvalue of the $|\nu_i\rangle$. As a direct consequence of this, the probability of for a given a initial flavor neutrino eigenstate $|\nu_\mu\rangle$ to change to the $|\nu_\tau\rangle$ neutrino flavor state, it is not zero but:

$$P(\nu_\mu \to \nu_\tau) = sen^2\,(2\theta_{23}) sen^2\left(\frac{\Delta m^2_{32} L}{2E}\right) , \tag{4}$$

where $\Delta m^2_{atm} = \Delta m^2_{32} \equiv m_3^2 - m_2^2$ is atmospheric squared mass difference. We also have the solar squared mas difference defined as $\Delta m^2_\odot = \Delta m^2_{21} \equiv m_2^2 - m_1^2$.

This is not zero if the mass eigenvalues are different of zero and not all degenerated, because if $\Delta m^2_{32} = 0$ there is no oscillation. Eq .(4) generates the solid lines in Fig .(1)that are in agreement with the muon-like neutrino data from SK.

B. Why to generalize to three generations?

As pointed by [7], a inspection of the first column in Fig. (1) with respect to electron-like neutrino events, a excess of events in the SK data at low energies ($P < 400$ GeV) when compared to predictions with the two neutrino oscillations taken account [9]. So we generalize the oscillation model to three neutrino generations and allow the possibility of oscillation between the electron neutrino and the others.

Because of the asymmetry in the leptonic content in usual matter, only the electronic neutrino scatters with its charged

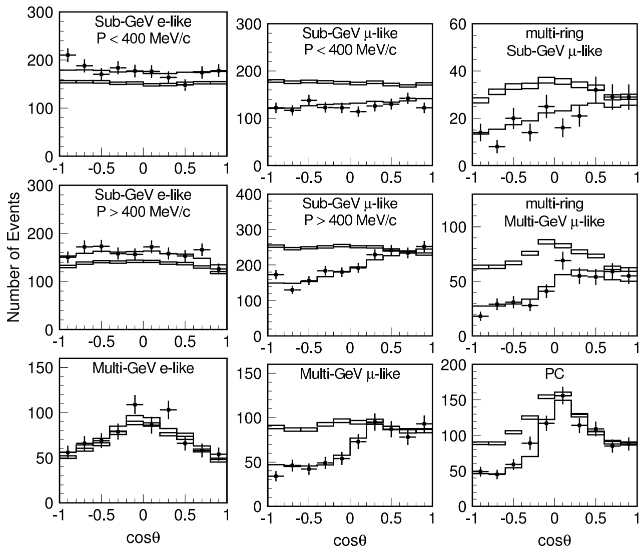

FIG. 1: SuperKamiokande results [8] for the zenith dependence of the atmospheric data (points) compared with the theoretical simulations (little boxes) from Honda *et. al* [2]. The prediction with oscillation is denoted by a solid line. From top to bottom, respectively three charged lepton momentum bins, as can be seen in the plots. From left to right, we have the different samples: electron-like, muon-like and muon-like multi-ring events

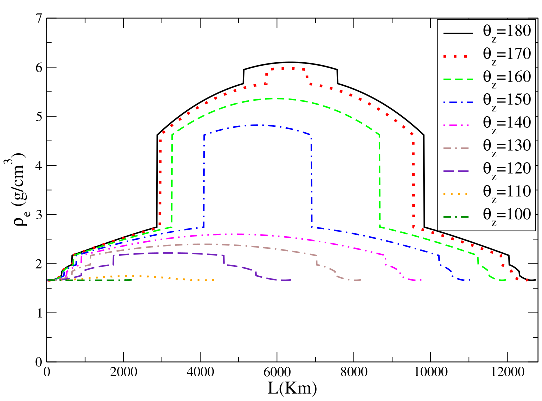

FIG. 2: In the picture are shown the density filled by ν_e as a function of distance L. In the legend, from the top to bottom we have $\theta_z = 180^o \rightarrow 100^o$.

lepton to interact by weak charged current (CC), in which the mediator boson is massive and electrically charged, $W^{\pm}$. This interaction gives rise to a effective potential due to electrons in a medium. For the Earth's interior a simple calculation [10] gives

$$V_e = \sqrt{2} G_F N_e \sim 3.10^{-14} \left(\frac{\rho}{10g/cm^3} \right) \quad \text{eV} \,, \tag{5}$$

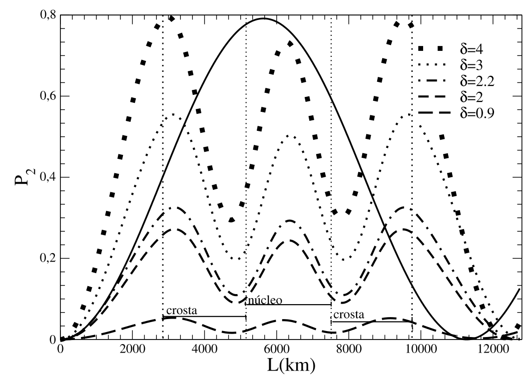

FIG. 3: Oscillation probabilities as a function of the distance L for different values of δ parameter in units of (10^{-11}) eV.

where G_F is the Fermi constant, N_e is the number of electrons in the medium, and ρ_e is the electronic number density [11, 12]. In Fig. (2) we show ρ_e as a function of the distance traveled in Earth, $L = -2R\cos\theta_z$, where R is the Earth's radius, for different values of the zenith angle θ_z.

As shown in ref. [10], for anti-neutrinos the signal of V_{CC} is opposite for the signal to neutrinos. The changes on oscillations patterns depend on the local electronic density in Earth's interior. We called it as medium effect.

C. How to generalize to three neutrino generations ?

In the presence of a medium, the effective Hamiltonian may be written as,

$$H = \left(\frac{UM^2U^\dagger}{2E} + V \right), \tag{6}$$

where the mass matrix M^2 is $M^2 = diag\left(0, \Delta m^2_{21}, \Delta m^2_{31}\right)$, and $V = (V_e, 0, 0)$. So in three neutrino generations and taking into account the medium effects, the evolutions of $|\nu_\alpha\rangle$ is given by,

$$i\frac{d}{dt}|\nu_\alpha\rangle = \left(\frac{U_{23}U_{13}U_{12}M^2U^\dagger_{12}U^\dagger_{13}U^\dagger_{23}}{2E} + V \right) |\nu_\alpha\rangle, \tag{7}$$

where $|\nu_\alpha\rangle$ is a combination of three mass eigenstates,

$$|\nu_\alpha\rangle = \sum_{i=1}^{3} U^*_{\alpha i} |\nu_i\rangle \,. \tag{8}$$

The neutrino phenomenology gives to us the values of the oscillation parameters, squared mass differences and mixing angles. For squared mass differences we have

$\Delta m^2_{21} = \Delta m^2_{\odot} = 8\ 10^{-5}$ eV ^{2} [1, 13], and $\Delta m^2_{32} = \Delta m^2_{ATM} = 2.5\ 10^{-5}$ eV ^{2} [9, 14] so in our calculations we use the approximation $\Delta m^2_{\odot} << \Delta m^2_{atm}$. For mixing angles we have $tan^2(\theta_{\odot}) = 0.4$, $sen^2(\theta_{atm}) = 1/2$, and $sen^2(\theta_{13}) < 0.1$. Therefore θ_{13} is a small angle and the others two are large.

Now we apply two rotations in the eigenstate base to transform the 3×3 system in other two, one 2×2 and a second 1×1. The first rotation is $|\nu_\alpha\rangle = U_{23}U_{13}|\nu'_\alpha\rangle$. In the basis $|\nu'_\alpha\rangle$,

$$H' = \begin{pmatrix} s^2_{12}\delta + V_e c^2_{13} & s^2_{12}c^2_{12}\delta & V_e s_{13}c_{13} \\ s^2_{12}c^2_{12}\delta & s^2_{12}\delta & 0 \\ V_e s_{13}c_{13} & 0 & \Delta + V_e s^2_{13} \end{pmatrix}. \quad (9)$$

Here we have defined:

$$\frac{\Delta m^2_{21}}{2E} = \delta, \qquad \frac{\Delta m^2_{31}}{2E} = \Delta\,, \quad (10)$$

where δ and Δ are the inverse of the oscillation lengths of solar and atmospheric neutrinos.

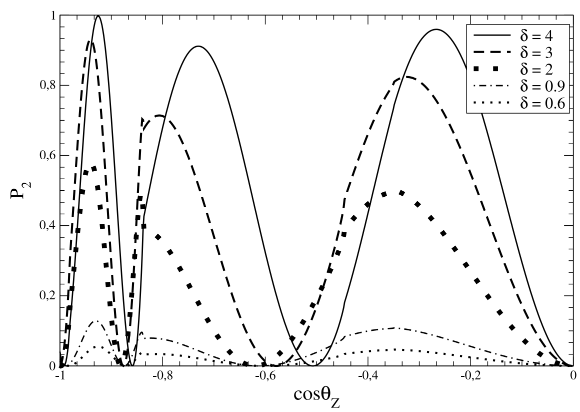

FIG. 4: Oscillation probabilities as a function of $cos\theta_Z$ for different values of δ parameter in units of (10^{-11}) eV.

The second rotation is $|\nu'_\alpha\rangle = U'_{13}|\nu''_\alpha\rangle$, where the the rotation angle θ'_{13} is such that,

$$s'_{13} = \frac{s_{13}c_{13}2EV_e}{2EV_e s^2_{13} + \Delta m^2_{31}} \sim \frac{2\theta_{13}EV_e}{\Delta m^2_{31} + 2EV_e\theta^2_{13}}\,, \quad (11)$$

then θ' is a small angle also.

In the propagation basis the Hamiltonian have the approximated form, because the hierarchies of solar and atmospheric scale and smallness of θ_{13}.

$$H'' \sim \begin{pmatrix} s^2_{12}\delta + V_e c^2_{13} & s_{12}c_{12}\delta & \sim 0 \\ s_{12}c_{12}\delta & c^2_{12}\delta & \sim 0 \\ 0 & 0 & \Delta + V_e s^2_{13} \end{pmatrix}, \quad (12)$$

that implies in the decoupling of the tau neutrino state in the rotate basis,ν''_τ. So by these two rotations we may write the 2×2 sub-system as,

$$i\frac{d}{dt}\begin{pmatrix} \nu_{e''} \\ \nu_{\mu''} \end{pmatrix} = \begin{pmatrix} s^2_{12}\delta + V_e c^2_{13} & s_{12}c_{12}\delta \\ s_{12}c_{12}\delta & c^2_{12}\delta \end{pmatrix}\begin{pmatrix} \nu_{e''} \\ \nu_{\mu''} \end{pmatrix} \quad (13)$$

and the sub-system 1×1

$$i\frac{d}{dt}\nu''_\tau = (s'_{13}s_{13}c_{13}V_e + \Delta + V_e s^2_{13})\nu''_\tau \quad (14)$$

Physically speaking, the two oscillation scales represent different orders of magnitude, i.e. the atmospheric one oscillates faster than the solar one. When the oscillation due atmospheric scale is influencing the system the solar scale still not manifest. When the solar scale turns relevant the atmospheric mechanism had oscillated so many times that the best we can observe is a average value.

In the propagation basis the S'' matrix of amplitudes of probabilities have the form,

$$\bar{S}'' = \begin{pmatrix} A''_{ee}e^{i\phi_1} & A''_{\mu e}e^{i\phi_2} & 0 \\ A''_{\mu e}e^{i\phi_2} & A''_{\mu\mu}e^{i\phi_4} & 0 \\ 0 & 0 & A''_{\tau\tau}e^{i\phi_3} \end{pmatrix}. \quad (15)$$

where we obtain the coefficients $A''_{\alpha\beta}$ by numerical evaluation of Eq .(13), and the probabilities are given as $P''_{\alpha\to\beta} = |A''_{\alpha\beta}|^2$, and ϕ_i are oscillation phases, i=1,2,3.

III. PRELIMINARY RESULTS

We define $P_2 \equiv P_{\nu_e\to\nu_\mu} = |A''_{e\mu}|^2$ as the probability of flavor transition in the propagation basis. The Fig. (3) show P_2 as a function of the distance L traveled in the Earth for $cos(\theta_z) = -1.0$, that means, for neutrinos that cross the entire Earth. We show the changes in the oscillation pattern when neutrinos cross regions with have different densities. In the Fig. (4) we show P_2 for the same values of δ but as a function of the zenith angle ($cos\theta_z$). For values of $cos\theta_z < -0.85$ neutrino crosses the Earth's core and in this region all the curves are in phase, that means that the oscillation length in this region is not dependent of the δ parameter.

A. Results in flavor basis

In the flavor basis the S matrix has the form,

$$S = U_{23}U_{13}U'_{13+13'}S''U^\dagger_{13+13'}U^\dagger_{23}\,, \quad (16)$$

and we will define the probabilities as $P(\nu_\alpha \to \nu_\beta) = P_{\alpha\to\beta}$, and the probabilities may be written as,

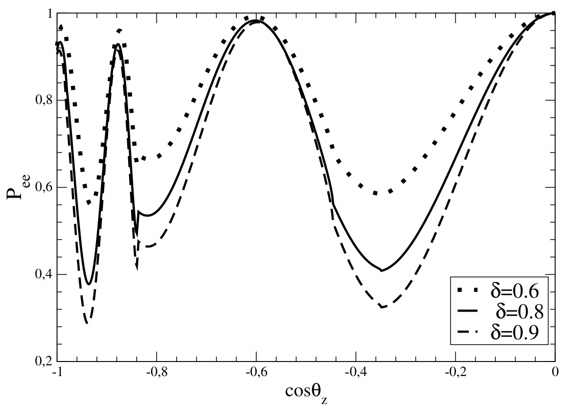

FIG. 5: Probability of electronic neutrino survival as a function of $cos\theta_z$ for different values of δ in units of (10^{-11}) eV.

$$\begin{aligned}
P_{e\to e} &= 1 - P_2\,(c''_{13})^4 - 2(c''_{13})^2(s''_{13})^2\left(1-\sqrt{1-P_2}\,cos\,\phi_{23}\right)\\
P_{e\to\mu} &= (c''_{13})^2(s''_{13})^2 s_{23}^2\left(1-2\sqrt{1-P_2}\,cos\,\phi_{23}+(1-P_2)\right)\\
&+ 2(c''_{13})^2 s''_{13}s_{23}c_{23}\left(\sqrt{P_2}\,cos\,\phi_{13}-\sqrt{(1-P_2)P_2}\,cos\phi_{12}\right)\\
&+ (c''_{13})^2 c_{23}^2\,P_2\\
P_{e\to\tau} &= (c''_{13})^2(s''_{13})^2 c_{23}^2\left(1-2\sqrt{1-P_2}\,cos\,\phi_{23}+(1-P_2)\right)\\
&+ 2(c''_{13})^2 s''_{13}s_{23}c_{23}\left(\sqrt{(1-P_2)P_2}\,cos\,\phi_{12}-\sqrt{P_2}\,cos\,\phi_{13}\right)\\
&+ (c''_{13})^2 s_{23}^2\,P_2, \qquad (17)
\end{aligned}$$

where $\phi_{ij} = \phi_i - \phi_j$.

We plot in Fig. (5) the probability of survival of electronic neutrino given by Eq .(17) as a function of $cos\theta_z$. Again we note that for $cos\theta_z < -0.85$ the phase of oscillation patterns is enhanced due to interaction with the electrons in the medium.

IV. CONCLUSIONS

This paper contains the our primary results that are summarized below.

Upon generalizing of the neutrino flavor oscillation model from two to three neutrino flavors, we verify also oscillations between $\nu_e \to \nu_\mu$ which depends on Δm_{21}^2 and θ_{12} that are parametrized by by solar neutrino scale. This oscillations also dependents of θ_{13}, and have their pattern changed due to interaction of ν_e with the electrons in the medium, when the ν_e realizes transitions between zones with densities in the Earth's interior.

We conclude remembering that this medium effects plays an important rule in evolution of ν_e in the Earth's interior, as shown in Figs .(3, 4, 5). Since for low energies initially there are approximately two times more muon neutrinos that electron neutrinos, the medium effects would enhance the oscillation and causes the excess of electron neutrino in Sub-GeV region for $cos\theta_z \to -1$ that was referred above in this work. From Eqs .(17) we see a dependence of the medium effects with respect to θ_{23} and θ_{13}, such that this medium effects would be used to determinate the octant of θ_{23} and impose limits on θ_{13}, as pointed by [7].

[1] The SuperKamiokande Collaboration, S. Fukuda *et al.*, Phys. Rev. Lett. **81**, 1158 (1999); *ibid*, **85**, 3999 (2000).
[2] M. Honda, T. Kajita, arXiv:astro-ph/0404457.
[3] T. K. Gaisser and M. Honda, Ann. Rev. Nucl. Part. Sci. **52**, 153 (2002)
[4] B. Pontecorvo, J. Exptl. Theoret. Phys. **33**, 549 (1957); *ibid.* **34**, 247 (1958).
[5] Z. Maki, M. Nakagawa, and S. Sakata, Prog. Theor. Phys. **28**, 870 (1962).
[6] Particle Data Group, S. Eidelman *et al.*, Phys. Lett. B **592**, 1 (2004).
[7] O. L. G. Peres, A. Yu. Smirnov, Nucl. Phys. B **680**, 479 (2004), and references cited there.
[8] SuperKamiokande Collaboration, Y. Ashie *et al.*, Phys. Rev. D **71**, 112005, (2005).
[9] The Super-Kamiokande Collaboration, S.Fukuda *et al.*, arXiv:hep-ex060411.
[10] C. W. Kim, A. Pevsner. *Neutrinos in Physics and Astrophysics,* Hardwood, (1995).
[11] A.M. Dziewonski, D.L. Anderson, Physics of the Earth and Planetary Interiors, **25**, 297 (1981).
[12] E. Lisi, D. Montanino, Phys. Rev. D **56**, 1792 (1997).
[13] SNO Collaboration, B. Aharmim *et al.*, Phys. Rev. C **72**, 055502 (2005).
[14] MINOS Collaboration, D. G. Michael *et. al* , hep-ex/0607088 (2006).