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GENERAL AND APPLIED PHYSICS

Solitonic, Periodic and Quasiperiodic Behaviors of Dust Ion Acoustic Waves in Superthermal Plasmas

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Abstract The solitonic, periodic, and quasiperiodic behaviors of dust ion acoustic waves in superthermal plasmas with q-nonextensive electrons are studied using the bifurcation theory of planar dynamical systems through direct approach. Using a *Galilean* transformation, model equations are transformed to a Hamiltonian system involving electrostatic potential. The existence of solitary and periodic waves is shown for the unperturbed Hamiltonian system. Analytical forms of these waves are presented depending on physical parameters q and μ. The effects of q and μ are studied on characteristics of nonlinear dust ion acoustic solitary and periodic waves. It is observed that parameters q and μ significantly influence the characteristics of nonlinear dust ion acoustic solitary and periodic structures. Considering an external periodic perturbation, the quasiperiodic behavior of the perturbed Hamiltonian system for dust ion acoustic waves is studied. It is seen that the unperturbed Hamiltonian system has the solitary and periodic wave solutions whereas the perturbed Hamiltonian system has quasiperiodic motion for same values of parameters q, μ and v.

Keywords Solitary wave · Periodic wave · Dusty plasma · Nonextensive electron · Bifurcation · Quasiperiodic motion

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1 Introduction

During last few decades, there is a rapidly growing interest in dusty plasma because of its significant role in astrophysical as well as space environments, such as Planetary rings, Cometary tails, Interstellar media, Mercury, *Solar wind*, and in the Magnetosphere of the Earth [1–6]. Shukla and Silin [7] investigated theoretically the existence of low frequency dust-ion-acoustic (DIA) waves in a dusty plasma with mobile ions, Boltzmann electrons, and negatively charged stationary dust particles for the first time in the literature of dusty plasma. Because of the presence of different types of dust charged grains in a plasma, many types of different wave modes are introduced, such as dust acoustic mode [8], dust ion acoustic mode [9], dust lattice mode [10], shukla-varma mode [11], dust Bernstein-Greene-Kruskal mode [12], and dust drift mode [13]. Recently, the theoretical and experimental investigations on the dust ion acoustic solitary waves (DIASWs) have been made by a large numbers of researchers [16–20]. Ghosh et al. [21] studied the head-on collision of dust acoustic solitary waves in a dusty plasma consisting of dust particles and q-nonextensive electrons. They showed the effect of the nonextensivity on the phase shift. Ghorui et al. [22] investigated the *head-on collision* of DIASWs in a magnetized quantum dusty plasma. Ashraf et al. [23] studied the basic features of obliquely propagating DIASWs in a nonextensive magnetized dusty plasma containing nonextensive electrons, inertial ions, and negatively charged stationary dust. Pakzad and Javidan [24] studied dust acoustic solitary and shock waves in strongly coupled dusty plasmas with nonthermal ions. They derived the Korteweg-de Vries-Burgers (KdV-Burgers) equation and modified KdV-Burgers equation and showed the effects of important parameters such as ion nonthermal parameter, temperature, density, and velocity on the properties of shock waves and solitary waves. Applying the reductive perturbation technique, Pakzad [25] studied the propagation of nonlinear waves in warm dusty plasmas with variable dust charge, two-temperature ions, and nonthermal electrons. Das et al. [26] studied large-amplitude double layers in a dusty plasma with an arbitrary streaming ion beam. Recently, Chatterjee et al. [27] investigated the head-on collision between two dust ion acoustic (DIA) solitons in dusty plasmas. Very recently, Ferdousi and Mamun [28] studied nonlinear dust-acoustic shock waves in a dusty plasma with negatively charged mobile dust, nonextensive electrons of two distinct temperatures, and Maxwellian ions. Alam et al. [29] investigated the nonplanar DIA shock waves in an unmagnetized dusty plasma with inertial ions, negatively charged immobile dust, and superthermal electrons of two distinct temperatures.

It is well known that Maxwell distribution is valid for the macroscopic ergodic equilibrium state, and it may be inadequate to investigate the long range interactions in unmagnetized collisionless plasma having the non-equilibrium stationary state. This type of state may exist due to a number of physical mechanisms, for examples, external force field present in natural space plasma environments, wave-particle interaction, and turbulence. Furthermore, space plasma observations clearly indicate the presence of ion and electron populations which are far away from their thermodynamic equilibrium [14, 15]. Renyi proposed a new statistical approach [31] which is based on the derivation of Boltzmann-Gibbs-Shannon (BGS) entropic measure to investigate the different cases in which Maxwell distribution is not appropriate. It was first acknowledged by Renyi [31] and afterward proposed by Tsallis [32], where the entropic index q characterizes the degree of nonextensivity of the system. The Tsallis distribution is also known as q-Gaussian, and it is basically a probability distribution which arises from the optimization of the Tsallis entropy. Tsallis [32] has described a model for nonextensivity by considering a composition law in the sense that the entropy of the composition $(A + B)$ of two independent systems A and B is equal to $S_q^{(A+B)} = S_q^{(A)} + S_q^{(B)} + (1-q)S_q^{(A)}S_q^{(B)}$, where the parameter q that underpins the generalized entropy of Tsallis is linked to the underlying dynamics of the system and presents a measure of the degree of its correlation. In statistical mechanics and thermodynamics, systems which are characterized by the property of nonextensivity are systems in which the entropy of the whole is different from the sum of the entropies of the respective parts, i.e., the generalized entropy of the whole is greater than the sum of the entropies of the parts if $q < 1$ and it is known as superextensivity, whereas the generalized entropy of the system is smaller than the sum of the entropies of the parts if $q > 1$, which is known as subextensivity. The q-entropy may represent a suitable frame for the analysis of many astrophysical scenarios, such as, stellar polytropes, solar neutrino problem, and peculiar velocity distribution of galaxy clusters. It is important to note that the q-distribution is not normalizable for $q < -1$. In the extensive limiting case, when $q \to 1$, the q-distribution transforms to the well-known Maxwell-Boltzmann velocity distribution.

Recently, Samanta et al. [33] investigated bifurcations of dust ion acoustic nonlinear waves in a magnetized dusty plasma with q-nonextensive electrons applying bifurcation theory of planar dynamical systems for the first time. A number of works [34–42] on bifurcations of nonlinear waves in plasmas were investigated through perturbative and nonperturbative approaches. Saha and Chatterjee [43] studied propagation and interaction of dust acoustic multi-soliton in dusty plasmas with q-nonextensive electrons and ions. Very recently, Saha et al. [44] investigated the dynamic behavior of ion acoustic waves in electron-positron-ion magnetoplasmas with superthermal electrons and positrons in the framework of perturbed and non-perturbed Kadomtsev-Petviashili (KP) equations. Sahu et al. [45] studied the quasi periodic behavior in quantum plasmas due to the presence of Bohm potential. Zhen et al. [46] investigated the dynamic behavior of the quantum ZK equation in dense quantum magnetoplasma. But, there is no work related to quasiperiodic motion of dust ion acoustic waves in classical plasmas to the best of our knowledge.

In this work, our aim is to study dust ion acoustic solitary waves and periodic waves in an unmagnetized plasma with q-nonextensive electrons through direct approach applying the bifurcation theory of planar dynamical systems. Two analytical solutions of the solitary and periodic waves are derived depending on the system parameters. From these solutions, we have shown the significant effects of the parameter q and μ on characteristics of dust ion acoustic solitary and periodic waves. Considering an external periodic perturbation, the quasiperiodic motion of the perturbed Hamiltonian system is investigated.

The remaining part of the paper is organized as follows: In Section 2, we consider basic equations and corresponding Hamiltonian system. In Section 3, we consider phase portrait analysis. Two analytical solutions are derived in Section 4. We present the parametric effects in Section 5. We consider quasiperiodic behavior of the perturbed Hamiltonian system in Section 6, and Section 7 is kept for conclusions.

2 Basic Equations and Hamiltonian System

In this paper, we consider a three component unmagnetized plasma which contains cold inertial ions, negatively charged stationary dust, and non-inertial electrons that follow q-nonextensive distribution. Then, at equilibrium condition, we get $n_{e0} = n_0 - Z_d n_{d0}$, where Z_d is the number of electrons residing onto the dust grain surface and n_0, n_{e0} and n_{d0} denote the equilibrium ion, electron, and dust number densities, respectively. The dynamics of nonlinear dust ion acoustic waves, whose phase speed is much smaller than the electron thermal speed and larger than the ion thermal speed, is described by the following basic equations:

$$\frac{\partial n}{\partial t} + \frac{\partial (nu)}{\partial x} = 0, \quad (1)$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -\frac{\partial \phi}{\partial x}, \quad (2)$$

$$\frac{\partial^2 \phi}{\partial x^2} = (1-\mu)n_e - n + \mu, \quad (3)$$

where $\mu = \frac{Z_d n_{d0}}{n_0}$. In order to model an electron distribution with nonextensive particles, we consider the following nonextensive electron distribution function [47]

$$f_e(v) = C_q \left\{1 + (q-1)\left[\frac{m_e v^2}{2k_B T_e} - \frac{e\phi}{k_B T_e}\right]\right\}^{\frac{1}{(q-1)}},$$

where ϕ is the electrostatic potential and the other variables or parameters obey their usual meaning. It is important to note that $f_e(v)$ is the special distribution which maximizes the Tsallis entropy and, thus, conforms to the laws of thermodynamics. Then, the constant of normalization is given below

$$C_q = n_{e0}\frac{\Gamma\left(\frac{1}{1-q}\right)}{\Gamma\left(\frac{1}{1-q} - \frac{1}{2}\right)}\sqrt{\frac{m_e(1-q)}{2\pi k_B T_e}} \text{ for } -1 < q < 1$$

and

$$C_q = n_{e0}\frac{1+q}{2}\frac{\Gamma\left(\frac{1}{q-1} + \frac{1}{2}\right)}{\Gamma\left(\frac{1}{q-1}\right)}\sqrt{\frac{m_e(q-1)}{2\pi k_B T_e}} \text{ for } q > 1.$$

Integrating $f_e(v)$ over all velocity space, we can obtain the following q-nonextensive electron number density as:

$$n_e(\phi) = n_{e0}\left\{1 + (q-1)\frac{e\phi}{k_B T_e}\right\}^{1/(q-1)+1/2}.$$

Thus, the normalized electron number density [47] takes the form:

$$n_e(\phi) = \{1 + (q-1)\phi\}^{1/(q-1)+1/2}. \quad (4)$$

where the parameter q is a real number greater than -1, and it stands for the strength of nonextensivity.

Here, n and n_e denote the number densities of the ions and electrons, respectively, normalized by equilibrium value of ion density n_0. In this case, u and ϕ are the ion fluid velocity and electrostatic potential, respectively, normalized by the ion acoustic speed $C_s = (K_B T_e/m)^{1/2}$ and $k_B T_e/e$, where e is the electron charge, k_B is the Boltzmann constant and m is the mass of ions. The time t and space variable x are normalized by the inverse of ion plasma frequency $\omega_p^{-1} = (m/4\pi e^2 n_0)^{1/2}$ and the Debye length $= (k_B T_e/4\pi e^2 n_0)^{1/2}$, respectively.

To study all dust ion acoustic traveling wave solutions of the system, we consider a *Galilean transformation* $\chi = x - vt$, where v denotes the speed of dust ion acoustic traveling wave. Applying this transformation and initial conditions $u = 0, n = 1$ and $\phi = 0$, we can easily obtain the number density of ions from (1) and (2) as

$$n = \frac{v}{\sqrt{v^2 - 2\phi}}. \quad (5)$$

Substituting (4) and (5) into (3) and considering the terms of ϕ up to second degree, we get

$$\frac{d^2\phi}{d\chi^2} = \left[\frac{(1-\mu)(1+q)(3-q)}{8} - \frac{3}{2v^4}\right]\phi^2 + \left[\frac{(1-\mu)(1+q)}{2} - \frac{1}{v^2}\right]\phi. \quad (6)$$

Then (6) is equivalent to the Hamiltonian system:

$$\begin{cases} \frac{d\phi}{d\chi} = z, \\ \frac{dz}{d\chi} = \left[\frac{(1-\mu)(1+q)(3-q)}{8} - \frac{3}{2v^4}\right]\phi^2 + \left[\frac{(1-\mu)(1+q)}{2} - \frac{1}{v^2}\right]\phi. \end{cases} \quad (7)$$

The system (7) is a planar dynamical system with parameters q, μ, and v, and the corresponding Hamiltonian function is given by:

$$H(\phi, z) = \frac{z^2}{2} - \left[\frac{(1-\mu)(1+q)(3-q)}{8} - \frac{3}{2v^4}\right]\frac{\phi^3}{3} - \left[\frac{(1-\mu)(1+q)}{2} - \frac{1}{v^2}\right]\frac{\phi^2}{2} = h, \text{ say.} \quad (8)$$

It is really important to note that the phase orbits defined by the vector fields of (7) will determine all traveling wave solutions of (6). We study the bifurcations of phase portraits of (7) in the (ϕ, z) phase plane depending on the parameters q, μ, and v. A solitary wave solution of (6) corresponds to a homoclinic orbit of (7). A periodic orbit of (7) corresponds to a periodic traveling wave solution of (6).

3 Phase Portrait Analysis

In this section, we obtain all possible periodic orbits and homoclinic orbits defined by the vector field (7) when the parameters q and v are varied. When $ab \neq 0$, then there exist two equilibrium points at $E_0(\phi_0, 0)$ and $E_1(\phi_1, 0)$, with $\phi_0 = 0$ and $\phi_1 = -\frac{a}{b}$, where $a = \left[\frac{(1-\mu)(1+q)}{2} - \frac{1}{v^2}\right]$ and $b = \left[\frac{(1-\mu)(1+q)(3-q)}{8} - \frac{3}{2v^4}\right]$. If we consider that $M(\phi_i, 0)$ is the coefficient matrix of the linearized system of (7) at an equilibrium point $E_i(\phi_i, 0)$, then one can obtain the following determinant

$$J = detM(\phi_i, 0) = -a - 2b\phi_i. \tag{9}$$

By the theory of planar dynamical systems [48, 49], we know that an equilibrium point $E_i(\phi_i, 0)$ of the planar dynamical system is a saddle point when $J < 0$ and the equilibrium point $E_i(\phi_i, 0)$ of the planar dynamical system is a center when $J > 0$.

Applying systematic analysis, we have presented different phase portraits of (7) depending on parameters q, μ, and v, shown in Figs. 1–2.

For the phase portrait given by Fig. 1, the parameters q and v connected with the relations $\frac{(1-\mu)(1+q)(3-q)}{8} < \frac{3}{2v^4}$ and $\frac{(1-\mu)(1+q)}{2} > \frac{1}{v^2}$. For the phase portrait given by Fig. 2, the parameters q and v connected with the relations $\frac{(1-\mu)(1+q)(3-q)}{8} > \frac{3}{2v^4}$ and $\frac{(1-\mu)(1+q)}{2} < \frac{1}{v^2}$.

4 Analytical Solutions

In this section, applying the planar dynamical system (7) and the Hamiltonian function (8) with $h = 0$, we obtain two analytical traveling wave solutions of (6) depending on the physical parameters q, μ, and v, which are solitary wave solution and periodic traveling wave solution.

1. Corresponding to the homoclinic orbit at the equilibrium point $E_0(\phi_0, 0)$ in Fig. 1, the system (6) has a dust ion acoustic compressive solitary wave solution:

$$\phi = -\frac{3\left[\frac{(1-\mu)(1+q)}{2} - \frac{1}{v^2}\right]}{2\left[\frac{(1-\mu)(1+q)(3-q)}{8} - \frac{3}{2v^4}\right]} \times sech^2\left(\frac{1}{2}\sqrt{\left[\frac{(1-\mu)(1+q)}{2} - \frac{1}{v^2}\right]}\chi\right). \tag{10}$$

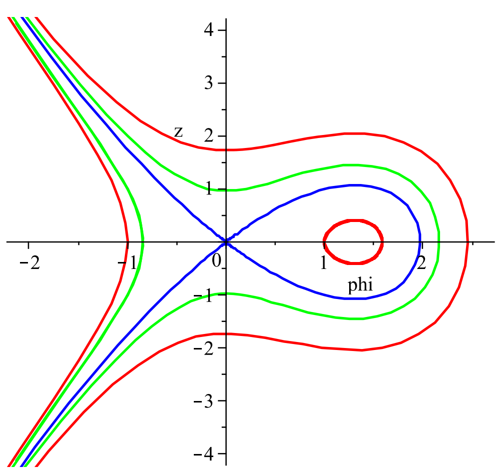

Fig. 1 Phase portrait of (7) for $q = 5.7$, $\mu = 0.34$, and $v = 2.33$

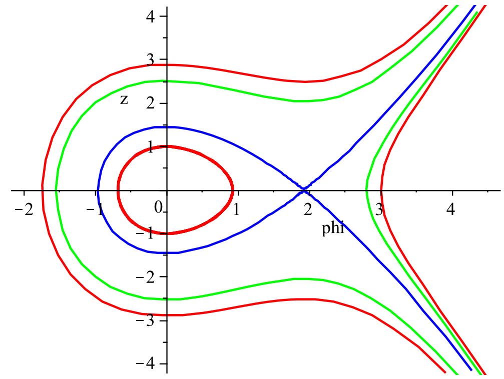

Fig. 2 Phase portrait of (7) for $q = -0.43$, $\mu = 0.5$, and $v = 2.01$

2. Corresponding to the family of periodic orbits about the equilibrium point $E_0(\phi_0, 0)$ in Fig. 2, the system (6) has dust ion acoustic periodic traveling wave solution:

$$\phi = -\frac{3\left[\frac{(1-\mu)(1+q)}{2} - \frac{1}{v^2}\right]}{2\left[\frac{(1-\mu)(1+q)(3-q)}{8} - \frac{3}{2v^4}\right]} \times sec^2\left(\frac{1}{2}\sqrt{-\left[\frac{(1-\mu)(1+q)}{2} - \frac{1}{v^2}\right]}\chi\right). \tag{11}$$

5 Effects of Parameters

In this section, we shall discuss the effects of parameters q, μ, and v on characteristics of dust ion acoustic solitary waves (Figs. 3 and 4) and periodic waves (Figs. 5 and 6).

In Fig. 3, we have shown variation of dust ion acoustic solitary wave profile for different ranges of nonextensive parameter q with fixed values of other parameters μ and v. In this case, ϕ has been plotted against χ for $\mu = 0.34$, $v = 2.33$ with $q = 4.6$ (black long dashed curve),

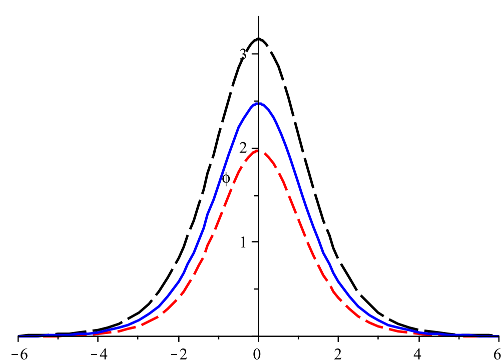

Fig. 3 Solitary wave solutions of (6) have been plotted for $\mu = 0.34$, $v = 2.33$ with $q = 4.6$ (*black long dashed curve*), $q = 5.1$ (*blue continuous curve*), and $q = 5.7$ (*red dashed curve*)

$q = 5.1$ (blue continuous curve), and $q = 5.7$ (red dashed curve). Here, we have chosen parameters in such a way that they satisfy the conditions $\frac{(1-\mu)(1+q)(3-q)}{8} < \frac{3}{2v^4}$ and $\frac{(1-\mu)(1+q)}{2} > \frac{1}{v^2}$. It is clear from Fig. 3 that amplitude and width of dust ion acoustic solitary waves decrease with increase in nonextensive parameter q. In other words, dust ion acoustic solitary wave is noticed to diminish as electrons evolve far away from their Maxwell-Boltzmann equilibrium.

In Fig. 4, we have shown variation of dust ion acoustic solitary wave profile for different values of parameter μ with fixed values of q and v. In this case, ϕ has been plotted against χ for $q = 5.7$, $v = 2.33$ with $\mu = 0.2$ (black long dashed curve), $\mu = 0.45$ (blue continuous curve), and $\mu = 0.7$ (red dashed curve). Here, we have chosen parameters in such a way that they satisfy the conditions $\frac{(1-\mu)(1+q)(3-q)}{8} < \frac{3}{2v^4}$ and $\frac{(1-\mu)(1+q)}{2} > \frac{1}{v^2}$. It is clear from Fig. 4 that amplitude of dust ion acoustic solitary wave decreases and width of dust ion acoustic solitary wave increases with increase in μ. In other words, dust ion acoustic solitary waves become smooth as the parameter μ increases.

In Fig. 5, we have shown variation of dust ion acoustic periodic traveling wave profile for different ranges of the nonextensive parameter q with fixed values of other parameters μ and v. Here, ϕ is plotted against χ for $\mu = 0.5$, $v = 2.01$ with $q = -0.43$ (*black long dashed curve*), $q = -0.38$ (*blue continuous curve*), and $q = -0.33$ (*red dashed curve*). In this case, we have chosen parameters q, μ, and v in such a way that they satisfy conditions $\frac{(1-\mu)(1+q)(3-q)}{8} > \frac{3}{2v^4}$ and $\frac{(1-\mu)(1+q)}{2} < \frac{1}{v^2}$. It is clear from Fig. 5 that amplitude and width of dust ion acoustic periodic traveling waves increase with increase in nonextensive parameter q. In other words, dust ion acoustic periodic wave is noticed to diminish as electrons evolve far away from their Maxwell-Boltzmann equilibrium.

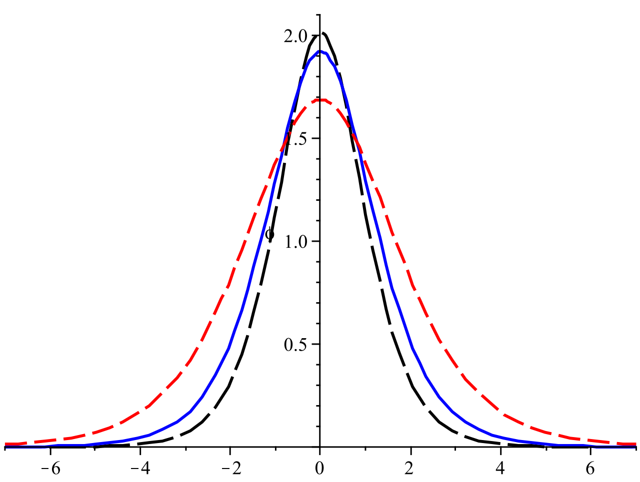

Fig. 4 Solitary wave solutions of (6) have been plotted for $q = 5.7$, $v = 2.33$ with $\mu = 0.2$ (*black long dashed curve*), $\mu = 0.45$ (*blue continuous curve*), and $\mu = 0.7$ (*red dashed curve*)

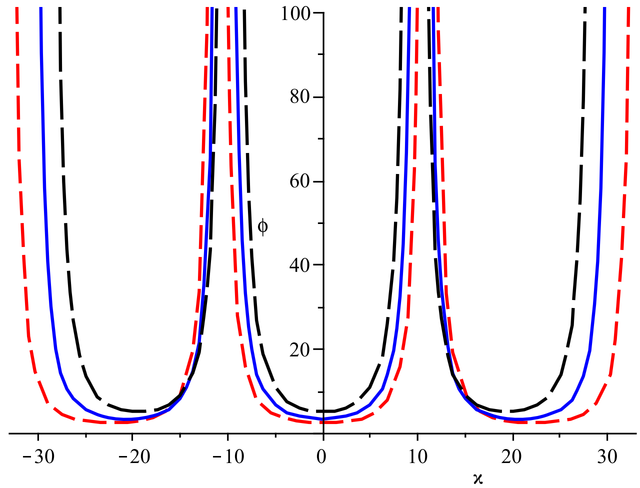

Fig. 5 Periodic traveling wave solutions of (6) have been plotted for $\mu = 0.5$, $v = 2.01$ with $q = -0.43$ (*black long dashed curve*), $q = -0.38$ (*blue continuous curve*), and $q = -0.33$ (*red dashed curve*)

In Fig. 6, we have shown variation of dust ion acoustic periodic traveling wave profile for different ranges of the parameter μ with fixed values of the other parameters q and v. Here, ϕ is plotted against χ for $q = -0.43$, $v = 2.01$ with $\mu = 0.4$ (black long dashed curve), $\mu = 0.45$ (blue continuous curve), and $\mu = 0.5$ (red dashed curve). In this case, we have chosen parameters q, μ, and v in such a way that they satisfy conditions $\frac{(1-\mu)(1+q)(3-q)}{8} > \frac{3}{2v^4}$ and $\frac{(1-\mu)(1+q)}{2} < \frac{1}{v^2}$. It is clear from the Fig. 6 that amplitude and width of dust ion acoustic periodic traveling waves decrease with increase in μ. In other words, dust ion acoustic periodic wave is noticed to diminish as the parameter μ increases.

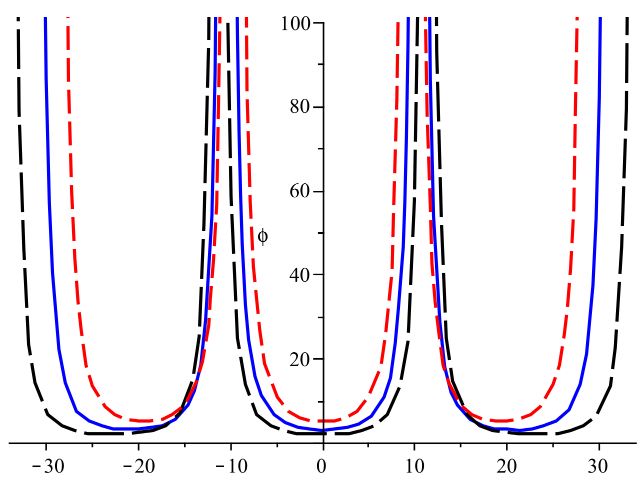

Fig. 6 Periodic traveling wave solutions of (6) have been plotted for $q = -0.43$, $v = 2.01$ with $\mu = 0.4$ (*black long dashed curve*), $\mu = 0.45$ (*blue continuous curve*), and $\mu = 0.5$ (*red dashed curve*)

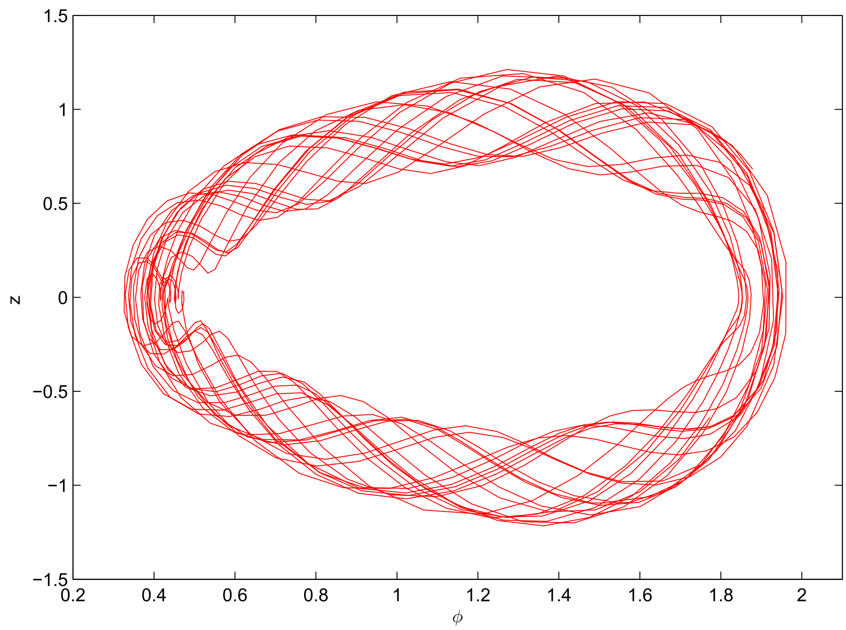

Fig. 7 Phase portrait of (12) for $q = 5.7$, $\mu = 0.34$, $v = 2.33$, $f_0 = 1.11$, and $\omega = 5.1$ (Initial condition $\phi = 0.3$, $z = 0.034$)

6 Quasiperiodic Behavior

In this section, we study the quasi-periodic behavior of the perturbed Hamiltonian system:

$$\begin{cases} \frac{d\phi}{d\chi} = z, \\ \frac{dz}{d\chi} = \left[\frac{(1-\mu)(1+q)(3-q)}{8} - \frac{3}{2v^4}\right]\phi^2 \\ \qquad + \left[\frac{(1-\mu)(1+q)}{2} - \frac{1}{v^2}\right]\phi + f_0\, cos(\omega\chi), \end{cases} \tag{12}$$

where $f_0\, cos(\omega\chi)$ is the external periodic perturbation, f_0 is the strength of the periodic perturbation and ω is the frequency. The difference between the system (7) and the system (12) is that a external periodic perturbation is added with the system (12).

In Fig. 7, we have presented the phase portrait of the perturbed system (12) for $q = 5.7$, $\mu = 0.34$, $v = 2.33$, $f_0 = 1.11$, and $\omega = 5.1$ with initial condition $\phi = 0.3, z = 0.034$. In Fig. 8, we have plotted z vs. χ with same values of

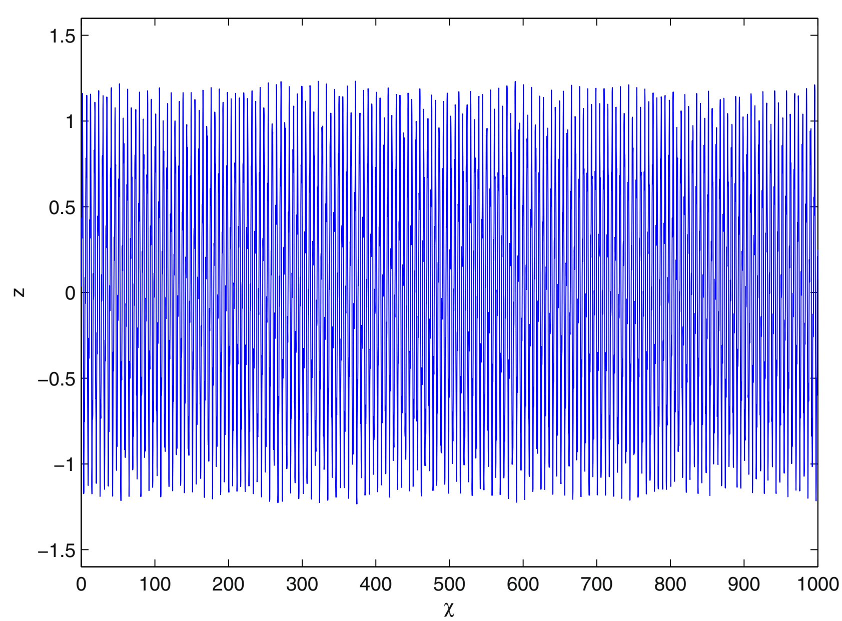

Fig. 8 Plot of z vs. χ with same values of parameters as Fig. 7

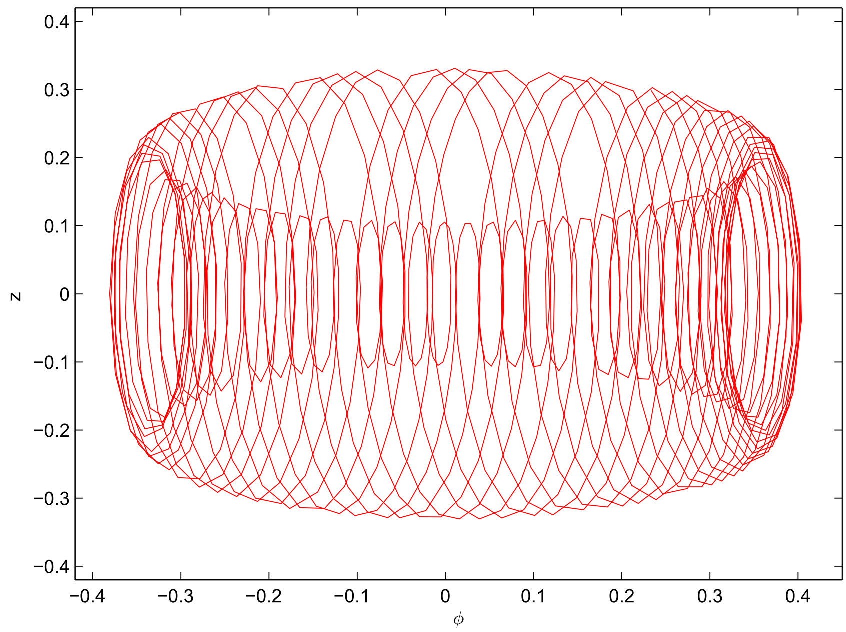

Fig. 9 Phase portrait of (12) for $q = -0.43$, $\mu = 0.5$, $v = 2.01$, $f_0 = 1.11$, and $\omega = 5.1$ (Initial condition $\phi = 0.3, z = 0.034$)

parameters as Fig. 7. In Fig. 9, we have presented phase portrait of the perturbed system (12) for $q = -0.43$, $\mu = 0.5$, $v = 2.01$, $f_0 = 1.11$, and $\omega = 5.1$ with initial condition $\phi = 0.3, z = 0.034$. In Fig. 10, we have plotted z vs. χ with same values of parameters as Fig. 9. A quasi periodic motion of the system (12) is observed with incommensurable periodic motions and the trajectory in the phase space winds around a torus filling its surface densely. It is easily seen that stable oscillatory behavior is possible in the system (12) for different values of q. The presence of slow and fast frequency components are visible. From Figs. 7–10, it is found that the perturbed system (12) has the quasi-periodic behavior but not chaotic motion in presence of an external periodic perturbation.

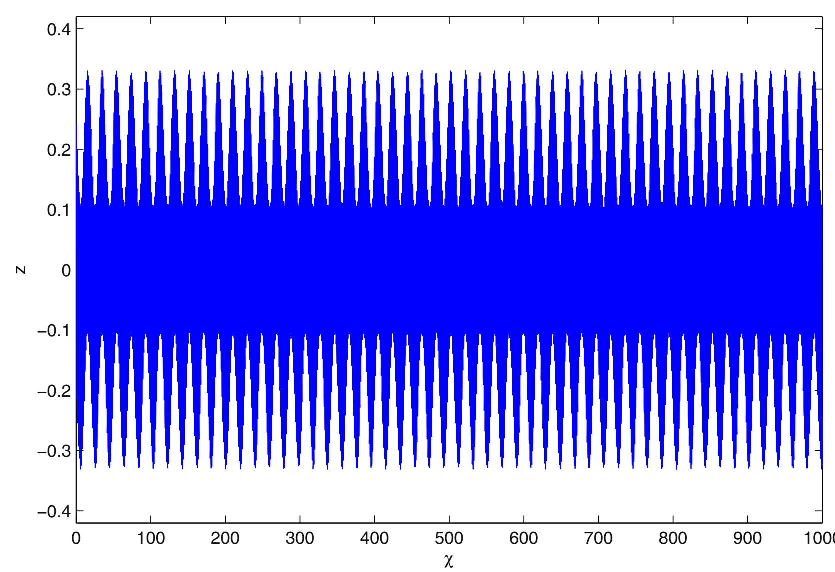

Fig. 10 Plot of z vs. χ with same values of parameters as Fig. 9

7 Conclusions

Applying bifurcation theory of planar dynamical systems, we investigate dust ion acoustic solitary and periodic waves in an unmagnetized plasma containing cold inertial ions, negatively charged stationary dust and non-inertial q-nonextensive electrons through direct approach. Using *Galilean* transformation, basic equations are reduced to a Hamiltonian system involving electrostatic potential. Existence of solitary and periodic waves is presented with the help of phase portraits analysis for the unperturbed Hamiltonian system. Analytical forms of these waves are derived depending upon physical parameters q and μ. The effects of q and μ are shown on characteristics of nonlinear dust ion acoustic solitary and periodic waves. The parameters q and μ significantly influence the characteristics of nonlinear dust ion acoustic solitary and periodic structures. Considering an external periodic perturbation, the quasiperiodic behavior of the perturbed Hamiltonian system for dust ion acoustic waves is investigated through numerical simulations. It is important to note that the unperturbed Hamiltonian system has the solitary and periodic wave solutions whereas the perturbed Hamiltonian system represents quasiperiodic motion for same values of parameters q, μ and v. Our present study could be helpful in understanding the dust ion acoustic nonlinear wave features in laboratory plasmas as well as space plasmas, where q-nonextensive electrons are present.

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