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Nonplanar Positron-Acoustic Shock Waves in Astrophysical Plasmas

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Abstract The problem of nonlinear positron-acoustic shock waves (PASWs) in an unmagnetized, collisionless, dense plasma system (containing non-relativistic cold positrons, both non-relativistic and ultra-relativistic degenerate electron and hot positron fluids and positively charged static ions) is addressed. The combined effects of the non-relativistic and ultra-relativistic degenerate electron and hot positron fluids are organized in the study of the PASWs. By using the reductive perturbation method, modified Burgers equation is derived and numerically analyzed. For the non-relativistic limits in like manner for the ultra-relativistic limits, it is seen that the shock wave characteristics are modified significantly. The effects of kinematic viscosity, degenerate pressure, nonplanar geometries, and plasma particle number densities on the properties of PASWs are numerically analyzed. As time goes, PASWs propagating in cylindrical and spherical geometry are deformed. The fundamental features and the underlying physics of PASWs, which are concerned to some astrophysical compact objects (viz. neutron stars, white dwarfs, etc.), are concisely mentioned.

Keywords Positron-acoustic waves · Shock waves · Nonplanar geometry · Degenerate pressure · Relativistic effect · Compact objects

1 Introduction

The perusal of nonlinear wave phenomena in unmagnetized electron-positron-ion (EPI) plasma has drawn a great appeal in the last two decades among the researchers [1–10]. In the present day, the nonlinear phenomena (viz. solitons, shocks, double layers, etc.) correlated with positron-acoustic (PA) waves have received a considerable attention by a number of authors [2–4]. In contrary to regular two-component electron-ion (EI) plasma, it was found that the nonlinear waves in plasmas having a further positron component behave differently. Fundamentally, PA waves are nothing but normal IA waves in which the inertia is provided by the cold positron mass and restoring force appears from the degenerate pressure of hot positrons and electrons. Tribeche et al. [3] addressed the nonlinear PA solitary waves involving the dynamics of mobile cold positrons.

Recently, a great attention has been attracted in comprehending the principal properties of matter under extreme conditions [11–14], that are found in several astrophysical compact objects. The astrophysical compact objects support themselves by degenerate electron-positron pressure. This pressure has a pivotal role in the study of the electrostatic perturbation in matter existing in extreme conditions [15–20]. For compact objects, the degenerate fermion number density is so high (in white dwarfs, it could be of the order of 10^{30}cm^{-3}, even more) [21–24]. The equation of state for degenerate fermions was mathematically clarified by Chandrasekhar [19] for two limits, called non-relativistic and ultra-relativistic limits. In case of such astrophysical compact objects, the degenerate pressure for ion fluid could be given by the following equation

$$P_i = K_i n_i^{\alpha}, \tag{1}$$

where

$$\alpha = \frac{5}{3}; \quad K_i = \frac{3}{5}\left(\frac{\pi}{3}\right)^{\frac{1}{3}} \frac{\pi \hbar^2}{m} \simeq \frac{3}{5}\Lambda_c \hbar c, \tag{2}$$

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for the non-relativistic limit (where $\Lambda_c = \pi\hbar/mc = 1.2 \times 10^{-10}$ cm, and $\hbar$ is the Planck constant divided by 2π). While for the electron fluid,

$$P_e = K_e n_e^{\gamma}, \tag{3}$$

and while for the positron fluid

$$P_p = K_p n_p^{\gamma}, \tag{4}$$

where for non-relativistic limit [11, 12, 15, 17, 19]

$$\gamma = \alpha = \frac{5}{3}; K_e = K_p \tag{5}$$

and for the ultra-relativistic limit [11, 12, 15, 17, 19]

$$\gamma = \frac{4}{3}; \quad K_e = K_p = \frac{3}{4}\left(\frac{\pi^2}{9}\right)^{\frac{1}{3}} \hbar c \simeq \frac{3}{4}\hbar c, \tag{6}$$

In present days, there are a number of investigations that have been made of the nonlinear propagation of electrostatic waves in degenerate quantum plasma [8, 25–28]. Eliasson and Shukla [29] thought of a super-dense quantum plasma composed of relativistic degenerate electrons and fully ionized ions and studied the formation of electrostatic shock structures. Zeba et al. [25] considered a collisionless EPI quantum plasma with ultra-relativistic degenerate electrons and positrons and studied the existence regions for ion solitary pulses. A quantum plasma with non-relativistic and ultra-relativistic degenerate electron fluids and strongly coupled degenerate ion fluids was considered by Mamun and Shukla [26] and they rigorously investigated the salient features of shock waves. The electrostatic solitons in unmagnetized hot EPI plasmas were examined by Asif et al. [30] and they observed that both the lower and upper branches of the Langmuir waves can propagate in such plasmas. Masood et al. [31] found that the strength and the steepness of the quantum ion acoustic shock wave increase with decreasing stretched time coordinate by assuming an unmagnetized quantum plasma consisting of electrons, positrons, and ions employing the quantum hydrodynamic (QHD) model. Roy et al. [13] considered an EPI plasma and examined the principal features of solitary waves and double layers. Nejoh [4] carefully examined the nonlinear propagation of PA waves in an e-p plasma with an electron beam and examined closely that the maximum amplitude of the wave decreases as the positron temperature increases and the region of PA waves spreads as the positron temperature increases. Zobaer et al. considered [32–34] a relativistic degenerate plasma containing non-relativistic degenerate cold ion and both non-relativistic and ultra-relativistic degenerate electron fluids and analyzed the fundamental features of electrostatic shock structures. By considering four-component plasma model with two-temperature positrons, electrons, and ions, Tribeche [3] rigorously studied the small-amplitude PA double layers.

Recently, Hossen et al. [7] considered ion acoustic waves in a relativistic degenerate plasma and investigated the 1D planar shock structures. But in our present work, we have taken different wave modes, i.e., PAWs and geometries. If anyone gives a close look in our present model and parametric investigation section then finds a significant outcomes.

Most of these works are limited to 1D (planar) geometry and may not be a realistic situation in space and laboratory devices. There are numerous cases of practical importance where planar geometry does not work and one would have to consider a nonplanar geometry. Hereby, in our present work, we attempt to study the basic features of nonplanar PA shock waves by deriving the modified Burgers equation in a degenerate plasma system containing non-relativistic cold positrons, both non-relativistic and ultra-relativistic hot positrons and electrons and positively charged static ions. Up to the best of our knowledge, no theoretical investigations have been made to study the matter under extreme conditions by considering PA waves in both non-relativistic and ultra-relativistic limits. Hence, it is worthy to begun a first study for the PA waves where degenerate plasma pressure, nonplanar geometry, and relativistic effects play a pivotal role.

2 Governing Equations

An unmagnetized collisionless four-component degenerate plasma system containing non-relativistic cold positrons, both non-relativistic and ultra-relativistic degenerate electron and hot positron fluids, and positively charged static ions have been considered. Therefore, we have found $n_{\rm pco} + n_{\rm pho} + n_{i0} = n_{e0}$ where $n_{\rm pco}$, $n_{\rm pho}$, n_{e0}, and n_{i0} are the cold positron, hot positron, electron, and ion number densities, respectively, at equilibrium. The dynamics of nonlinear PASWs in our plasma system is governed by the following momentum equations,

$$\frac{\partial \phi}{\partial r} - \frac{K_2}{n_{ph}} \frac{\partial {n_{ph}}^{\gamma}}{\partial r} = 0, \tag{7}$$

$$\frac{\partial \phi}{\partial r} - \frac{K_3}{n_e} \frac{\partial {n_e}^{\gamma}}{\partial r} = 0, \tag{8}$$

and the generalized viscoelastic cold positron equations composed of the positron continuity and cold positron momentum equation are given by

$$\frac{\partial n_{pc}}{\partial t} + \frac{1}{r^{\nu}} \frac{\partial}{\partial r}(r^{\nu} n_{pc} u_{pc}) = 0, \tag{9}$$

$$\frac{\partial u_{pc}}{\partial t} + u_{pc}\frac{\partial u_{pc}}{\partial r} + \frac{\partial \phi}{\partial r} + \frac{K_1}{n_{pc}} \frac{\partial {n_{pc}}^{\alpha}}{\partial r} - \eta \frac{1}{r^{\nu}} \frac{\partial}{\partial r}\left(r^{\nu} \frac{\partial u_{pc}}{\partial r}\right) = 0, \tag{10}$$

The equation that is closed by Poisson's equation,

$$\frac{1}{r^\nu}\frac{\partial}{\partial r}(r^\nu \frac{\partial \phi}{\partial r}) = \beta n_e - \sigma n_{ph} - n_{pc} - (\beta - \sigma - 1), \quad (11)$$

where n_s is the plasma number density of the species s (s = e for electron, pc for cold positron, and ph for hot positron) normalized by its equilibrium value n_{so}, u_s is the plasma species fluid speed normalized by $C_{pm} = (m_e c^2/m_p)^{1/2}$ with m_e (m_p) being the electron (positron) mass and c being the speed of light in vacuum, ϕ is the electrostatic wave potential normalized by $m_e c^2/e$ with e being the magnitude of the charge of an electron, the time variable (t) is normalized by $\omega_{pi} = (4\pi n_{pc0} e^2/m_p)^{1/2}$, and the space variable (r) is normalized by $\lambda_m = (m_e c^2/4\pi n_{pc0} e^2)^{1/2}$. Here, β ($=n_{e0}/n_{pc0}$) is the ratio of the number density of electrons and cold positrons, σ ($=n_{\mathrm{pho}}/n_{\mathrm{pco}}$) is the ratio of number density of hot positrons and cold positrons, μ is the ratio of ions to cold positrons number densities. We have defined as $K_1 = n_{pc0}^{\alpha-1} K_i/m_e c^2$, $K_2 = n_{ph0}^{\gamma-1} K_e/m_p c^2$ and $K_3 = n_{e0}^{\gamma-1} K_e/m_e c^2$.

3 Derivation of Modified Burgers Equation

A dynamical equation for the nonlinear propagation of the PA shock waves is derived by using (7)–(11). In this regard, we employ a reductive perturbation technique to study the electrostatic perturbations propagating in the relativistic degenerate dense plasma on account of the effect of dissipation; we first introduce the stretched coordinates [35, 36]

$$\zeta = -\epsilon(r + V_p t), \quad (12)$$
$$\tau = \epsilon^2 t, \quad (13)$$

where V_p is the wave phase speed (ω/k with ω being angular frequency and k being the wave number of the perturbation mode) and ϵ is a smallness parameter measuring the weakness of the dissipation ($0 < \epsilon < 1$). We then expand n_s, u_s, and ϕ in power series of ϵ:

$$n_s = 1 + \epsilon n_s^{(1)} + \epsilon^2 n_s^{(2)} + \cdots, \quad (14)$$
$$u_s = \epsilon u_s^{(1)} + \epsilon^2 u_s^{(2)} + \cdots, \quad (15)$$
$$\phi = \epsilon \phi^{(1)} + \epsilon^2 \phi^{(2)} + \cdots, \quad (16)$$

and develop equations in various powers of ϵ. To the lowest order in ϵ, using (12)–(16) into equations (7)–(11), we get, $u_{pc}^{(1)} = -V_p \phi^{(1)}/(V_p^2 - K_1')$, $n_{pc}^{(1)} = \phi^{(1)}/(V_p^2 - K_1')$, $n_{ph}^{(1)} = \phi^{(1)}/K_2'$, $n_e^{(1)} = \phi^{(1)}/K_3'$, and $V_p = \sqrt{\frac{K_2' K_3'}{\beta K_2' - \sigma K_3'} + K_1'}$ (which represents the dispersion relation for the PA type electrostatic waves in the degenerate plasma under consideration).

We are eager to examine the nonlinear propagation of these dissipative PA-type electrostatic waves in a degenerate dense plasma. To the next higher order in ϵ, we obtain a set of equations

$$\frac{\partial n_{pc}^{(1)}}{\partial \tau} - V_p \frac{\partial n_{pc}^{(2)}}{\partial \zeta} + \frac{\partial}{\partial \zeta}\left[u_{pc}^{(2)} + n_{pc}^{(1)} u_{pc}^{(1)}\right] + \frac{\nu u_{pc}^{(1)}}{V_p \tau} = 0, \quad (17)$$

$$\frac{\partial u_{pc}^{(1)}}{\partial \tau} - V_p \frac{\partial u_{pc}^{(2)}}{\partial \zeta} + u_{pc}^{(1)} \frac{\partial u_{pc}^{(1)}}{\partial \zeta} + \frac{\partial \phi^{(2)}}{\partial \zeta} + K_1' \frac{\partial}{\partial \zeta}\left[n_{pc}^{(2)} + \frac{(\alpha - 2)}{2}(n_{pc}^{(1)})^2\right] - \eta \frac{\partial^2 u_{pc}^{(1)}}{\partial \zeta^2} = 0, \quad (18)$$

$$\frac{\partial \phi^{(2)}}{\partial \zeta} - K_2' \frac{\partial}{\partial \zeta}\left[n_{ph}^{(2)} + \frac{(\gamma - 2)}{2}(n_{ph}^{(1)})^2\right] = 0, \quad (19)$$

$$\frac{\partial \phi^{(2)}}{\partial \zeta} - K_3' \frac{\partial}{\partial \zeta}\left[n_{e}^{(2)} + \frac{(\gamma - 2)}{2}(n_{e}^{(1)})^2\right] = 0, \quad (20)$$

$$\beta n_e^{(2)} - \sigma n_{ph}^{(2)} - n_{pc}^{(2)} = 0. \quad (21)$$

Now, combining (17)–(21), we deduce a modified Burgers equation

$$\frac{\partial \phi^{(1)}}{\partial \tau} + A\phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \zeta} + \frac{\nu \phi^{(1)}}{2\tau} = B \frac{\partial^2 \phi^{(1)}}{\partial \zeta^2} \quad (22)$$

where the values of A and B are given by

$$A = \frac{(V_p^2 - K_1')^2}{2V_p}\left[\frac{(3V_p^2 + K_1'(\alpha - 2))}{(V_p^2 - K_1')^3} + \frac{\beta(\gamma - 2)}{K_3'^2} - \frac{\sigma(\gamma - 2)}{K_2'^2}\right], \quad (23)$$

$$B = \frac{\eta}{2}. \quad (24)$$

4 Parametric Investigations

The nonlinear propagation of cylindrical and spherical PASWs in degenerate plasma system has been studied

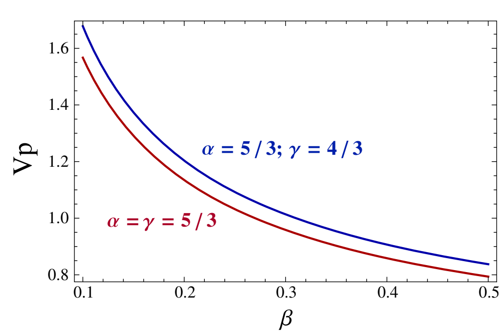

Fig. 1 *Color online* showing the variation of phase speed V_p with β for $u_0 = 0.01$ and $\sigma = 0.02$. The *blue solid* line represents the ultra-relativistic case and the *red solid* one represents the non-relativistic case

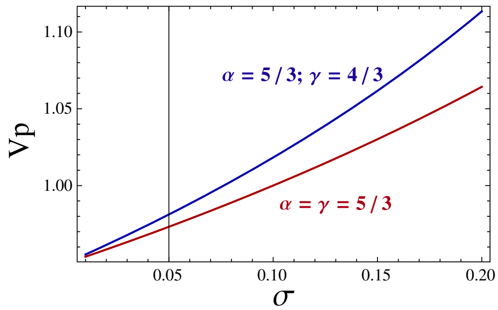

Fig. 2 *Color online* showing the variation of phase speed V_p with σ for $u_0 = 0.01$ and $\beta = 0.3$. The *blue solid* line represents the ultra-relativistic case and the *red solid* one represents the non-relativistic case

numerically. In our numerical analysis, we first analyzed the solution of Burgers equation (22). It is important to note that for large value of τ, the term $\frac{\nu\phi^{(1)}}{2\tau}$ is negligible, i.e., $\frac{\nu\phi^{(1)}}{2\tau} \to 0$. So, in our numerical analysis, we start with a large value of τ (viz. $\tau = -20$), and at this large (negative) value of τ, the stationary shock wave solution of (22) [without the term $\frac{\nu\phi^{(1)}}{2\tau}$] has been chosen. The stationary shock wave solution of standard Burgers equation is obtained by considering a frame $\zeta = \xi - u_0\tau$ (moving with speed u_0) and the solution is

$$\phi^{(1)}_{(\nu\to 0)} = \phi_m \left[1 - \tanh\left(\frac{\xi}{\Delta}\right)\right], \qquad (25)$$

where the amplitude, $\phi_m = u_0/A$, and the width, $\Delta = (2B/u_0)$.

Now, we investigate the effects of the different intrinsic parameters of our model (namely the electron-to-cold positron number density ratio β, hot-to-cold positron number density ratio σ, fluid speed u_0, and the kinematic viscosity η) on the dynamical properties (linear as well as nonlinear) of cylindrical and spherical PAWs.

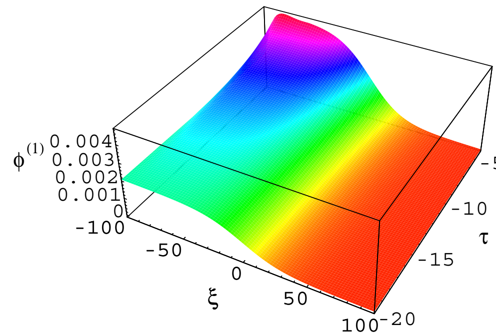

Fig. 4 *Color online* effects of cylindrical geometry ($\nu = 1$) on PA shock waves for $\beta = 0.4$, $u_0 = 0.01$, $\sigma = 0.02$, and $\eta = 0.3$, when both electron and hot positron being non-relativistic degenerate

4.1 Linear Properties

Figure 1 shows the variation of phase speed with electron-to-cold positron number density β for both non-relativistic and ultra-relativistic degenerate plasma system. We observed that the phase speed increases with the decreasing values of β (as it is expected from the expression of V_p). Then, we have also shown the variation of V_p with σ (see Fig. 2). It is observed that the phase speed increases with the increasing values of σ. It is important to mention that the damping (via kinematic viscosity) term does not have any effect on the linear wave propagation.

4.2 Nonlinear Properties

Effect of Electron-to-Cold Positron Number Density Ratio (via β) The effect of electron-to-cold positron number density ratio β on the cylindrical and spherical shock profiles is depicted in Figs. 3, 4, 5, and 6 for both non-relativistic and ultra-relativistic limit. This shows that the amplitude of shock structures increases with the increasing values of β. Actually, this happens because the increase of β causes the decrease of the nonlinearity coefficient A. It is also observed that the amplitude of these shocks is higher for

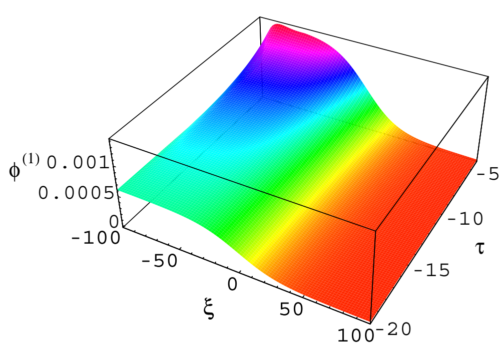

Fig. 3 *Color online* effects of cylindrical geometry ($\nu = 1$) on PA shock waves for $\beta = 0.3$, $u_0 = 0.01$, $\sigma = 0.02$, and $\eta = 0.3$, when both electron and hot positron being non-relativistic degenerate

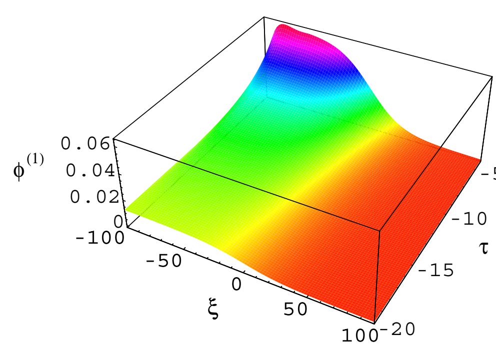

Fig. 5 *Color online* effects of spherical geometry ($\nu = 2$) on PA shock waves for $\beta = 0.3$, $u_0 = 0.01$, $\sigma = 0.02$, and $\eta = 0.3$, when both electron and hot positron being ultra-relativistic degenerate

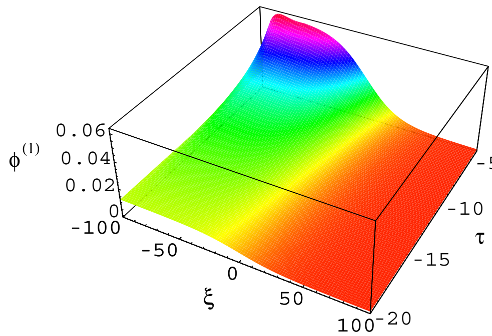

Fig. 6 *Color online* effects of spherical geometry ($\nu = 2$) on PA shock waves for $\beta = 0.4$, $u_0 = 0.01$, $\sigma = 0.02$, and $\eta = 0.3$, when both electron and hot positron being ultra-relativistic degenerate

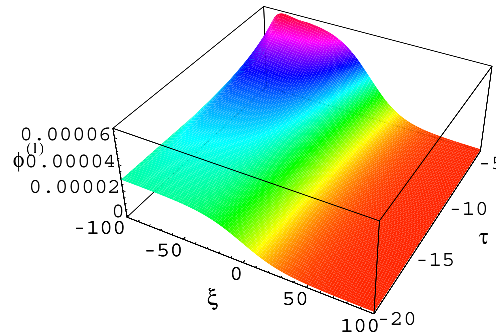

Fig. 8 *Color online* effects of cylindrical geometry ($\nu = 1$) on PA shock waves for $\sigma = 0.03$, $\beta = 0.3$, $u_0 = 0.01$, and $\eta = 0.3$, when both electron and hot positron being non-relativistic degenerate

ultra-relativistic case than for non-relativistic case. We have found that as time decreases the amplitude of the shock waves in cylindrical and spherical geometry increases. It is also examined that in spherical geometries, the amplitude is always distinctly higher than cylindrical geometries, which indicates that the density compression can be more effectively obtained in a spherical geometry.

Effect of Hot-to-Cold Positron Number Density Ratio (via σ) The effect of hot-to-cold positron number density ratio (σ) on the cylindrical and spherical PA shock profiles is illustrated in Figs. 7 and 8. It is found that the amplitude of the shock profiles decreases with the increasing values of σ. It happens on the basis of the driving force of the PA wave, as the driving force for the PA wave is provided by positrons inertia. Actually, increase in positron concentration (depopulation of electrons) causes decrease in the driving force, which is provided by the positron inertia, and consequently shock wave enervates.

Kinematic Viscosity Effect (via η) Figures 9 and 10 illustrate the effect of kinematic viscosity on the cylindrical PA shock profiles. It is found that the shock height increases with the increasing values of η. Actually, this happens because the increase of η increases the dissipative coefficient B. We also observed the same results for spherical geometry.

It is also important to mention that the Burgers equation derived here is valid only for the cases $A \neq 0$, namely, $A > 0$ and $A < 0$. The cylindrical and spherical PA waves are seen to be modified when cold positrons, hot positrons, and electrons being non-relativistic degenerate ($\alpha = \gamma = \frac{5}{3}$) than hot positrons and electrons being ultra-relativistic degenerate ($\alpha = \frac{5}{3}$; $\gamma = \frac{4}{3}$). It is noteworthy that, if we compare the non-relativistic and ultra-relativistic cases for PAWs then the amplitude variation shock profiles are found in every case that were displayed in Figs. 1, 2, 3, 4, and 5. The ranges ($u_0 = 0.01 - 1$, $\beta = 0.1 - 0.4$, $\sigma = 0.1 - 0.3$, and $\mu = 0.1 - 0.6$) [2, 37, 38] of plasma parameters used in this numerical analysis are very wide and correspond to space and laboratory plasma situations.

5 Discussion

In this manuscript, we have studied the nonlinear propagation of PA shock waves in an unmagnetized, collisionless degenerate quantum plasma (containing non-relativistic

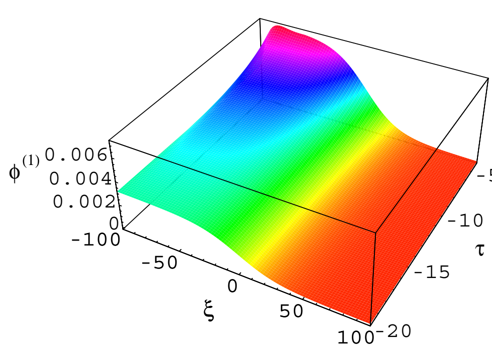

Fig. 7 *Color online* effects of cylindrical geometry ($\nu = 1$) on PA shock waves for $\sigma = 0.01$, $\beta = 0.3$, $u_0 = 0.01$, and $\eta = 0.3$, when both electron and hot positron being non-relativistic degenerate

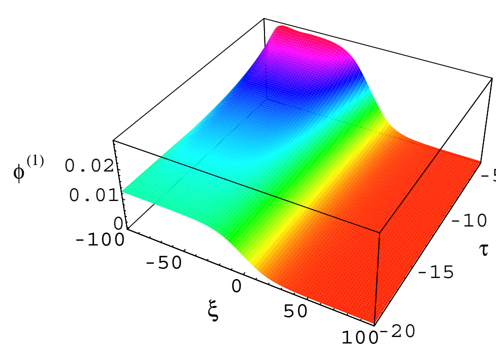

Fig. 9 *Color online* effects of cylindrical geometry ($\nu = 1$) on PA shock waves for $\eta = 0.2$, $\sigma = 0.02$, $\beta = 0.3$, and $u_0 = 0.01$ when both electron and hot positron being ultra-relativistic degenerate

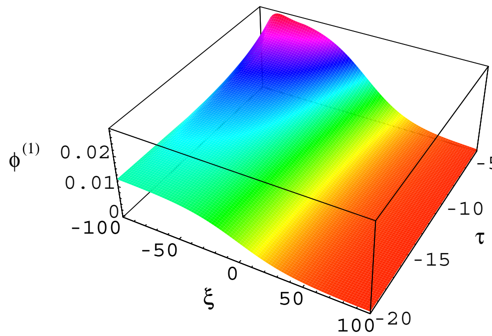

Fig. 10 (Color online) Effects of cylindrical geometry ($\nu = 1$) on PA shock waves for $\eta = 0.4$, $\sigma = 0.02$, $\beta = 0.3$, $u_0 = 0.01$ when both electron and hot positron being ultra-relativistic degenerate

cold positrons, both non-relativistic and ultra-relativistic degenerate electrons, and hot positron fluids and positively charged static ions). The positively charged static ions participate only in maintaining the quasi-neutrality condition at equilibrium. We have derived Burgers equation by using reductive perturbation technique and then analyzed the basic features of the cylindrical and spherical shock profiles. In this plasma system, cylindrical and spherical geometries as well as existence of both non-relativistic and ultra-relativistic degenerate electron and hot positron play a pivotal role in the fundamental properties (viz. amplitude, speed, width, etc.) of PASWs. Our present investigation would be useful to comprehend the shock wave properties and the behavior of several astrophysical objects (viz. neutron stars [39], white dwarfs [16], pulsar magnetosphere [37, 40], active galactic nuclei [41], our own galaxy [42], black holes [43]) and laboratory devices [44–46]. The PASWs presented here are an advanced nonlinear wave mode, and this investigation predicts unique findings on positron-acoustic waves in EPI plasmas.

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