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GENERAL AND APPLIED PHYSICS

Heavy-Ion-Acoustic Solitary and Shock Waves in an Adiabatic Multi-Ion Plasma

M. A. Hossen[1] · M. M. Rahman[1] · M. R. Hossen[2] · A. A. Mamun[1]

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Abstract The standard reductive perturbation method has been employed to derive the Korteweg-deVries (K-dV) and Burgers (BG) equations to investigate the basic properties of heavy-ion-acoustic (HIA) waves in a plasma system which is supposed to be composed of nonthermal electrons, Boltzmann distributed light ions, and adiabatic positively charged inertial heavy ions. The HIA solitary and shock structures are found to exist with either positive or negative potential. It is found that the effects of adiabaticity of inertial heavy ions, nonthermality of electrons, and number densities of plasma components significantly modify the basic properties of the HIA solitary and shock waves. The implications of our results may be helpful in understanding the electrostatic perturbations in various laboratory and astrophysical plasma environments.

Keywords Heavy-ion-acoustic waves · Adiabaticity · Nonthermality · Solitary and shock waves

1 Introduction

The study of the nonlinear electrostatic waves has received a considerable attention for understanding the nonlinear phenomena in different plasma medium during the last two decades. An electron-ion (e-i) plasma is usually characterized as a fully ionized gas consisting of electrons and ions. It is widely thought that e-i plasmas have presumably appeared in the early universe, earth's environments, and are frequently encountered in active galactic nuclei, also ubiquitous in most of the astrophysical objects and space environments, such as white dwarfs, neutron stars, interstellar molecular clouds, planetary atmospheres, interstellar media, cometary tails, nebula, etc., because of their vital role in understanding the dynamics and fragmentation of interstellar molecular clouds, the star formation, the galactic structure and its evolution, etc. [1–8].

Ion acoustic (IA) waves have been investigated both theoretically and experimentally by numerous plasma physicists [9–16]. Cairns et al. [17] have considered a plasma consisting of nonthermally distributed electrons and cold ions, and shown that it is possible to obtain both positive (compressive) and negative (rarefactive) solitary waves. The IA shock structures have been experimentally observed by Anderson et al. [10]. The excitation of multiple IA shock-like structures has also been experimentally observed by Chan et al. [18] in a collisionless plasma. It is well known that in the presence of some sort of dissipative mechanism, the balance between nonlinearity and dissipation may lead to formation of the shock structures. Several different mechanisms which can be responsible for the formation of the shock waves have been studied by a number of authors in different plasma models [15, 18–22].

The heavy ions play an important role in many astrophysical phenomena and many attempts have been done by taking heavy ions as one of the components of many plasma models. Hossen et al. [23, 24] considered a plasma

✉ M. A. Hossen
armanplasma@gmail.com

1 Department of Physics, Jahangirnagar University, Savar, Dhaka-1342, Bangladesh

2 Department of Natural Sciences, Daffodil International University, Dhanmondi, Dhaka-1207, Bangladesh

system consisting of positively charged static heavy ions, degenerate cold ions, and electron fluids, and showed that HIA solitary structures are significantly modified by the number density of electron and inertial light ion fluids, and the positively charged static heavy ions. Wu et al. [25] have discussed the spontaneous generation of Alfvén waves by deriving a quasilinear theory and showed that the heavy ions in this region can play crucial roles.

Several authors have considered the effects of finite ion temperature in their plasma models [13, 26, 27] by assuming adiabatic ions and non-adiabatic electrons following different types of distributions [17, 28–30]. In a dusty plasma, Eslami et al. [31] numerically analyzed the effects of the nonplanar geometry, nonextensive parameter, dust density, and ion temperature on dust-ion acoustic (DIA) solitary waves, whose constituents are adiabatic ion fluid, nonextensive electrons, and negatively charged static dust particles. Mamun and Jahan [32] considered a dusty plasma containing adiabatic inertial electrons and ions, and negatively charged static dust, and found that the combined effects of adiabatic electrons and negatively charged static dust significantly modify the basic properties of the DIA K-dV solitons. Choi et al. [33] considered a plasma system containing nonthermal electrons and heavy ions, and found that the nonthermality of electrons determines the existence of the double layer solution. They inferred that the amplitude of the electrostatic solitary waves is enhanced by the density of heavy ions. To the best of our knowledge, no theoretical investigation has been made to study the HIA solitary and shock waves in e-i plasmas containing nonthermal electrons, Boltzmann light ions, and adiabatic positively charged inertial heavy ions. Therefore, our present attempt is to investigate the roles of adiabaticity of heavy ions and nonthermality of electrons on the propagation of HIA waves.

The manuscript is organized as follows. The model equations are provided in Section 2. The derivation of the K-dV and BG equations, and their solutions are kept in Sections 3 and 4, respectively. Numerical analysis is given in Section 5, and a brief discussion is provided in Section 6.

2 Model Equations

We consider the propagation of HIA waves in an unmagnetized, collisionless plasma containing nonthermal electrons, Boltzmann distributed light ions, and adiabatic positively charged inertial heavy ions. Thus, at equilibrium, we have $Z_i n_{i0} + Z_h n_{h0} = n_{e0}$, where n_{i0}, n_{h0}, and n_{e0} are the unperturbed ion, heavy ion, and electron number density, Z_h represents the heavy ion charge state, whereas Z_i denotes the light ion charge state.

The nonthermal distribution of electrons [17] and the Maxwell-Boltzmann distribution of ions are given by the following equations:

$$n_e = n_{e0}\left(1 - \beta\phi + \beta\phi^2\right) exp\left(\frac{e\phi}{T_e}\right), \tag{1}$$

$$n_i = n_{i0} exp\left(-\frac{e\phi}{T_i}\right), \tag{2}$$

where β is the nonthermal parameter, n_e (n_i) is the number density of the perturbed electron (ion), and T_e (T_i) is the temperatures of electron (ion) (in the energy units), respectively. The range of the nonthermal parameter β is 0.1 $\leq \beta \leq$ 0.9 [34].

We consider the propagation of a low phase speed (in comparison with the electron thermal speed) electrostatic perturbation mode, whose nonlinear dynamics is described by

$$\frac{\partial n_j}{\partial t} + \frac{\partial}{\partial x}(n_j u_j) = 0, \tag{3}$$

$$\frac{\partial u_h}{\partial t} + u_h\frac{\partial u_h}{\partial x} = -\frac{\partial \phi}{\partial x} - \frac{\sigma}{n_h}\frac{\partial p_h}{\partial x} + \eta\frac{\partial^2 u_h}{\partial x^2}, \tag{4}$$

$$\frac{\partial p_h}{\partial t} + u_h\frac{\partial p_h}{\partial x} + \gamma_j p_h\frac{\partial u_h}{\partial x} = 0, \tag{5}$$

$$\frac{\partial^2 \phi}{\partial x^2} = -n_h + \mu n_e - \alpha n_i, \tag{6}$$

where the number density of the plasma species (n_j) (j = i, e, and h; i for ion, e for electron, and h for heavy ion), heavy ion number density (n_h), heavy ion thermal pressure (p_h), heavy ion fluid speed (u_h), and electrostatic potential (ϕ) are normalized by ion/electron equilibrium number density n_{j0}, heavy ion number density at equilibrium multiplied by their charge state $Z_h n_{h0}$, $Z_h n_{h0} T_h$, effective heavy ion acoustic velocity $C_h = \sqrt{T_e/m_h}$, and the quantity T_e/e, where e is the magnitude of the electron charge. The space and time variables are normalized by the Debye radius $\lambda_D = \sqrt{T_e/4\pi Z_h n_{h0} e^2}$ and the reciprocal heavy ion plasma frequency $\omega_h{}^{-1} = (4\pi e^2 Z_h n_{h0}/m_h)^{1/2}$. Furthermore, $\sigma = (T_h/T_e)$, $\alpha = (Z_i n_{i0}/Z_h n_{h0})$, and $\mu = (n_{e0}/Z_h n_{h0})$. It should be mentioned here that for an isothermal process $\gamma_s = 1$ and $p_s = n_s$ with constant $T_s (i.e., T_s = T_{s0})$, where s is the corresponding plasma species taken as adiabatic in the corresponding model. For adiabatic ion fluid, one should consider $\gamma_s = 3$ [32].

3 Derivation of K-dV Equation

The well known reductive perturbation technique [9] is employed to derive K-dV equation for our considered plasma system. Now, we first introduce the stretched coordinates [9]

$$\zeta = \epsilon^{1/2}(x - V_p t), \tag{7}$$

$$\tau = \epsilon^{3/2} t, \tag{8}$$

where V_p is the wave phase speed (ω/k with ω being angular frequency and k being the wave number of the perturbation mode), and ϵ is a smallness parameter measuring the weakness of the dispersion ($0 < \epsilon < 1$). We then expand n_j, u_h, p_h, and ϕ in power series of ϵ, viz.,

$$n_j = 1 + \epsilon n_j^{(1)} + \epsilon^2 n_j^{(2)} + \epsilon^3 n_j^{(3)} + \cdots, \tag{9}$$

$$u_h = 0 + \epsilon u_h^{(1)} + \epsilon^2 u_h^{(2)} + \epsilon^3 u_h^{(3)} + \cdots, \tag{10}$$

$$p_h = 1 + \epsilon p_h^{(1)} + \epsilon^2 p_h^{(2)} + \epsilon^3 p_h^{(3)} + \cdots, \tag{11}$$

$$\phi = 0 + \epsilon \phi^{(1)} + \epsilon^2 \phi^{(2)} + \epsilon^3 \phi^{(3)} + \cdots. \tag{12}$$

Now, substituting the values of ζ, ϵ, and expansions (9–12) into (3–6) and collecting the terms in the different powers of ϵ, to the lowest order in ϵ, we have $n_j^{(1)} = u_j^{(1)}/V_p$, $u_h^{(1)} = \phi^{(1)}/V_p + \sigma p_h^{(1)}/V_p$, $p_h^{(1)} = \gamma\phi^{(1)}/\left(V_p^2 - \gamma\sigma\right)$, and $V_p = \sqrt{[\gamma\sigma + \frac{1}{\mu(1-\beta)+\alpha\sigma_i}]}$, where $\sigma_i = T_e/T_i$, $\mu = 1 + \alpha$, and V_p represents the dispersion relation for the HIA type electrostatic waves in an e-i plasma under consideration.

To the next higher order of ϵ, i.e., equating the coefficients of $\epsilon^{5/2}$ from (3–5) and ϵ^2 from (6), one can derive the following set of equations

$$\frac{\partial n_j^{(1)}}{\partial\tau} - V_p\frac{\partial n_j^{(2)}}{\partial\zeta} + \frac{\partial u_j^{(2)}}{\partial\zeta} + \frac{\partial}{\partial\zeta}\left[u_j^{(1)} n_j^{(1)}\right] = 0, \tag{13}$$

$$\frac{\partial u_h^{(1)}}{\partial\tau} - V_p\frac{\partial u_h^{(2)}}{\partial\zeta} + u_h^{(1)}\frac{\partial u_h^{(1)}}{\partial\zeta} + \frac{\partial\phi^{(2)}}{\partial\zeta} + \sigma\frac{\partial p_h^{(2)}}{\partial\zeta} - n_h^{(1)}\sigma\frac{\partial p_h^{(1)}}{\partial\zeta} = 0, \tag{14}$$

$$\frac{\partial p_h^{(1)}}{\partial\tau} - V_p\frac{\partial p_h^{(2)}}{\partial\zeta} + u_h^{(1)}\frac{\partial p_h^{(1)}}{\partial\zeta} + \gamma\frac{\partial u_h^{(2)}}{\partial\zeta} + \gamma p_h^{(1)}\frac{\partial u_h^{(1)}}{\partial\zeta} = 0, \tag{15}$$

$$\frac{\partial^2\phi^{(1)}}{\partial\zeta^2} = -n_h^{(2)} - \mu\beta\phi^{(2)} + \mu\phi^{(2)} + \frac{1}{2}\left\{\phi^{(1)}\right\}^2 + \alpha\sigma_i - \frac{1}{2}\alpha\sigma_i{}^2. \tag{16}$$

Now combining (13–16), we obtain an equation of the form

$$\frac{\partial\phi^{(1)}}{\partial\tau} + A\phi^{(1)}\frac{\partial\phi^{(1)}}{\partial\zeta} + B\frac{\partial^3\phi^{(1)}}{\partial\zeta^3} = 0. \tag{17}$$

Equation (17) is the well-known K-dV equation describing the nonlinear propagation of the HIA solitary waves in the e-i plasma system under consideration, where the nonlinear coefficient A and the dispersion coefficient B are given by

$$A = \frac{(V_p^2 - \gamma\sigma)^2}{2V_p}\left[\frac{3V_p^2 + \sigma\gamma^2 - 2\gamma\sigma}{(V_p^2 - \gamma\sigma)^3} - \mu + \alpha\sigma_i{}^2\right], \tag{18}$$

$$B = \frac{(V_p^2 - \gamma\sigma)^2}{2V_p}. \tag{19}$$

It is obvious from (18), that for $A > (<)\ 0$, the plasma system under consideration supports compressive (rarefactive) HIA waves that are associated with a positive (negative) potential and that no solitary waves exist at $A = 0$. It is seen that A can be either positive or negative depending on the value of V_p, α, and σ. For the stationary solitary wave solution of (17), we assumed a special coordinate (moving with speed U_0) to transform the independent variables to $\zeta = \xi - U_0\tau$. The boundary conditions imposed here are as follows, $\phi^{(1)} \to 0$, $\partial\phi^{(1)}/\partial\zeta \to 0$, and $\partial^2\phi^{(1)}/\partial\zeta^2 \to 0$ at $\zeta \to \pm\infty$. Thus, one can express the stationary solitary wave solution of the K-dV equation (17) as

$$\phi^{(1)} = \phi_m \operatorname{sech}^2\left(\frac{\xi}{\Delta}\right), \tag{20}$$

where the amplitude ϕ_m, and the width Δ are given by

$$\phi_m = \frac{3U_0}{A}, \tag{21}$$

$$\Delta = \sqrt{\frac{4B}{U_0}}. \tag{22}$$

It is obvious from (18) and (23) that the plasma system supports the HIA solitary waves with either positive ($A > 0$) or negative ($A < 0$) potential. α_c is the critical value of α above (below) which solitary waves with a positive (negative) potential exist.

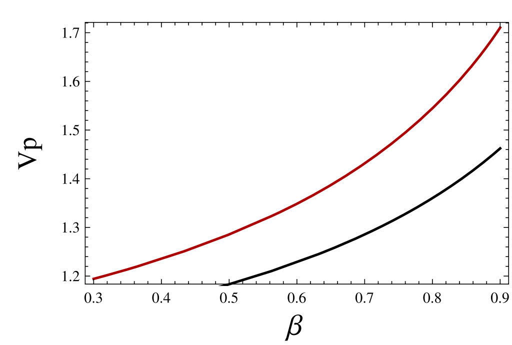

Fig. 1 Showing the variation of V_p with β for $\alpha = 0.3$ upper (*red*) curve, $\alpha = 0.5$ lower (*black*) curve, and σ_i=1.2

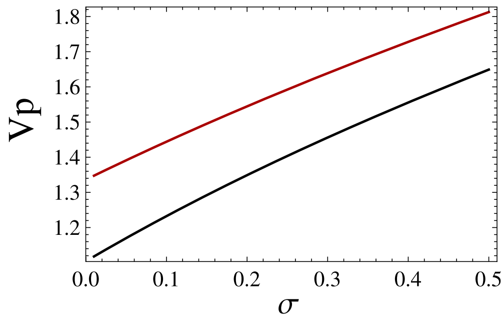

Fig. 2 Showing the variation of V_p with σ for $\alpha = 0.2$ upper (*red*) curve, $\alpha = 0.4$ lower (*black*) curve, and σ_i=1.2

4 Derivation of Burgers Equation

To derive a dynamical equation for the HIA shock waves from our basic Eqs. (1)–(4), we again applied the reductive perturbation technique [9] with another stretched coordinates [35]

$$\zeta = \epsilon(x - V_p t), \tag{23}$$
$$\tau = \epsilon^2 t. \tag{24}$$

Now, following the same procedures used in the derivation of the K-dV, we can take the first and second-order coefficients of ϵ. By applying same procedures, we found the following BG equation describing the considered dissipative medium as

$$\frac{\partial\phi^{(1)}}{\partial\tau} + A\phi^{(1)}\frac{\partial\phi^{(1)}}{\partial\zeta} = C\frac{\partial^2\phi^{(1)}}{\partial\zeta^2}, \tag{25}$$

where the nonlinear coefficient A is exactly the same appeared in (18) in simplified form, and the dissipation coefficient C is given by

$$C = \frac{\eta}{2}. \tag{26}$$

Equation (25) is the well-known BG equation describing the nonlinear propagation of the HIA shock waves in the e-i plasma system under consideration.

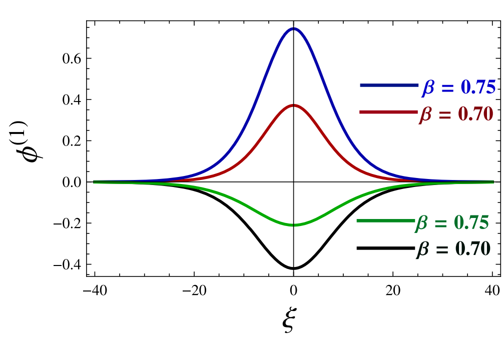

Fig. 3 Showing the positive solitary profile at $\alpha = 0.30$ and negative solitary profile at $\alpha = 0.15$ with different values of β, σ=0.01, σ_i=1.2, and U_0= 0.05

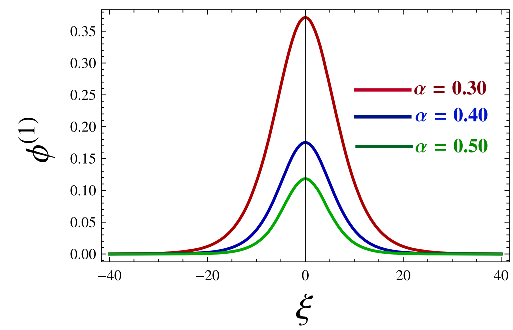

Fig. 4 Showing the variation of $\phi^{(1)}$ with different values of α, β = 0.7, σ_i=1.2, U_0= 0.05, and σ =0.05

In order to trace the influence of different plasma parameters on the propagation of electrostatic shock waves in e-i plasma, we derive the solution of Burgers Eq. (25). For the stationary shock wave solution of (25), we assumed a special coordinate (moving with speed U_0) to transform the independent variables to $\zeta = \xi - U_0\tau$. The boundary conditions imposed here are as follows, $\phi^{(1)} \to 0$, $\partial\phi^{(1)}/\partial\zeta \to 0$, and $\partial^2\phi^{(1)}/\partial\zeta^2 \to 0$ at $\zeta \to \pm\infty$. Thus, one can express the stationary shock wave solution of the BG equation (25) as,

$$\phi^{(1)} = \phi_{\mathrm{m}}\left[1 - \tanh\left(\frac{\xi}{\delta}\right)\right], \tag{27}$$

where the amplitude, $\phi_m = U_0/A$, and the width, $\delta = (2B/U_0)$.

5 Numerical Analysis

The nonthermality of electron β, the ratio of the electron to heavy ion number density μ, the heavy ion temperature to electron temperature σ, and the ratio of the ion number density to heavy ion number density α significantly modify the dispersion and dissipation properties of the HIA waves. The effects of α, β, and σ on V_p are shown in Figs. 1 and 2. The

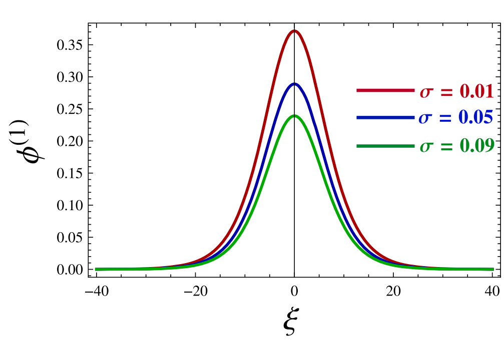

Fig. 5 Showing the variation of $\phi^{(1)}$ with different values of σ, U_0= 0.05, α=0.3, σ_i=1.2, and β = 0.7

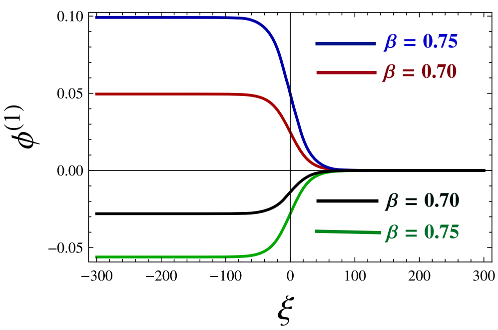

Fig. 6 Showing the positive shock profile at $\alpha = 0.30$ and negative shock profile at $\alpha = 0.15$ with different values of β, U_0= 0.01, η=0.3, σ_i=1.2, and σ =0.01

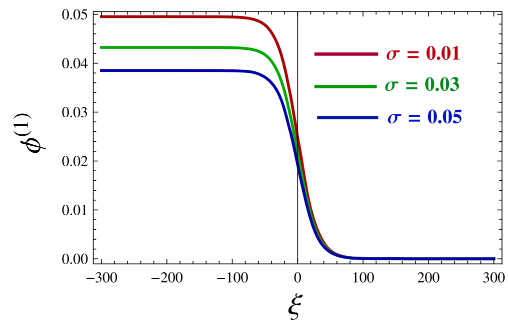

Fig. 8 Showing the variation of $\phi^{(1)}$ of shock profile with different values of σ, η=0.3, U_0= 0.01, σ_i=1.2, and α=0.3

inertia of the system increases with the increase of α, that is the reason for V_p decreasing in both cases. We graphically show in Fig. 3 that the positive (negative) solitary profile is found for $\alpha_c > 0.20$ ($\alpha_c < 0.20$). The nonthermality effect of electron is shown in Fig. 3. It is seen that with the increase of β, the amplitude $\phi^{(1)}$ increases for both negative and positive profiles. The effects of α and σ on the solitary profiles are shown in Figs. 4 and 5. With the increase of α the inertia increases, thus the amplitude of the HIA solitary waves decrease (displayed in Fig. 4). The increase of heavy ion temperature σ causes $\phi^{(1)}$ to decrease gradually, which is shown in Fig. 5. It is seen from Fig. 6 that the positive and negative profiles are also found for BG shock waves, as for K-dV solitary waves solution. It is found that with the increase of β, $\phi^{(1)}$ increases for both negative and positive shock profiles, similarly to what is seen in Fig. 3. We now investigate the effect of increasing the value of α (increasing light ion number density) on the amplitude of the shock waves, and the effect is visually shown in Fig. 7. With the increasing light ion number density, the inertia increases, thus the amplitude decreases with increasing α. The effect of σ on $\phi^{(1)}$ is illustrated in Fig. 8. It is seen that with increasing σ (heavy ion-fluid temperature), $\phi^{(1)}$ decreases gradually.

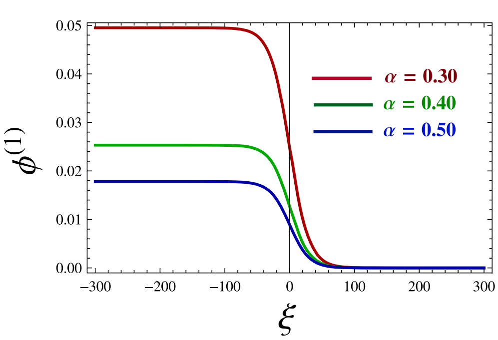

Fig. 7 Showing the variation of $\phi^{(1)}$ of positive shock profile with different values of α, η=0.3, U_0= 0.01, σ=0.05, σ_i=1.2, and β=0.7

6 Discussion

We have considered an e-i plasma (containing nonthermal electrons, Boltzmann distributed light ions, and adiabatic positively charged inertial heavy ions) and analyzed the HIA solitary and shock waves. We have numerically investigated the effects of different parameters on the basic features (viz. polarity, amplitude, and phase speed) of the HIA solitary and shock waves. The results of our present investigation can be summarized as follows:

1. The phase speed (V_p) of the HIA waves increases with increasing β but decreases with the increase of α.
2. Our considered plasma system supports small but finite amplitude HIA solitary and shock waves and have a strong dependence on different plasma parameters, particularly α, σ, β, and η.
3. The HIA solitary and shock structures are associated with either positive ($\alpha_c > 0.20$) or negative ($\alpha_c < 0.20$) potentials.
4. The amplitude of the HIA solitary and shock waves decreases with the increment of σ.

Our analysis clearly showed how the effects due to nonthermal electrons and adiabatic positively charged inertial heavy ions significantly modify the basic properties of the HIA solitary and shock waves. The ranges of plasma parameters used in our numerical analysis (α = 0.1–0.9, σ_i = 1.2–3, σ = 0.01–0.5, and β = 0.1–0.9 [34]) are relevant to both space and laboratory plasmas. This model is also precisely valid for large value of Z_h ($Z_h > 100$) [24]. Our results have an excellent similarity with those obtained in some recent works [36–38] considering the adiabatic plasma system. We can summarize our results stating that the amplitude decreases with the increase of the ion-fluid temperature σ, the amplitude also decreases with the increase of the heavy ion number density α, and the nonthermality β has a decisive influence on these types of existing solitary and shock waves. Finally, we stress that these results may be useful in investigating the collective phenomena related to the propagation of HIA waves in astrophysical plasmas as well as experimental plasmas where nonthermal energetic electrons, Boltzmann light ions, and adiabatic positively charged inertial heavy ions can be the major plasma species.

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