Brazilian Journal of Physics
ISSN: 0103-9733
luizno.bjp@gmail.com
Sociedade Brasileira de Física
Brasil

Sharrad, Fadhil I.; Hossain, I.; Ahmed, I. M.; Abdullah, Hewa Y.; Ahmad, S. T.; Ahmed, A. S.

U(5) Symmetry of Even 96,98 Ru Isotopes Under the Framework of Interacting Boson Model (IBM-1)

Brazilian Journal of Physics, vol. 45, núm. 3, junio, 2015, pp. 340-346
Sociedade Brasileira de Física
Sâo Paulo, Brasil

Braz J Phys (2015) 45:340–346
DOI 10.1007/s13538-015-0302-6

NUCLEAR PHYSICS

U(5) Symmetry of Even ^{96,98}Ru Isotopes Under the Framework of Interacting Boson Model (IBM-1)

Fadhil I. Sharrad · I. Hossain · I. M. Ahmed · Hewa Y. Abdullah · S. T. Ahmad · A. S. Ahmed

Received: 1 June 2014 / Published online: 11 March 2015

Abstract In this paper, the yrast bands of the even ^{96,98}Ru isotopes are studied within the framework of the interacting boson model-1 (IBM-1), using the MATLAB computer code (IBM-1.Mat). The theoretical energy levels are obtained for the ^{96,98}Ru isotopes, with neutron numbers 52 and 54, up to spin-parity 16^+ and 12^+, respectively. The ratio of the excitation energies of the first 4^+ to the first 2^+ excited states ($R_{4/2}$), the backbending curves and the potential energy surfaces are also calculated. The calculated and experimental $R_{4/2}$ values show that the ^{96,98}Ru nuclei have U(5) dynamic symmetry. The calculated energies of the yrast states are compared with experimental results and they are shown to be in good agreement with the data. The contour plots of the potential energy surfaces show two interesting nuclei having a slightly oblate but almost spherical vibrator-like character.

I. Hossain (✉) · A. S. Ahmed
Department of Physics, Rabigh College of Science and Arts, King Abdulaziz University, 21911 Rabigh, Saudi Arabia
e-mail: mihossain@kau.edu.sa

F. I. Sharrad
Department of Physics, College of Science, Kerbala University, 56001 Karbala, Iraq

H. Y. Abdullah
Department of Physics, College of Science Education, Salahaddin University, Erbil, KRG, Iraq

I. M. Ahmed
Department of Physics, College of Education, Mosul University, Mosul, Iraq

S. T. Ahmad
Department of Physics, Faculty of Science, Koya University, Koya, KRG, Iraq

Keywords Interacting boson model-1 · Even-even isotopes · Ruthenium · Energy level · Potential energy

PACS Nos. 21.10.-k · 42.40. Ht · 42.30. Kq

1 Introduction

The nuclear structure of neutron-rich ruthenium isotopes has been the focus of many experimental and theoretical works in recent years. The collective nuclear characters in the medium mass nuclei have been successfully described by Arima and Iachello using the interacting boson model-1 (IBM-1) [1]. There is no distinction between proton and neutron degree of freedom in the IBM-1. The microscopic anharmonic vibrator approach (MAVA) has been used to investigate the low-lying collective states in ruthenium isotopes [2].

The neutron-rich Ru (Z=44) isotopes are of great interest because they are near the magic number 50, which is found in the single closed shell Sn nucleus. The proton configuration of the ^{96,98}Ru isotopes, with Z=44, is $pg_{9/2}^{-6}$. These nuclei have six proton holes close to the magic number 50, and they have two and four neutron particles, respectively. These configurations have been used to calculate the yrast levels. The yrast level structure and the electromagnetic transition probabilities of even-even Ru isotopes have been investigated by many scientists [3–7].

Recently, we have studied the evolution of the yrast states of the even-even $^{100-110}$Pd isotopes [8]. The reduced electromagnetic transition probabilities of the even-even $^{104-112}$Cd isotopes were studied by Abdullah et al. [9]. Reduced electromagnetic transition probabilities of ground-state band of even-even $^{102-112}$Pd isotopes were studied within the framework of the interacting boson model (IBM-1) [10, 11].

The low-lying states of the ^{184}W and ^{184}Os nuclei were studied by Sharrad et al. [12].

These studies motivated the present work. We use the IBM to predict the yrast energies, the $R_{4/2}$ values, the backbending curves and the potential energy surfaces of $^{96,98}Ru$, with the aim of understanding the dynamical symmetries of these isotopes.

2 Theory

The interacting boson model (IBM-1) of Arima and Iachello [1] has been widely accepted as a tractable theoretical scheme of correlating, describing, and predicting low-energy collective properties of complex nuclei. The vibrational model uses geometric approach, the IBM employs a severely truncated model space, and as such, calculations are possible for nuclei with N nucleons, providing a quantitative mechanism to compare experimental results and calculated values [13]. In the first approximation of IBM-1, only pairs with angular momentum L=0 (called s bosons) and L=2 (called d bosons) are considered.

The Hamiltonian of the interacting bosons in IBM-1 is given by [14].

$$H = \sum_{i=1}^{N} \varepsilon_i + \sum_{i\langle j}^{N} V_{ij} \tag{1}$$

where ε is the intrinsic boson energy and V_{ij} is the interaction between bosons i and j.

The multipole form of the IBM-1 Hamiltonian is given by [15]

$$H = \varepsilon \hat{n}_d + a_0\left(\hat{P}.\hat{P}\right) + a_1\left(\hat{L}.\hat{L}\right) + a_2\left(\hat{Q}.\hat{Q}\right) + a_3\left(\hat{T}_3.\hat{T}_3\right) + a_4\left(\hat{T}_4.\hat{T}_4\right) \tag{2}$$

where

$$\hat{n}_d = \left(d^\dagger.\tilde{d}\right), \hat{P} = \frac{1}{2}\left(\tilde{d}.\tilde{d}\right) - \frac{1}{2}\left(\tilde{s}.\tilde{s}\right)$$
$$\hat{L} = \sqrt{10}\left[d^\dagger \times \tilde{d}\right]^{(1)}$$
$$\hat{Q} = \left[d^\dagger \times \tilde{s} + S^\dagger \times \tilde{d}\right]^{(2)} - \frac{1}{2}\sqrt{7}\left[d^\dagger \times \tilde{d}\right]^{(2)}$$
$$\widehat{T_3} = \left[d^\dagger \times \tilde{d}\right]^{(3)}, \widehat{T_4} = \left[d^\dagger \times \tilde{d}\right]^{(4)}$$

Here, $\hat{n}_d$ is the number of d boson, P is the pairing operator for the s and d bosons, L is the angular momentum operator, Q is the quadrupole operator, and T_3 and T_4 are the octupole and hexadecapole operators, respectively.

The Hamiltonian as given in Eq. (2) tends to reduce to three limits, the vibration U(5), γ-soft O(6), and the rotational SU(3) nuclei, starting with the unitary group U(6) and finishing with group O(2) [16]. In U(5) limit, the effective parameter is ε, in the γ-soft limit O(6); the effective parameter is the pairing a_0; and in the SU(3) limit, the effective parameter is the quadrupole a_2.

The Hamiltonian and eigen values for the three limits are given as follows [17]:U(5):

$$\left.\begin{array}{l}\hat{H}_{U(5)} = \varepsilon\hat{n}_d + a_1\hat{L}.\hat{L} + a_3\hat{T}_3.\hat{T}_3 + a_4\hat{T}_4.\hat{T}_4 \\ E(n_d, u, L) = \varepsilon\, n_d + K_1 n_d(n_d+4) + K_4 u(u+3) \\ \qquad + K_5 L(L+1)\end{array}\right\} \tag{3}$$

with

K_1 1/12 a_1
K_4 −1/10 a_1+1/7 a_3−3/70 a_4
K_5 −1/14 a_3+1/14 a_4

O(6):

$$\left.\begin{array}{l}\hat{H}_{O(6)} = a_0\hat{P}.\hat{P} + a_1\hat{L}.\hat{L} + a_3\hat{T}_3.\widehat{T_3} \\ E(\sigma,\nu,L) = K_3[(N(N+4)) - \sigma(\sigma+4)] \\ \qquad + K_4\nu(\nu+3) + K_5 L(L+1)\end{array}\right\} \tag{4}$$

with

K_3 1/4 a_0
K_4 1/2 a_3
K_5 −1/10 a_3+a_1

SU(3):

$$\left.\begin{array}{l}\hat{H}_{SU(3)=a1}\hat{L}.\hat{L}+_{a2}\hat{Q}.\hat{Q} \\ E(\lambda,\mu,L) = K_2\left(\lambda^2+\mu^2+\lambda\mu+3(\lambda+\mu)\right) \\ \qquad + K_5 L(L+1)\end{array}\right\} \tag{5}$$

with

K_2	1/2 a_2
K_5	a$_1$−3/8 a_2
K_1, K_2, K_3, K_4, and K_5	are other forms of strength parameters.

Then, using specific limit or symmetry (U(5), SU(3), or O(6)) to examine the structure of the set of nuclei or set of isotopes is more beneficial than employing the full

Hamiltonian of IBM-1. This is because this Hamiltonian contains multi-free parameters which make it easy to fit the structure of any isotope. Using a limited number of parameters or specific symmetry limit will emphasize the physics of the system such as the excitation type, the shape of the nucleus, and how isotopes change their shape and mode of excitation with the rise of neutron number in the shell which means an increase in neutron boson number where the nuclei started to sweep from the vibrational spectra (spherical shape) to rotational spectra (deformed shape).

Referring to the ratio $R_{E4/E2}$, the values $R\approx 2\rightarrow$U(5), $R\approx 2.5\rightarrow$O(6), and $R\approx 3.333\rightarrow$SU(3) [18, 19] are typical values which only exist when the nucleus absolutely and purely belongs to these limits, which means that this nucleus is an example of the related symmetry. Unfortunately, we have very few nuclei which own this privilege. While the majority of the isotopes are considered to be transitional nuclei which contain properties of two or three of these symmetries, here, we refer to what we call Casten triangle [20] or symmetry triangle, which means that the R value of nucleus to which these three typical values is closest will indicate the assignment of the symmetry of this nucleus.

The relation between the moment of inertia (ϑ) and gamma energy E_γ is given by

$$2\vartheta/\hbar^2 = \frac{2(2I-1)}{E(I)-E(I-2)} = \frac{4I-2}{E_\gamma} \tag{6}$$

And the relation between E_γ and ħω is given by

$$\hbar\omega = \frac{E(I)-E(I-2)}{\sqrt{I(I+1)}-\sqrt{(I-2)(I-1)}} = \frac{E_\gamma}{\sqrt{I(I+1)}-\sqrt{(I-2)(I-1)}} \tag{7}$$

3 Results and Discussion

In the framework of IBM-1, the ^{96}Ru and ^{98}Ru nuclei with neutron numbers 52 and 54 have proton boson hole number 3 and neutron boson particle numbers 1 and 2, respectively. Therefore, the total numbers of bosons of ^{96}Ru and ^{98}Ru nuclei are 4 and 5, respectively. IBM-1 calculations have been performed with no distinction made between the neutron and proton bosons. The symmetry shape of a nucleus can be predicted from the energy ratio $R=E4_1^+/E2_1^+$, where $E4_1^+$ is the energy level at 4_1^+ and $E2_1^+$ is the energy level at 2_1^+. The R has a limit value of ≈2 [18, 19] for the vibration nuclei U(5), ≈2.5 for γ-unstable nuclei O(6), and ≈3.33 for rotational

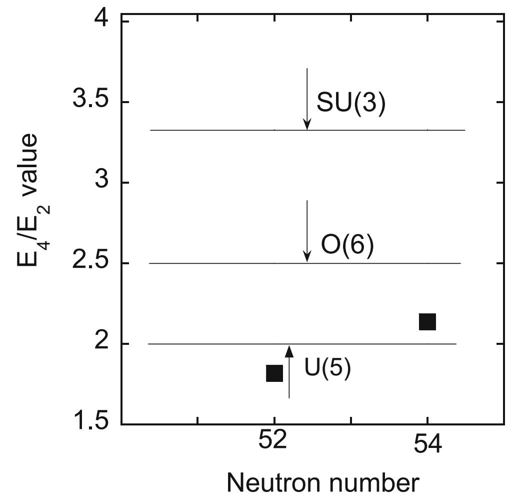


Fig. 1 $E(4_1^+)/E(2_1^+)$ values calculated for experimental data [21] of the isotopes ^{96,98}Ru isotopes are compared with those yielded by the U(5), O(6), and SU(3) limits

nuclei SU(3). The R values of low-lying energy levels of ^{96}Ru and ^{98}Ru isotopes are 1.8 and 2.1, respectively, and it is shown in Fig. 1. From this figure, we have identified U(5) symmetry in even-even ^{96}Ru and ^{98}Ru isotopes.

For the analysis of the yrast states in the ^{96,98}Ru nuclei up to 16^+ states, we tried to keep the number of free parameters in the Hamiltonian to a minimum. Overall best fit was archived for the yrast-state bands of double even isotopes ^{96,98}Ru. Furthermore, these parameters can be determined from the Eq. (3). The experimental eigen values ($E(n_d, \upsilon, L)$) are used to determine these parameters, where n_d, υ, and L are quantum numbers. In addition, the fitted parameters have been change in the limitation of it in the range of error of the energy levels to get the best fit.

Each nucleus at the evolving states is determined using Eq. (3). Table 1 shows the values of the parameters used to calculate the energy of the yrast states for the isotopes under study. The energy level fits with IBM-1 are presented in Table 2. It is clear from the conversion of the set of parameters ε, k_1, k_4, and k_5 to ε, a_0, a_1, a_2, a_3, and a_4 that a_0 and a_2 which are the characteristic parameters of O(6) and SU(3), respectively, are vanished and are equal to zero. It can be concluded that the Ru-96 and Ru-98 in their ground-state bands are good candidates of U(5) symmetry, with domination not less than 90 % according to the weight of fitting parameters.

Table 1 Boson number and calculated parameters in kilo-electron-volt (keV) for even $^{96–98}$Ru isotopes

Isotopes	N	ε (keV)	K_1 (keV)	K_4 (keV)	K_5 (keV)
^{96}Ru	4	597.69±2.78	14.14	10.39	−2.26
^{98}Ru	5	549.41±4.30	19.73	15.99	−2.53

Table 2 Excitation energies, moment of inertia, and square of rotational frequency for even $^{96-98}$Ru isotopes [21]

Nucl	I	$E_{exp}(I)$ keV	$E_{cal.}$ keV	Transition	E_γ Exp MeV	E_γ Cal MeV	$2\vartheta/\hbar^2$ Exp MeV^{-1}	$2\vartheta/\hbar^2$ Cal MeV^{-1}	$(\hbar\omega)^2$ MeV^2
^{96}Ru	2	832.6	796.4	$2^+\rightarrow0^+$	0.833	0.796	7.20	7.54	0.116
	4	1518.1	1423.8	$4^+\rightarrow2^+$	0.685	0.627	20.44	22.33	0.273
	6	2149.8	2182.2	$6^+\rightarrow4^+$	0.632	0.758	34.81	29.02	0.099
	8	2950.4	2971.7	$8^+\rightarrow6^+$	0.801	0.790	37.45	37.97	0.159
	10	3816.7	3792.1	$10^+\rightarrow8^+$	0.866	0.820	43.88	46.34	0.187
	12	4417.6	4643.6	$12^+\rightarrow10^+$	0.601	0.852	76.54	53.99	0.090
	14	5679.8	5526.1	$14^+\rightarrow12^+$	1.262	0.883	42.79	61.16	0.398
	16	6440.5	6439.7	$16^+\rightarrow14^+$	0.761	0.913	81.47	67.90	0.144
^{98}Ru	2	652.4	696.8	$2^+\rightarrow0^+$	0.652	0.697	9.20	8.61	0.071
	4	1397.8	1444.8	$4^+\rightarrow2^+$	0.745	0.748	18.79	18.72	1.36
	6	2222.5	2244.0	$6^+\rightarrow4^+$	0.825	0.799	26.67	27.53	0.169
	8	3190.2	3094.3	$8^+\rightarrow6^+$	0.968	0.850	30.99	35.29	0.233
	10	4000.8	3995.7	$10^+\rightarrow8^+$	0.811	0.901	46.86	42.18	0.164
	12	4914.0	4948.3	$12^+\rightarrow10^+$	0.913	0.953	50.38	48.27	0.208

For Ru-96 and Ru-98, the fitting parameters are as shown in Table 1. However, we may come out with an interesting point for Ru-96, ε which, representative of pure U(5) symmetry, is the dominating parameter in this calculation with uncertainty less than 0.47 %, since the closest parameter which still belongs to U(5) is k_1, which is only 2.36 % of ε. The k_4 parameter is only 10.39 keV, 1.74 % of ε. The k_5 parameter is only −2.26 keV, 0.38 % of ε. For Ru-98, ε which, representative of pure U(5) symmetry, is the dominating parameter in this calculation with uncertainty less than 0.78 % since the closest parameter which still belongs to U(5) is k_1, which is only 3.59 % of ε. The k_4 parameter is only 15.99 keV, 2.91 % of ε. The k_5 parameter is only −2.53 keV, 0.46 % of ε.

Nuclei with a small number of bosons near to closed shells may be practicing a different type of excitation than the rotation, which will be single-particle excitations which may push the first excited state in Ru-96 up a little, while in Ru-98 where the boson number is greater than one, collectivity appears to play a role and the 2_1^+ is lower in terms of energy than its counterpart in Ru-96.

Figures 2 and 3 show the yrast states of ^{96}Ru and ^{98}Ru isotopes as a function of angular momentum. The agreement

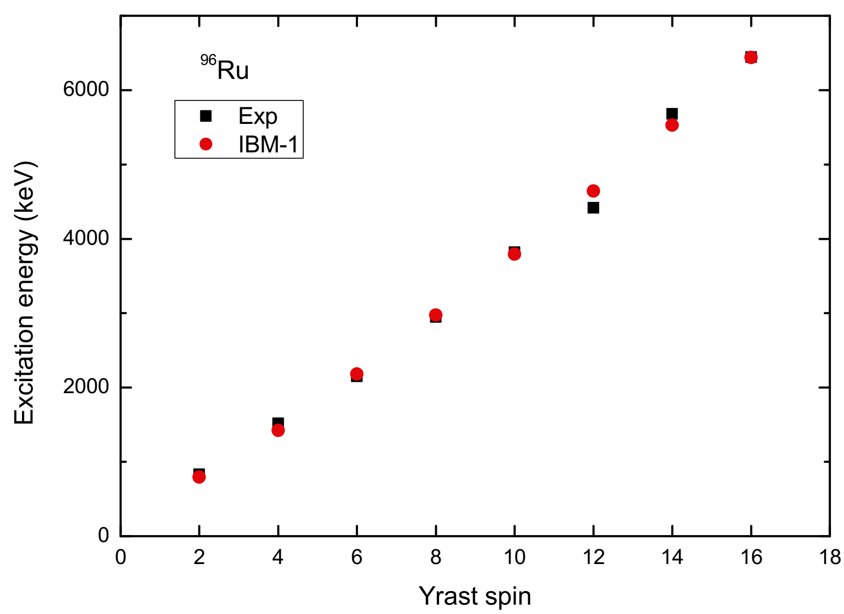


Fig. 2 Yrast states as a function of angular momentum for ^{96}Ru nucleus

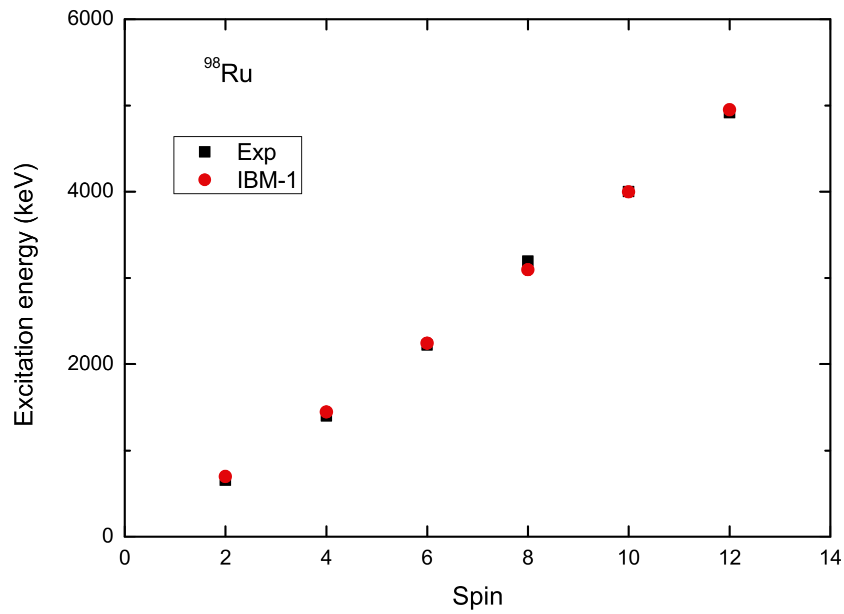


Fig. 3 Yrast states as a function of angular momentum for ^{98}Ru nucleus

between the calculations and the experimental yrast states is good and reproduced well. The values of the first excited state $E2_1^+$ and the ratio $R=E4_1^+/E2_1^+$ show that ^{96,98}Ru isotopes are vibrational nuclei.

To measure the evolution of nuclear collectivity, Fig. 4 gives the comparisons of the ratios $R_L=E(L^+)/E(2_1^+)$ as a function of angular momentum (L) in the ground-state band for ^{96}Ru and ^{98}Ru isotopes. We present energies of the yrast sequences using IBM-1 (normalized to the energy of their respective $2_1{}^+$ levels) in both nuclei and have compared them with previous experimental values [21]. From Fig. 4a, b, we can see that IBM-1 calculation fit the U(5) predictions generally. However, the comparison between the calculations and the experimental R_L values are increased toward a higher spin.

The positive parity yrast levels are connected by a sequence of stretched E2 transition with energies which increase smoothly except around the backbends. The transition energy $\Delta E_{I,I-2}$ should increase linearly with I for the constant rotor as $\Delta E_{I,\ I-2}=I/2\vartheta\ (4I-2)$ does not increase, but decrease for certain I values. The moment of inertia $2\vartheta/\hbar^2$ and rotational frequency $\hbar\omega$ have been calculated from Eqs. (6) and (7), respectively. The ground-state bands up to 16 and 12 units of angular momentum are investigated for a moment of inertia in even ^{96,98}Ru isotopes. The moments of inertia are plotted versus spin in Fig. 5. It is shown that $2\vartheta/\hbar^2$ as a function of spin do not change up to spin 10 theoretically as well as experimentally. $2\vartheta/\hbar^2$ as a function of the square of rotational energy in even ^{96,98}Ru nuclei are plotted in Fig. 6. In the lowest order according to variable moment of inertia (VMI) model, this should give a straight line in the plot of inertia $2\vartheta/\hbar^2$ as a function of ω^2. It is seen that the backbending behavior changes from one nucleus to another, ^{96}Ru and ^{98}Ru isotopes show back bending at $I=4^+$ and $I=8^+$, respectively. Results are presented on collective $\Delta I=2$ ground band level sequence for the variation of shapes for Ru isotopes with even neutron $N=52$ and 54. The backbending phenomena appear clearly in the diagram

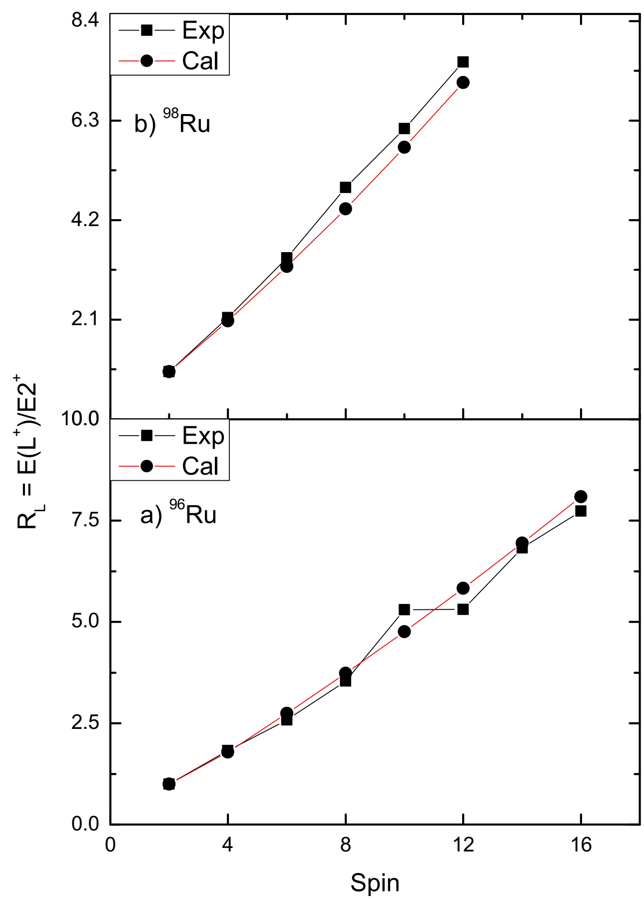


Fig. 4 Yrast ratio $R_L=E(L^+)/E(2_1^+)$ as a function of spin for **a** ^{96}Ru and **b** ^{98}Ru isotopes. $L=2, 4, 6, \ldots, 16$

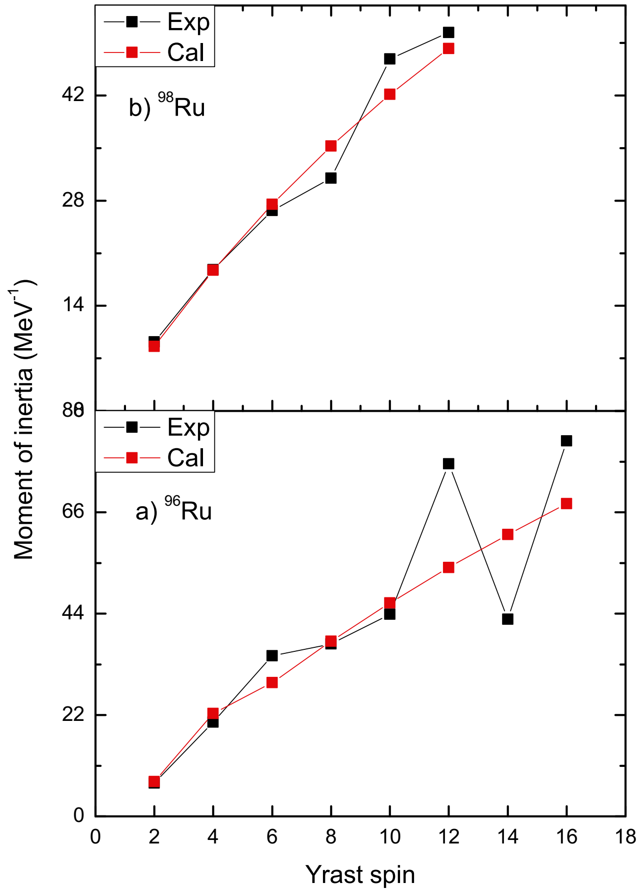


Fig. 5 Moment of inertia as a function of spin for **a** ^{96}Ru and **b** ^{98}Ru isotopes

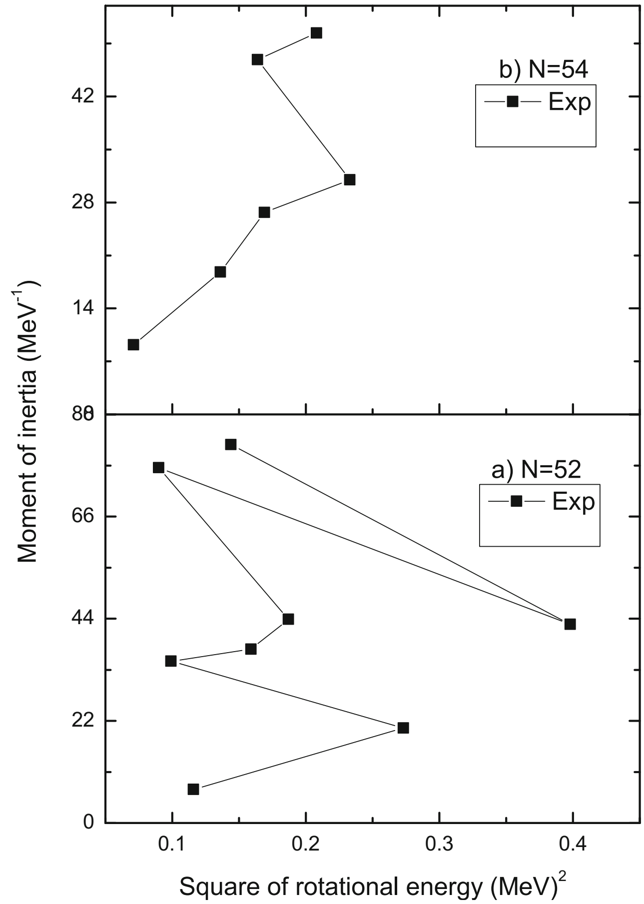


Fig. 6 Moment of inertia as a function of square of rotational energy for **a** ^{96}Ru and **b** ^{98}Ru isotopes

$2\vartheta/\hbar^2$ vs $(\hbar\omega)^2$. The backbending phenomenon can be phenomenological reproduced as an effect due to the crossing of two bands. Let us assume that two bands have different moments of inertia and cross each other in a certain region of angular momentum I. Because of the residual interaction, such a crossing does not take place, and therefore, the yrast band, which is defined as a line of states with the lowest energy for each I, changes discontinuously at the moment of inertia. Based on this definition, we may assume that if double or triple back endings exist in ^{96}Ru, the shape or the symmetry of this nucleus will change two or three times (in other words, we have bands with different moments of inertia and crossing each other many times).

One should, however, keep in mind that this kind of phenomenological description only gives a classification of the spectra and does not say anything about their physical origin. One of the theoretical interpretations of the backbending is that, in the second band, the nucleus has a different deformation, for instance, a triaxial one. This means that backbending is caused by a sudden change of deformation—typically the phase transition from axial symmetry to triaxiality.

The use of potential energy surface (PES) in IBM model is an attempt to seek microscopic and geometric roots through reflecting the three main symmetries as correspondent deformation shapes (spherical, oblate and prolate, and γ independent (γ soft)). However, the PES indicates us about the shape of nuclei, its symmetry, and the depth of the minimum and the change of the shape if it happens.

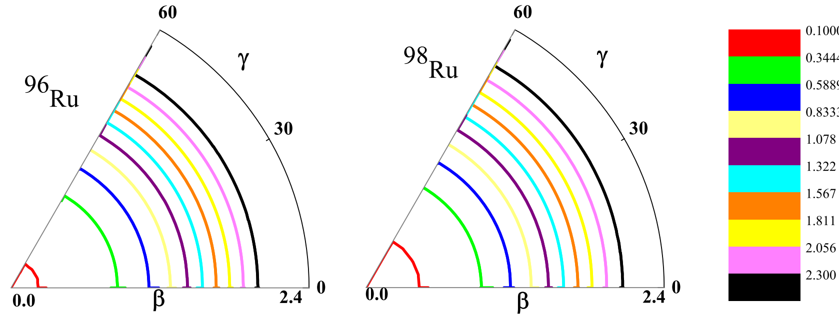
PES by the Skyrme mean field method was mapped onto the PES of the IBM Hamiltonian [22, 23]. The expectation value of the IBM-1 Hamiltonian with the coherent state $|N,\beta,\gamma\rangle$ is used to create the IBM energy surface [24]. The state is a product of the boson creation operators $b_c^\dagger$ with

$$\left|N,\beta,\gamma\right\rangle = \frac{1}{\sqrt{N!}}\left(b_c^\dagger\right)^N\left|0\right\rangle, \tag{8}$$

$$b_c^\dagger = \left(1+\beta^2\right)^{-1_2}\left\{s^\dagger + \beta\left[\cos\gamma\left(d_0^\dagger\right) + \sqrt{1/2}\sin\gamma\left(d_2^\dagger + d_{-2}^\dagger\right)\right]\right\}. \tag{9}$$

The energy surface as a function of β and γ, has been given by [1]

Fig. 7 Potential energy surfaces for even ^{96}Ru and ^{98}Ru isotopes



$$E(N,\beta,\gamma) = \frac{N\varepsilon_d\beta^2}{(1+\beta^2) + \frac{N(N-1)}{(1+\beta^2)^2(\alpha_1\beta^4 + \alpha_2\beta^3\cos\gamma + \alpha_3\beta^2 + \alpha_4)}} \quad (10)$$

where the α_is are related to the Cassimir coefficients C_L, ν_2, ν_0, u_2, and u_0 [20]. β is a measure of the total deformation of nucleus, where β= 0, the shape is spherical, and is distorted when≠0 and γ is the amount of deviation from the focus symmetry and correlates with the nucleus, if γ=0, the shape is prolate, and if γ=60, the shape becomes oblate. Then, we can rewrite Eq. (8) for U(5) limit as[20]: $E(N,\beta,\gamma) = \varepsilon_d N \frac{\beta^2}{1+\beta^2}, U(5),$

From above, the potential surfaces are approximately independent of gamma only. The contour plots in the γ-β plane resulting from $E(N,\beta,\gamma)$ are shown for ^{96}Ru and ^{98}Ru isotopes are shown in Fig. 7. In this figure, the color lines represent the potential energy surface values in the unit mega-electron-volt (MeV), and it is shown that the equilibrium shape of the nuclei is always slightly oblate, but almost spherical β_{minm}=0. Furthermore, the $E(N,\beta,\gamma)$ independent on the γ. Thus, the mapped IBM energy surfaces, which are the nuclei, are slightly oblate, but almost spherical shapes for the considered Ru nuclei.

4 Summary

The yrast states of the even-even $^{96-98}$Ru isotopes have been investigated using the interacting boson model-1. Energy levels up to 16$^+$, for ^{96}Ru, and up to 12$^+$, for ^{98}Ru, were obtained using the best-fitted values of the parameters in the IBM-1 Hamiltonian. The analyses of the calculated energies of the low-lying states with positive parity show a satisfactory agreement between the IBM-1 and the experimental data for the ground-state band. The behavior of the moment of inertia of the even-even Ru isotopes as a function of the square of the rotational angular velocity indicates the nature of backbending properties. The potential energy surfaces of the ^{96}Ru and ^{98}Ru nuclei where also calculated. Within the framework of the interacting boson approximation, the Ru isotopes studied here, with neutron numbers 52 and 54, can be considered as vibrational nuclei with symmetry close to the U(5) limit.

Acknowledgments Authors give thanks to King Abdulaziz University in providing the fund to complete the project.

References

1. A. Aritma and F. Iachello, The interacting boson model, (Cambridge Univ. Press, 1987)
2. J. Kotila, J. Suhonen, D.S. Delion, Phys. Rev. C **68**, 054322 (2003)
3. X.L. Che et al., Chin. Phys. Lett. **21**, 1904 (2004)
4. X.L. Che et al., Chin. Phys. Lett. **23**, 328 (2006)
5. Y. Luo et al., Int. J. Mod. Phys. E **18**, 1717 (2009)
6. A. Frank, P. Van Isacker, D.D. Warner, Phys. Lett. B **197**, 474 (1987)
7. D. Troltenier, J.A. Maruhm, W. Greiner, V.A. Velazquez, P.O. Hess, J.H.Z. Hamilton, Phys. A **338**, 261 (1991)
8. I.M. Ahmed et al., Int. J. Mod. Phys. E **21**, 1250101 (2012)
9. H.Y. Abdullah et al., Indian J. Phys. **87**, 571 (2013)
10. I. Hossain, M.A. Saeed, N.N.A.M.B. Ghani, H. Saadeh, M. Hussein, H.Y. Abdullah, Indian J. Phys. **88**, 5 (2014)
11. I. Hossain, H.Y. Abdullah, I.M. Ahmed, M.A. Saeed, Chin. Phys. C **38**, 024104 (2014)
12. F.I. Sharrad, H.Y. Abdullah, N. Al-Dahan, N.M. Umran, A.A. Okhunov, H. Abu Kassim, Chin. Phys. C **37**, 034101 (2013)
13. L.K. Green, Nuclear structure of ^{112}Cd through studies of β decay. A M.Sc. thesis. The University of Green, (2009)
14. O. Scholten et al., Ann. Phys. **115**, 325 (1978)
15. K.A. Al-Maqtary, Jordan J. Phys. **6**, 95 (2013)
16. F. Lachello, in *Nuclear structure*, ed. by K. Abraham, K. Allaart, A.E.L. Dieperink (Plenum, New York, 1981)
17. P. Van Isacker, The interacting boson model, nuclear structure and decay data: theory and evaluation workshop, Trieste - Italy, 04–15 April (2005)
18. V.N. Zamfir, R.F. Casten, Proc. Rom. Acad. Ser. A **4**, 1 (2003)
19. M.I.T.T.A.L. Harish Mohan, D.E.V.I. Vidya, Turk. J. Phys. **36**, 117 (2012)
20. R.F. Casten, D.D. Warner, Rev. Mod. Phys. **60**, 389 (1988)
21. B. Richard & Firestone, Table of isotopes (John Willey & Sons, 1999)
22. L.M. Robledo et al., J. Phys. G: Nucl. Part Phys. **36**, 115104 (2009)
23. K. Nomura et al., J. Phys. Conf. Ser. **267**, 012050 (2009)
24. R.F. Casten, N.V. Zamfir, Phys. Lett. **87**, 052503 (2001)