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Low-Frequency Electrostatic Shock Excitations in a Multi-Component Dusty Plasma

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Abstract Dust-acoustic shock waves are investigated in a four-component plasma consisting of arbitrarily charged inertial dusts, Boltzmann distributed negatively charged heavy ions, positively charged light ions, and electrons. The reductive perturbation technique is employed in order to derive the nonlinear time evolution Burgers-type equation. The properties of dust-acoustic shock waves are analysed via the solution of Burgers equation. It is observed that the basic features of dust-acoustic shock waves are significantly modified due to the influence of arbitrarily charged dusts, Maxwellian electrons, number density and temperatures of heavier and lighter ions, and dust kinematic viscosity. Both polarity (positive and negative potential) shock waves are also found to exists in the plasma under consideration in this manuscript. The findings of this investigation may be used in understanding the dust-acoustic wave properties in both laboratory and space plasmas.

Keywords Dust-acoustic waves · Maxwellian ions · Maxwellian electrons · Multi-ion plasma · Reductive perturbation method · Burgers equation

1 Introduction

The study of dusty plasmas has received a great deal of interest because of its scope in a variety of fields such as astrophysics, semiconductor manufacturing, and fusion reactors. Dust is omnipresent in nebulas, in asteroid zones, in planetary magnetospheres, in interstellar clouds, in cometary environments (e.g. cometary comae and tails), on the surfaces of the Mars' and Earth's moons, and in the Earth's polar mesosphere [1–5]. A dusty plasma is an electron-ion plasma with an additional component of small micron-sized charged dust [5]. It is noted that the inclusion of massive, charged dust component [5] modifies the existing linear wave modes and introduces new waves such as dust-acoustic (DA) wave, dust-ion-acoustic (DIA) wave, dust lattice waves, etc. The dust grains acquire a negative charge by the collection of electrons [5, 6] because the thermal speed of the electrons is much higher than that of the ions. On the other hand, dust grains may become positively charged also due to a variety of processes including photoelectron emission by UV photons [6], thermoionic emission induced by radiative heating [1, 5], secondary electron production [7], etc. Thus, arbitrarily (negatively or positively) charged dust particles are found to exist in many space and laboratory dusty plasma system [5, 8–11]. The DA wave (DAW) is a very low-frequency mode in which the wave is supported by the inertia of the dust particles, with the restoring force being provided by the pressure of both the electrons and ions. Rao et al. [12] first theoretically predicted the existence of DAWs in an unmagnetized dusty plasma. After 5 years, Barkan et al. [13] experimentally studied the DAWs and verified the theoretical prediction of Rao et al. [12]. In addition to DAWs, there may also be the associated nonlinear structures such as dust-acoustic shock waves (DASHWs), which arise due to the balance between the nonlinear effect and the dissipation. The dissipation arises due to Landau damping, kinematic viscosity among the plasma species, wave particle interaction, etc., which is responsible to form the shock structures in a plasma system [14]. The shock structures were found by Andersen et al. [15] in laboratory experiment such as Q-machine experiment. First observation of DASHWs was reported by

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Samsonov et al. [16] in a three-dimensional dusty plasma under microgravity condition.

Over the last few decades, a great deal of attention has been devoted to the study of the wave propagation in dusty multi-ion plasmas (positive ions and negative ions) because of its vital role in both space [5, 17] and laboratory environments [13, 18]. The plasma in environments such as in the Earth's ionosphere [19] and cometary comae contains both negative ion and positive ion species in addition to electrons. Positive-negative ion plasmas are also found in plasma processing reactors [20], neutral beam sources [21], and low temperature laboratory experiments [22]. The multi-ion plasmas have been investigated by many scientists both theoretically and experimentally [23–26]. The role of negative ions in experiments with dusty plasmas has been discussed by Klumov et al. [27]. The presence of a fraction of negative ions in a dusty plasma changes the plasma composition and plasma transport properties [27], as well as the dust charges [28–30]. Kim and Merlino [29] reported the conditions under which dust grains could be positively charged in an electron-ion plasma with both positive and negative ions. The effect of positive and negative dust grains on the IA wave instability in a plasma with negative and positive ions has been examined by Rosenberg and Merlino [31].

Duha [32] have considered negative and positive dust charging currents, where negative ions are not in Boltzmann equilibrium [32, 33] and current fluctuation associated with them has been neglected. On the other hand, it has been predicted by a number of authors [33, 34] that negative ions in such electronegative plasmas are in Boltzmann equilibrium. This prediction has been conclusively verified by a laboratory experiment of Ghim and Hershkowitz [35]. Mamun et al. [36] have considered dusty electronegative plasma system containing Boltzmann electrons, Boltzmann negative ions, cold mobile positive ions, and stationary negatively charged dust and have studied the formation of solitary waves and double layers. To the best of our knowledge, we have found in literature that no work has been carried out to analyse the properties of DASHWs in a plasma system consisting of arbitarily charged mobile dusts, Boltzmann distributed electrons, heavy negative ions, and light positive ions. Our aim here is to investigate the shock wave properties and also to analyse their basic features (polarity, amplitude, width, etc.) in such a dusty multi-ion plasma.

The manuscript is organized as follows. The governing equations are provided in Section 2. In Section 3, we have derived planar Burgers equation using the reductive perturbation method. The numerical solution of Burgers equation is presented in Section 4. A brief discussion is finally presented in Section 5.

2 Governing Equations

We consider a collisionless four-component unmagnetized dusty plasma containing arbitrarily charged mobile dusts particles, Boltzmann distributed electrons, positively charged light ions, and negatively charged heavy ions. Thus, the equilibrium condition reads $n_{i0} - Z_h n_{h0} - n_{e0} - jZ_d n_{d0} = 0$, where n_{s0} is the unperturbed number density of the species s (here $s = i, h, e, d$ for positive light ion, negative heavy ion, electron, and mobile dust, respectively), Z_d is the number of electrons residing onto the dust grain surface and $j = \pm 1$ (+ for positively and − for negatively charged dust). The number densities of heavy ions and light ions following Boltzmann distribution are

$$n_e = n_{e0} e^{\frac{e\phi}{T_e}}, \ n_h = n_{h0} e^{\frac{e\phi}{T_h}}, \ n_i = n_{i0} e^{-\frac{e\phi}{T_i}}. \tag{1}$$

The nonlinear dynamics of these low-frequency electrostatic DAWs in such a dusty plasma system, whose phase speed is much smaller than the electron and ion thermal speeds but greater than the dust thermal speed, are governed by the following normalized equations

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x}(n_d u_d) = 0, \tag{2}$$

$$n_d \frac{\partial u_d}{\partial t} + n_d u_d \frac{\partial u_d}{\partial x} = -j n_d \frac{\partial \psi}{\partial x} - \frac{\partial n_d}{\partial x} + \eta n_d \frac{\partial^2 u_d}{\partial x^2}, \tag{3}$$

$$\frac{\partial^2 \psi}{\partial x^2} = \mu_e e^{\sigma_e \psi} - \mu_i e^{-\sigma_i \psi} + \mu_h e^{-\sigma_h \psi} - j n_d, \tag{4}$$

where n_d is the dust particle number density normalized by its equilibrium value n_{d0}, u_d is the dust fluid speed normalized by C_d, ψ is the electrostatic wave potential normalized by $T_d/Z_d e$, and η is the viscosity coefficient normalized by $m_d n_{do} \omega_{pd} \lambda_{Dm}^2$. The time variable t is normalized by $\omega_{pd} = (4\pi e^2 Z_d^2 n_{do}/m_d)^{1/2}$ and the space variable x is normalized by $\lambda_{Dm} = (T_d/4\pi e^2 Z_d^2 n_{do})^{1/2}$. We have defined the parameters arising in (4) as $\mu_e = n_{eo}/Z_d n_{do}$, $\mu_i = n_{io}/Z_d n_{do}$, $\mu_h = Z_h n_{ho}/Z_d n_{do}$, $\sigma_e = T_d/Z_d T_e$, $\sigma_i = T_d/Z_d T_i$, and $\sigma_h = T_d/Z_d T_h$, where T_e is the electron temperature, T_i is the positive light ion temperature, and T_h is the negative heavy ion temperature.

To conclude this section, we should note that the quasi-neutrality condition (i.e. $n_{i0} - Z_h n_{h0} - n_{e0} - jZ_d n_{d0} = 0$) may lead to a plasma system having same charge state (Z_d) and number density (n_{d0}) of the dust particles (either dust is positively or negatively charged). That is, one may consider a plasma having same numerical values of Z_d and n_{d0} for positively ($j = +1$) as well as for negatively ($j = -1$) charged dust grains.

3 Shock Waves

To derive a dynamical equation for the electrostatic DASHWs from our basic system of equations (2)–(4), we employ the reductive perturbation technique. We first introduce the stretched coordinate [37, 38] as

$$\zeta = \epsilon(x - V_p t), \quad \tau = \epsilon^2 t, \tag{5}$$

where ϵ is a smallness parameter measuring the weakness of the dispersion and V_p is the phase speed of the DAWs. We can expand the perturbed quantities n_d, u_d, and ψ about the equilibrium values in power series of ϵ as

$$n_d = 1 + \epsilon n_d^{(1)} + \epsilon^2 n_d^{(2)} + \cdots, \tag{6}$$

$$u_d = 0 + \epsilon u_d^{(1)} + \epsilon^2 u_d^{(2)} + \cdots, \tag{7}$$

$$\psi = 0 + \epsilon \psi^{(1)} + \epsilon^2 \psi^{(2)} + \cdots, \tag{8}$$

and develop equations in various powers of ϵ. To the lowest order in ϵ, (2)–(4) give

$$u_d^{(1)} = \frac{j V_p \psi^{(1)}}{V_p^2 - 1}, \tag{9}$$

$$n_d^{(1)} = \frac{j \psi^{(1)}}{V_p^2 - 1}, \tag{10}$$

$$V_p = \sqrt{1 + \frac{j^2}{\mu_e \sigma_e + \mu_i \sigma_i + \mu_h \sigma_h}}. \tag{11}$$

Equation (11) describes the phase speed of DAWs regarding the dusty plasma under consideration. To the next higher order of ϵ, i.e. taking the coefficients of ϵ^3 from both sides of (2) and (3), and ϵ^2 from both sides of (4), one may obtain another set of simultaneous equations for $\psi^{(1)} = \psi$, $\psi^{(2)}$, $n_d^{(2)}$, and $u_d^{(2)}$. After some algebraic calculation (omitted here), one may obtain the nonlinear Burgers type equation as

$$\frac{\partial \psi}{\partial \tau} + A\psi \frac{\partial \psi}{\partial \zeta} = B \frac{\partial^2 \psi}{\partial \zeta^2}, \tag{12}$$

where the nonlinear coefficient A and the dissipative coefficient B are given by

$$A = \frac{(V_p^2 - 1)^2}{2j^2 V_p}\left[\frac{2j^2 V_p^2}{(V_p^2 - 1)^3} + \frac{j^3}{(V_p^2 - 1)^2} - \mu_e \sigma_e^2 - \mu_h \sigma_h^2 + \mu_i \sigma_i^2\right], \tag{13}$$

$$B = \frac{\eta}{2}. \tag{14}$$

Equation (12) is the well-known Burgers equation.

4 Steady-State Solution of the Burgers Equation

The stationary shock wave solution of the Burgers equation (12) is obtained by transforming the independent variables ζ and τ to $\xi = \zeta - U_0 \tau'$ and $\tau' = \tau$, where U_0 is the speed of the shock waves, and imposing the appropriate boundary conditions, viz. $\psi \to 0$, $d\psi/d\zeta \to 0$, $d^2\psi/d\zeta^2 \to 0$ at $\zeta \to \pm\infty$. Thus, one can express the stationary shock wave solution of the Burgers equation (12) as

$$\psi = \psi_m[1 - \tanh(\xi/\Delta)], \tag{15}$$

where the amplitude ψ_m, and the width Δ are given by

$$\psi_m = U_0/A, \tag{16}$$

$$\Rightarrow \quad U_0 = \psi_m A, \tag{17}$$

$$and \quad \Delta = 2B/U_0. \tag{18}$$

We see in (17) that the shock wave velocity U_0 depends on amplitude ψ_m and also on the nonlinear term A. It is obvious from (15)–(18) that for vanishing nonlinear effect (i.e. for $A = 0$) the amplitude of the shock waves approaches to infinity. This means that our theory is not valid when $A \sim 0$ which makes the amplitude extremely large and breaks down the validity of the reductive perturbation method. Thus, $A = 0$ gives the critical value of the plasma parameters above/below which positive/negative potential structures may exists. We note that the nonlinearity coefficient A is a function of μ_h, μ_i, μ_e, σ_e, σ_h, and σ_i for the model under consideration in this manuscript. So, to find the parametric regimes corresponding to $A = 0$, we have to express one (viz. μ_i) of these parameters in terms of the others (viz. μ_h, μ_e, σ_e, σ_h, and σ_i). Therefore, $A(\mu_i) = 0$ leads to the critical value of μ_i (long expression $\to$ omitted here), let us say $\mu_i = \mu_{cp}$ for $j = 1$ and $\mu_i = \mu_{cn}$ for $j = -1$.

We find the critical value $\mu_i = \mu_{cp} = 0.66$ for positively charged dust ($j = 1$) and $\mu_i = \mu_{cn} = 1.66$ for negatively charged dust ($j = -1$), for a set of plasma parameters (viz. $\mu_h = 0.2$, $\mu_e = 0.01$, $\sigma_e = 0.03$, $\sigma_h = 0.8$, and $\sigma_i = 0.05$). The parametric regime for this set of values is shown in Fig. 1. Figure 2 shows the variation of the width $\triangle$ of the SHWs with kinematic viscosity η. The amplitude of positive and negative potential SHWs with σ_h for positively (negatively) charged dust is shown in Fig. 3 (Fig. 4). Figure 5 shows the amplitude of positive and negative potential SHWs with σ_i for negatively charged dust, keeping parameters fixed at $\sigma_e = 0.03$, $\sigma_h = 0.5$, $\mu_h = 0.2$, $\mu_e = 0.01$, $\eta = 0.1$, and $U_0 = 0.01$. Figure 6 shows the variation of the amplitude with μ_e and σ_e for $j = 1$ with $\sigma_h = 0.5$, $\sigma_i = 0.05$, $\mu_h = 0.2$, $\eta = 0.1$, and $U_0 = 0.01$. Figure 7 shows the amplitude of positive potential SHWs

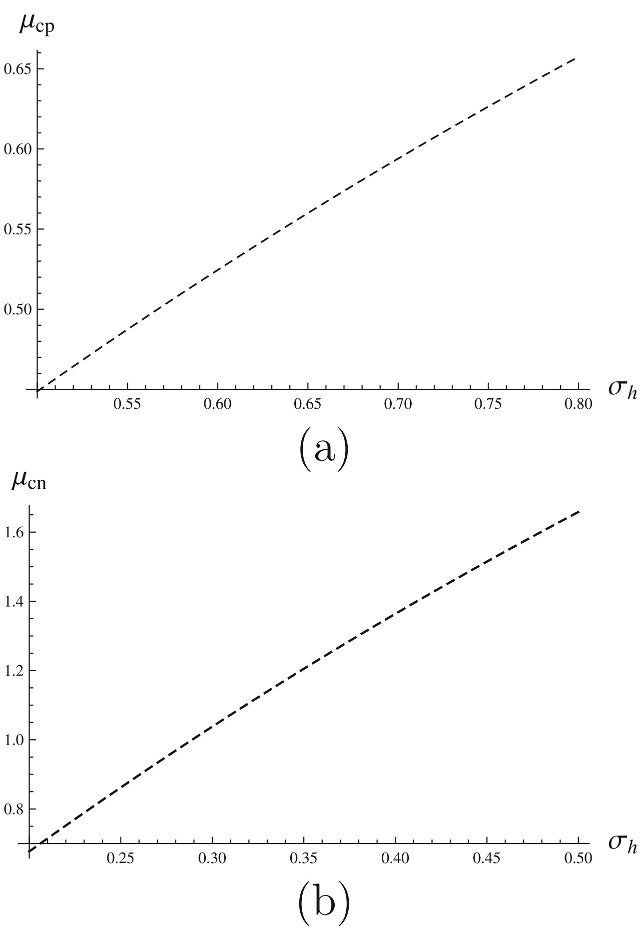

Fig. 1 The $A = 0$ graph which represents the variation of the critical value of light ion-to-dust number density ratio μ_c with dust-to-negatively charged heavy ion temperature ratio σ_h for (**a**) $j = 1$, where $\mu_c = \mu_{cp}$, and (**b**) $j = -1$, where $\mu_c = \mu_{cn}$

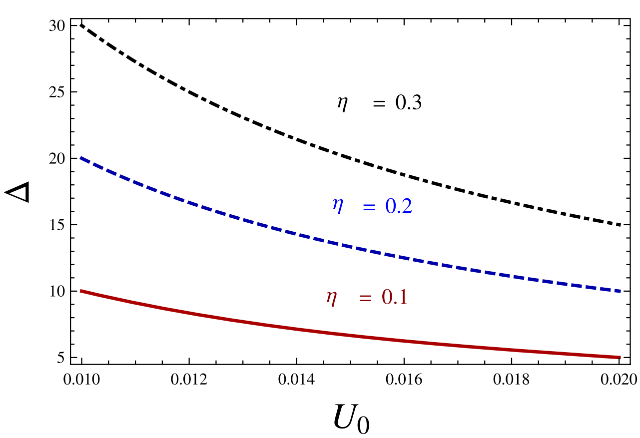

Fig. 2 Variation of the shock wave width $\triangle$ with U_0 for different viscosity coefficient η

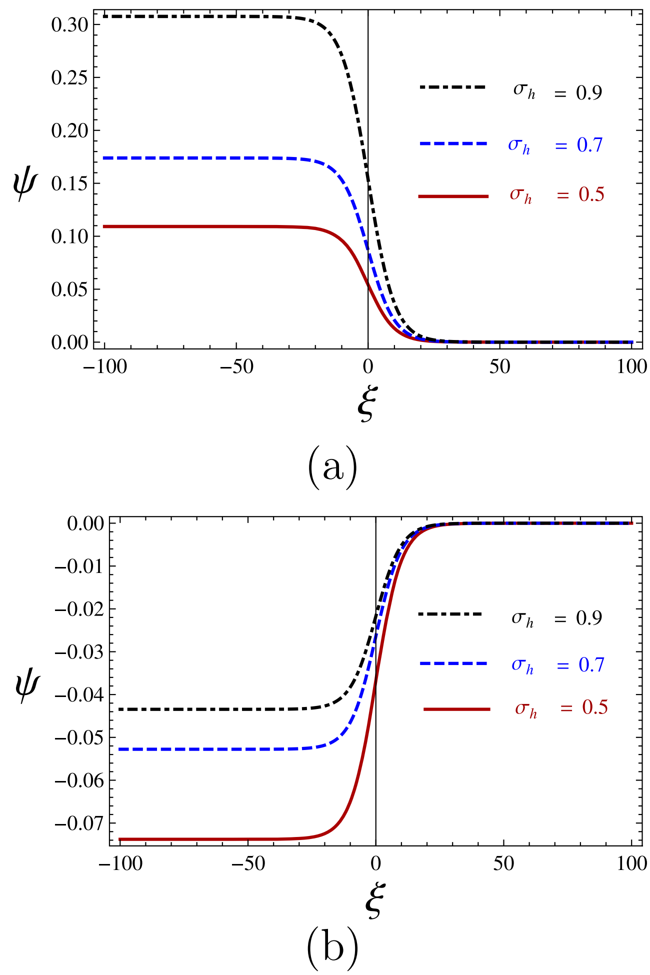

Fig. 3 Variation of the amplitude of the shock profile with ξ for different values of σ_h for $j = 1$, where (**a**) $\mu_i > \mu_{cn}$ and (**b**) $\mu_i < \mu_{cn}$. The other plasma parameters are fixed at $\sigma_e = 0.03$, $\sigma_i = 0.05$, $\mu_h = 0.2$, $\mu_e = 0.01$, $\eta = 0.1$, and $U_0 = 0.01$

with shock wave velocity U_0 for $j = \pm 1$, keeping other parameters fixed at $\sigma_h = 0.5$, $\sigma_i = 0.05$, $\mu_h = 0.2$, and $\eta = 0.1$. Similar effect has been found for the negative potential SHWs (figure omitted here).

5 Discussion

We have studied the nonlinear propagation of DASHWs in an unmagnetized four-component dusty plasma system consisting of arbitrarily charged mobile dusts fluid, Maxwellian negatively charged heavy ions, positively charged light ions, and electrons. The propagation of the small amplitude DASHWs in dusty plasmas has been considered by analysing the solution of the Burgers equation. It should also be noted that the Burgers equation derived here is valid [37, 38] only for the limits $A \neq 0$, $A > 0$, and $A < 0$. The

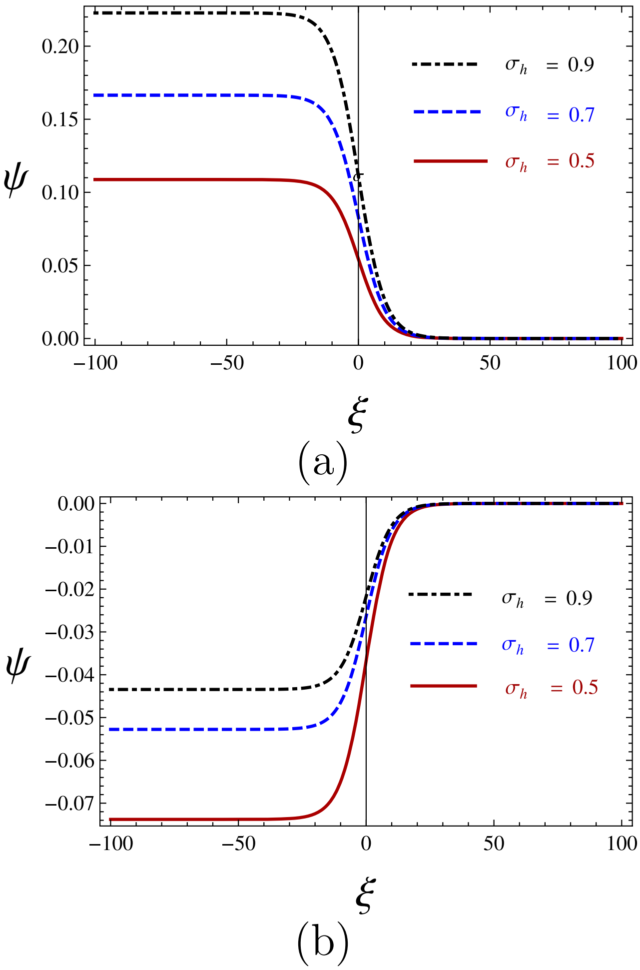

Fig. 4 Variation of the amplitude of the shock profile with ξ for different values of σ_h for $j = -1$, where **(a)** $\mu_i > \mu_{cp}$ and **(b)** $\mu_i < \mu_{cp}$. The other plasma parameters are fixed at $\sigma_e = 0.03$, $\sigma_i = 0.05$, $\mu_h = 0.2$, $\mu_e = 0.01$, $\eta = 0.1$, and $U_0 = 0.01$

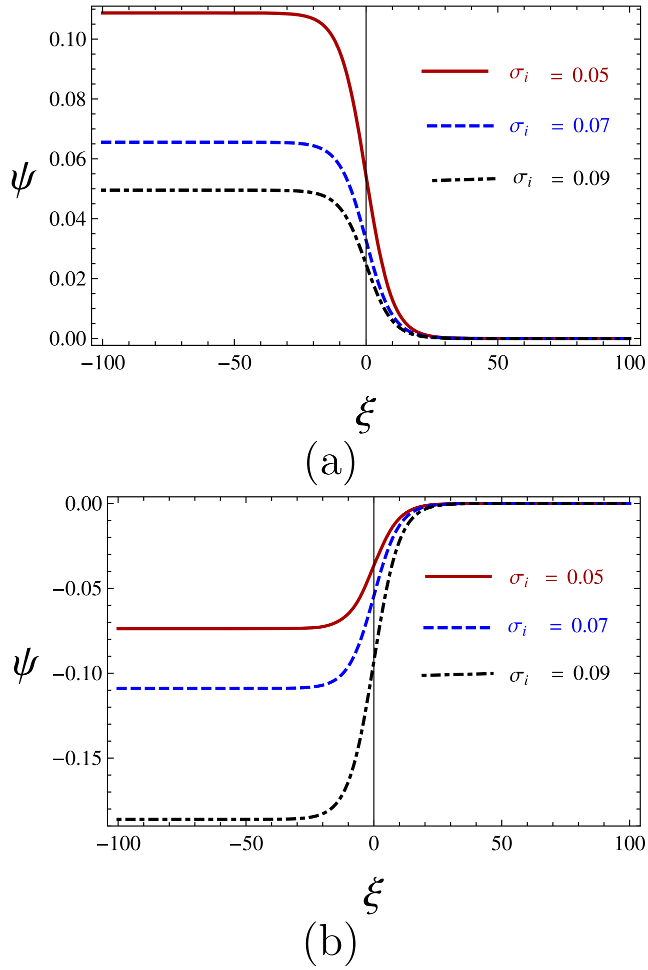

Fig. 5 Variation of the amplitude of the shock profile with ξ for different values of σ_i for $j = -1$, where **(a)** $\mu_i > \mu_{cn}$ and **(b)** $\mu_i < \mu_{cn}$. The other plasma parameters are fixed at $\sigma_e = 0.03$, $\sigma_h = 0.5$, $\mu_h = 0.2$, $\mu_e = 0.01$, $\eta = 0.1$, and $U_0 = 0.01$

results which have been found from this investigation can be pinpointed as follows:

1. The dusty multi-ion plasma under consideration supports finite amplitude shock structures whose basic features (polarity, amplitude, width, speed, etc.) strongly depend on different plasma parameters, particularly electron-to-dust number density ratio (via μ_e), light ion-to-dust number density ratio (via μ_i), heavy ion-to-dust number density ratio (via μ_h), dust-to-electron temperature ratio (via σ_e), dust-to-light ion temperature ratio (via σ_i), dust-to-heavy ion temperature ratio (via σ_h), and η.
2. We have obtained the critical value $\mu_i = \mu_{cp} = 0.66$ for $j = 1$ and $\mu_i = \mu_{cn} = 1.66$ for $j = -1$ for a fixed set of parametric values (viz. $\mu_h = 0.2$, $\mu_e = 0.01$, $\sigma_e = 0.03$, $\sigma_h = 0.8$, and $\sigma_i = 0.05$) (shown in Fig. 1).
3. The width $\triangle$ of the shock waves increases with the increase of kinematic viscosity η but the width of the shock structures decreases with the increase of heavy ion fluid speed U_0 (shown in Fig. 2).
4. For positively charged dust ($j = 1$), the DASHWs with positive (negative) potential are formed at $\mu_i > \mu_{cp}$ ($\mu_i < \mu_{cp}$) as shown in Fig. 3. Again, for negatively charged dust ($j = -1$), the DASHWs with positive (negative) potential are formed at $\mu_i > \mu_{cn}$ ($\mu_i < \mu_{cn}$) as presented in Fig. 4.
5. For $j = 1$, the magnitude of the amplitude of positive and negative potential SHWs increases with the increase of σ_h, as shown in Fig. 3. For $j = -1$ also,

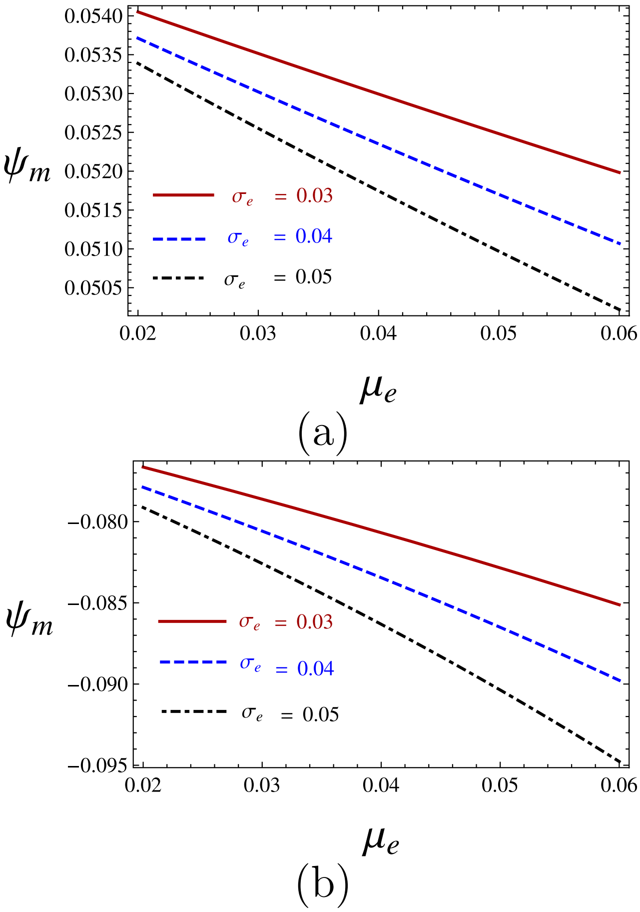

Fig. 6 Variation of the amplitude of the shock profile with μ_e for different values of σ_e for $j = 1$, where (**a**) $\mu_i > \mu_{cp}$, and (**b**) $\mu_i < \mu_{cp}$. The other plasma parameters are fixed at $\sigma_h = 0.5$, $\sigma_i = 0.05$, $\mu_h = 0.2$, $\eta = 0.1$, and $U_0 = 0.01$

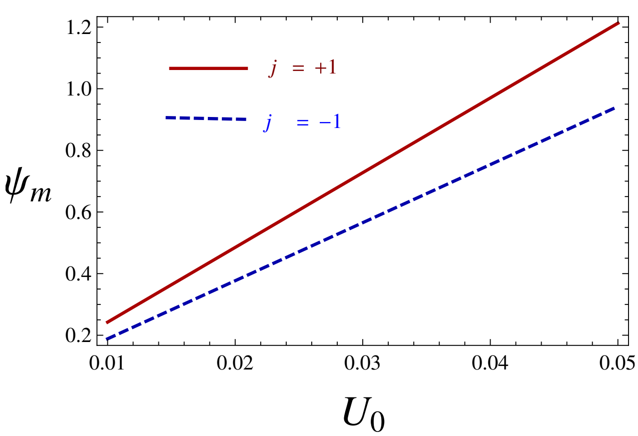

Fig. 7 Variation of the amplitude of the shock profile with U_0 for $j = \pm 1$, where $\mu_i > \mu_{cp,n}$. The other plasma parameters are fixed at $\mu_h = 0.2$, $\mu_e = 0.01$, $\sigma_e = 0.03$, $\sigma_h = 0.8$, and $\sigma_i = 0.05$, and $\eta = 0.1$

the magnitude of the amplitude of positive and negative potential SHWs increases with the increase of σ_h as presented in Fig. 4.

6. For $j = -1$, it is observed that the amplitude of positive and negative potential SHWs decreases with the increase of σ_i (see Fig. 5). Again for $j = 1$ also, the amplitude of positive and negative potential SHWs decreases with the increase of σ_i (figure omitted here).
7. The height of the positive and negative potential SHWs gradually increases with the increase of μ_e and σ_e for $j = 1$ as shown in Fig. 6.
8. The amplitude of the positive and negative potential SHWs for positively as well as for negatively charged dust with shock wave velocity U_0 is shown in Fig. 7. It is seen in our investigation that a shock profile of constant amplitude propagates faster in a plasma with positively charged dusts than in a plasma with negatively charged dusts. It is also found that taller shock moves faster in both positively and negatively charged dusty plasma.

It can be noted here that the analysis of shock structures in such dusty plasmas in the presence of external magnetic field are also problems of great importance but outside the scope of our present work. Laboratory experiments on SHWs in different plasma models have been performed by a number of authors. Luo et al. [39] have examined the shock formation in negative ion plasma. Nakamura [40] has examined DIASHWs in a homogeneous unmagnetized dusty double-plasma device. To conclude, we propose to perform a new laboratory experiment to verify the results of theory (i.e. to observe such DASHWs with Maxwellian electrons and Maxwellian ions (heavy and light ions), in a laboratory plasma) that is presented in this manuscript by using the experimental set up of Luo et al. [39] or Nakamura [40].

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