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GENERAL AND APPLIED PHYSICS

Dynamic Motions of Ion Acoustic Waves in Plasmas with Superthermal Electrons

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Abstract The dynamic motions of ion acoustic waves an unmagnetized plasma with superthermal (q-nonextensive) electrons are investigated employing the bifurcation theory of planar dynamical systems through direct approach. Using traveling wave transformation and initial conditions, basic equations are transformed to a planar dynamical system. Using numerical computations, all possible phase portraits of the dynamical system are presented. Corresponding to homoclinic and periodic orbits of the phase portraits, two new analytical forms of solitary and periodic wave solutions are derived depending on the nonextensive parameter q and speed v of the traveling wave. Considering an external periodic perturbation, the quasiperiodic and chaotic motions of ion acoustic waves are presented. Depending upon different ranges of nonextensive parameter q, the effect of q is shown on quasiperiodic and chaotic motions of ion acoustic waves with fixed value of v. It is seen that the unperturbed dynamical system has the solitary and periodic wave solutions, but the perturbed dynamical system has the quasiperiodic and chaotic motions with same values of parameters q and v.

Keywords Solitary wave · Periodic wave · Unmagnetized plasma · Quasiperiodic motion · Chaotic motion

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1 Introduction

Ion-acoustic waves (IAWs) have been investigated both theoretically and experimentally during the last few decades. Sagdeev [1] have considered fully nonlinear features of ion acoustic solitary waves (IASWs) using the pseudopotential technique for the first time. Later on, nonlinear ion acoustic solitary waves have received a remarkable attention both theoretically and experimentally [2–4]. It is important to note that only compressive ion-acoustic solitary waves (IASWs) involving electrostatic potential or density humps exist in unmagnetized two-component plasmas [3–5]. Depending upon the observations made by the Viking spacecraft [6] and Freja satellite [7], Cairns et al. [8] showed that the nature of IASWs may change and studied the existence of rarefactive localized structures (observed by Freja and Viking) by paying attention to the departure from the Boltzmann electron distribution to a non-thermal one. Trappert and Tagare [9, 10] extended the study of the nonlinear IASWs and investigated the effects of finite ion temperature. Schamel [11] studied ion acoustic solitary waves and presented the effects of trapped electrons on these waves.

It has been found that the high energy tails prevent strong deviation from simple Maxwellian [12, 13] for the anisotropy of the temperature and long range interactions which has been affected by the coupling between plasmas and external forces. These highly energetic particles are well known to describe the non-equilibrium stationary states, such as, long-range interactions [14] among the plasma species, global correlations [15] among the self-gravitating systems, long-time memory effects, or in a fractal-multi fractal space. These particles follow non-Maxwellian distribution function which is a particular class of Tsallis's velocity distribution [16]. This type of velocity distribution [16] is known as q-distribution, that characterizes the nonextensivity of any plasma species. Therefore, according to Tsallis' entropic principle, if A and B are two statistically independent subsystems, the q-entropy of $(A+B)$ is greater than or equal to $S_q(A)+S_q(B)$ when $q < 1$. Hence, $S_q(A+B)$ is superadditive, whereas when $q > 1$, $S_q(A+B) \leq S_q(A)+S_q(B)$, i.e., the entropy is subadditive. In the literature, the properties of superadditivity or subadditivity are frequently referred to as superextensivity or subextensivity, although they are different properties of a statistical system [17]. For this reason, the statistical mechanics spawned from Tsallis's entropic principle has become known as nonextensive (statistical mechanics). It should be noted that the q-distribution is unnormalizable when $q < -1$. In the extensive limiting case when $q \rightarrow 1$, the q-distribution reduces to the best known Maxwell-Boltzmann velocity distribution. After the seminal concept of nonextensive entropy, proposed by Renyi [18], and subsequently proposed by Tsallis [16], the subject of matter nonextensivity has been successfully employed in plasma physics [19–21]. The q-entropy may represent a suitable frame for the analysis of many astrophysical scenarios [22–25], for examples, stellar polytropes, solar neutrino problem, and peculiar velocity distribution of galaxy cluster. Saini and Shalini [26] studied nonlinear propagation of small amplitude ion acoustic solitary waves (IASWs) in a plasma with cold fluid ions and multi-temperature q-nonextensive electrons. In 2012, Saha [27] investigated topological 1-soliton solutions of the generalized Rosenau-KdV equation employing solitary wave ansatz method. Recently, many researchers [28–30] studied ion acoustic solitary waves in different plasma systems with superthermal electrons. Very recently, Ferdousi and Mamun [31] studied electrostatic shock structures in nonextensive plasma and the authors showed that there exist a critical value of nonextensivity for which shock structures transit from positive to negative potential depending on the plasma parameters. Alam et al. [32] studied ion-scale electrostatic nonplanar shock waves in dusty plasmas with two-temperature superthermal electrons. Hossen and Mamun [33] investigated the nonlinear propagation of cylindrical and spherical modified ion-acoustic (mIA) waves in an unmagnetized, collisionless, relativistic, degenerate multispecies plasma. Hossen et al. [34] studied cylindrical and spherical ion-acoustic shock waves in a relativistic degenerate multi-ion plasma. Hossen et al. [35] also studied the nonlinear propagation of one dimensional modified ion-acoustic waves in an unmagnetized electron-positron-ion (e-p-i) degenerate plasma containing relativistic electron and positron fluids, non-degenerate viscous positive ions, and negatively charged static heavy ions.

Recently, using bifurcation theory of planar dynamical systems, Samanta et al. [36] studied bifurcations of dust ion acoustic traveling waves in a magnetized dusty plasma with a q-nonextensive electron velocity distribution for the first time. Applying the same theory, a number of works [37–44] on bifurcations of nonlinear waves in plasmas have been reported through perturbative and non-perturbative approaches. Saha and Chatterjee [45] studied propagation and interaction of dust acoustic multi-soliton in dusty plasmas with q-nonextensive electrons and ions. It is important to be noted that the integrability of a system could be destroyed due to the effect of external periodic perturbations occurring in some real physical environments [46–48]. The type of the external periodic perturbation may change depending upon different physical environments. A remarkable attention is recently payed on the study of nonlinear evolution equations considering an external periodic perturbation, as a completely integrable nonlinear wave equation is unable to describe quasi-periodic or chaotic features. But the presence of an external periodic perturbation to a nonlinear integrable wave equation may lead to quasi-periodic or chaotic motions. Very recently, Saha et al. [49] reported the dynamic behavior of ion acoustic waves in electron-positron-ion magnetoplasmas with superthermal electrons and positrons on the frameworks of perturbed and non-perturbed Kadomtsev-Petviashvili (KP) equations. Sahu et al. [50] studied the quasi periodic behavior in quantum plasmas due to the presence of a Bohm potential. Zhen et al. [51] studied the dynamic behavior of the quantum Zakharov-Kuznetsov (ZK) equation in dense quantum magnetoplasma. Zhen et al. [52] also studied soliton solution and chaotic motion of the extended ZK equations in a magnetized dusty plasmas with Maxwellian hot and cold ions considering an external perturbation. In the work [53], Saha and Chatterjee presented ion acoustic solitary waves and periodic waves in an unmagnetized plasma with kappa distributed cool and hot electrons. The authors have transformed basic model equations to an ordinary differential equation involving the electrostatic potential ϕ considering the terms involving ϕ up to second degree in the Poisson equation. Using the bifurcation theory of planar dynamical systems, the authors proved the existence of compressive solitary wave solutions and periodic wave solutions depending on the homoclinic and periodic orbits in the phase portraits with two equilibrium points. They have derived two exact solutions of solitary and periodic waves, depending on the parameters, and presented the effects of the parameters density ratio (p), spectral index (κ), and temperature ratio (σ) on these solutions. But they did not study the quasiperiodic and chaotic motions of the nonlinear waves. It is really important to study quasi periodic and chaotic motions [50–52] of nonlinear waves in plasmas. That is why, in the present work, we have made an extension of the study [53] considering a two component plasma system containing cold ions and q-nonextensive electrons. In this case, we have considered the terms involving the electrostatic potential ϕ up to third degree in the Poisson equation, and obtained a dynamical system with three equilibrium points. We have presented new types of phase portraits with qualitatively different trajectories. Depending on these homoclinic and periodic orbits in the phase portraits, we have derived new analytical forms for solitary wave solutions of both compressive and rarefactive types and periodic wave solution involving Jacobian elliptic function. Furthermore, considering an external periodic perturbation, we study the quasiperiodic and chaotic behaviors of the perturbed system.

The remaining part of the paper is organized as follows: In Section 2, we consider basic equations. In Section 3, we obtain a planar dynamical system and corresponding phase portraits. New solitary and periodic wave solutions are derived in Section 4. We present quasiperiodic and chaotic behaviors of the perturbed system in Section 5, and Section 6 is kept for conclusions.

2 Basic Equations

In this work, we consider a two component unmagnetized plasma whose constituents are cold ions and q-nonextensive electrons. The normalized basic equations are as follows:

$$\frac{\partial n}{\partial t}+\frac{\partial (nu)}{\partial x}=0, \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}=-\frac{\partial \phi}{\partial x}, \tag{2}$$

$$\frac{\partial^2 \phi}{\partial x^2}=n_e-n. \tag{3}$$

In order to model an electron distribution with nonextensive particles, we use a nonextensive electron distribution function [25] given by

$$f_e(v)=C_q\{1+(q-1)[\tfrac{m_e v^2}{2T_e}-\tfrac{e\phi}{T_e}]\}^{\frac{1}{(q-1)}},$$

where ϕ denotes the electrostatic potential and the remaining variables or parameters obey their usual meaning. It is really important to note that $f_e(v)$ is the special distribution which maximizes the Tsallis entropy and, thus, conforms to the laws of thermodynamics. The constant of normalization is given by

$$C_q=n_{e0}\frac{\Gamma(\frac{1}{1-q})}{\Gamma(\frac{1}{1-q}-\frac{1}{2})}\sqrt{\frac{m_e(1-q)}{2\pi T_e}} \text{ for } -1<q<1,$$

and

$$C_q=n_{e0}\frac{1+q}{2}\frac{\Gamma(\frac{1}{q-1}+\frac{1}{2})}{\Gamma(\frac{1}{q-1})}\sqrt{\frac{m_e(q-1)}{2\pi T_e}} \text{ for } q>1.$$

Integrating $f_e(v)$ over all velocity space, we obtain the following nonextensive electron number density as:

$$n_e(\phi)=n_{e0}\{1+(q-1)\frac{e\phi}{T_e}\}^{1/(q-1)+1/2}.$$

Therefore, the normalized electron number density [25] is given by

$$n_e(\phi)=\{1+(q-1)\phi\}^{1/(q-1)+1/2}. \tag{4}$$

where the parameter q is a real number greater than -1, and it stands for the strength of nonextensivity.

Here, n and n_e denote the number densities of the ions and electrons, respectively, normalized by their unperturbed densities n_0 and n_{e0}. In this case, u and ϕ are the ion velocity and electrostatic potential, respectively, normalized by the ion acoustic speed $c=(T_e/m)^{1/2}$ and T_e/e, where e is the electron charge and m is the mass of ions. The time t and space variable x are normalized by inverse of ion plasma frequency $\omega^{-1}=(m/4\pi e^2 n_0)^{1/2}$ and the Debye length $\lambda=(T_e/4\pi e^2 n_0)^{1/2}$, respectively.

3 Formation of Dynamical System and Phase Portraits

In this section, we consider the traveling wave transformation $\xi=x-vt$, where v is the speed of the traveling wave. Using this transformation, we transform our model equations into a planar dynamical system, and we shall consider all possible phase portraits of the system. Using the variable ξ and the initial condition $u=0, n=1$ and $\phi=0$ in (1) and (2), one can easily obtain

$$n=\frac{v}{\sqrt{v^2-2\phi}}. \tag{5}$$

Substituting (4) and (5) into (3) and considering the terms of ϕ up to third degree, we obtain

$$\frac{d^2\phi}{d\xi^2}=a\phi+b\phi^2+c\phi^3, \tag{6}$$

where $a=\frac{(1+q)}{2}-\frac{1}{v^2}$, $b=\frac{(1+q)(3-q)}{8}-\frac{3}{2v^4}$, and $c=\frac{(1+q)(3-q)(5-3q)}{48}-\frac{5}{2v^4}$.

Then (6) is equivalent to the following dynamical system:

$$\begin{cases} \frac{d\phi}{d\xi}=z, \\ \frac{dz}{d\xi}=a\phi+b\phi^2+c\phi^3, \end{cases} \tag{7}$$

It is well known from classical mechanics that a system of planar equations $\frac{d\phi}{d\xi}=f_1(\phi,z)$, $\frac{dz}{d\xi}=f_2(\phi,z)$ is called a Hamiltonian system provided there exist a function $H(\phi,z)$ such that $f_1=\frac{\partial H}{\partial z}$ and $f_2=-\frac{\partial H}{\partial \phi}$. A necessary and sufficient condition for a planar system $\frac{d\phi}{d\xi} = f_1(\phi, z)$, $\frac{dz}{d\xi} = f_2(\phi, z)$ to be Hamiltonian is that $\frac{\partial f_1}{\partial \phi} + \frac{\partial f_2}{\partial z} = 0$.

The system (7) is a planar Hamiltonian system with the Hamiltonian function:

$$H(\phi, z) = \frac{z^2}{2} - a\frac{\phi^2}{2} - b\frac{\phi^3}{3} - c\frac{\phi^4}{4} = h, \tag{8}$$

which will be denoted as $H(\phi, z) = h$.

The system (7) is a planar dynamical system with parameters q and v. It is important to note that the phase orbits defined by the vector fields of (7) will determine all traveling wave solutions of (6). We study the bifurcations of phase portraits of (7) in the (ϕ, z) phase plane depending on the parameters q and v. A solitary wave solution of (6) corresponds to a homoclinic orbit of (7). A periodic orbit of (7) corresponds to a periodic traveling wave solution of (6).

We investigate the bifurcation set and phase portraits of the planar Hamiltonian system (7). Clearly, on the (ϕ, z) phase plane, the abscissas of equilibrium points of system (7) are the zeros of $f(\phi) = \phi(\phi^2 + \frac{b}{c}\phi + \frac{a}{c})$. Let $E_i(\phi_i, 0)$ be an equilibrium point of the dynamical system (7), where $f(\phi_i) = 0$. For $b^2 - 4ac > 0$, there exist three equilibrium points at $E_0(\phi_0, 0)$, $E_1(\phi_1, 0)$ and $E_2(\phi_2, 0)$, where $\phi_0 = 0$, $\phi_1 = \frac{-b+\sqrt{b^2-4ac}}{2c}$, and $\phi_2 = \frac{-b-\sqrt{b^2-4ac}}{2c}$. If $M(\phi_i, 0)$ is the coefficient matrix of the linearized system of the traveling system (7) at an equilibrium point $E_i(\phi_i, 0)$, then we get

$$J = det M(\phi_i, 0) = -cf'(\phi_i). \tag{9}$$

By the theory of planar dynamical systems ([54, 55]), it is known that an equilibrium point $E_i(\phi_i, 0)$ of the planar dynamical system (7) is a saddle point when $J < 0$ and it will be a center when $J > 0$.

In the work [53], the authors considered the terms of ϕ up to second degree in the Poisson equation. As a result, their dynamical system had two equilibrium points. Depending upon the parameters p, σ, κ and v in the work [53], they obtained phase portraits with qualitatively different trajectories. Because of two equilibrium points in the system, they found one homoclinic orbit at the saddle point and a family of periodic orbits about the center. But, in the present work, the dynamical system (7) has three equilibrium points with either two centers and one saddle point or one center and two saddle points. When the phase portrait has two centers and one saddle point, we get a pair of homoclinic orbits (see Figs. 2, 3, 4) at the saddle point and two families of periodic orbits about the two centers. On the other hand, if the phase portrait has one center and two saddle points, we get a homoclinic orbit (see Fig. 1) at a particular saddle point, and a family of periodic orbits about the center. If one considers $c = 0$ in the dynamical system (7), then one can obtain similar results as in the work [53]. Thus, it is seen that the inclusion of third degree terms in the Poisson equation represents a qualitative change in the dynamics of the dynamical system (7) described by the Hamiltonian function (8).

Applying systematic analysis of the physical parameters q and v, we have presented all phase portraits of the system (7) in the Figs. 1–4.

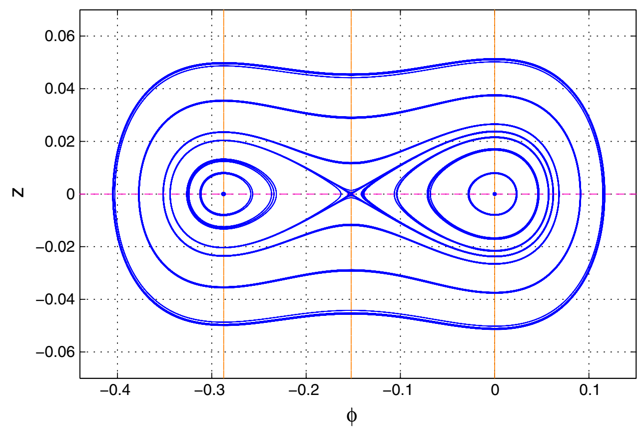

Fig. 2 Phase portrait of (7) for $q = 0.8$ and $v = 1$

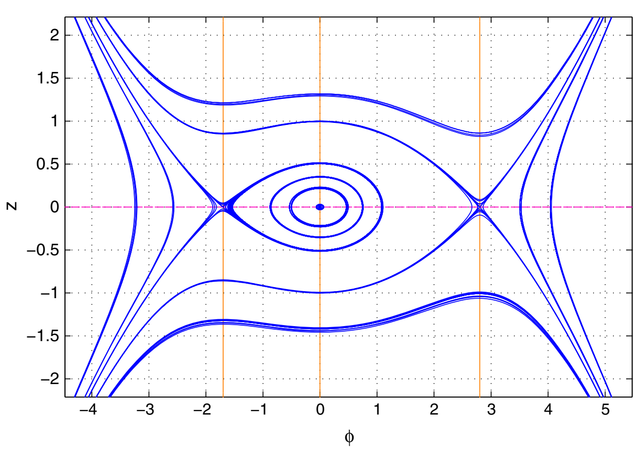

Fig. 1 Phase portrait of (7) for $q = -0.8$ and $v = 1.8$

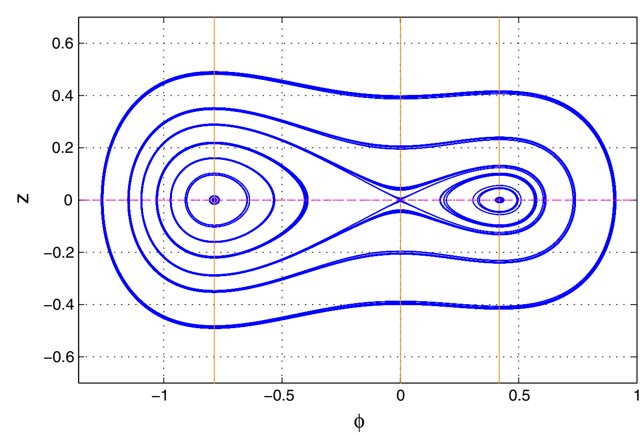

Fig. 3 Phase portrait of (7) for $q = 0.8$ and $v = 1.2$

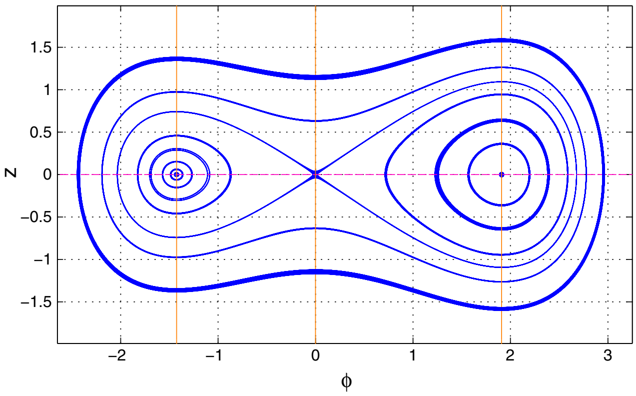

Fig. 4 Phase portrait of (7) for $q = 1.2$ and $v = 1.4$

(i) When $c > 0, ac < 0, bc < 0, b^2 - 4ac > 0$ and $2b^2 - 9ac > 0$, then the system (7) has three equilibrium points at $E_0(\phi_0, 0)$, $E_1(\phi_1, 0)$ and $E_2(\phi_2, 0)$, with $\phi_2 < 0 < \phi_1$, where $E_1(\phi_1, 0)$ and $E_2(\phi_2, 0)$ are saddle points, and $E_0(\phi_0, 0)$ is a center. There is a homoclinic orbit at $E_2(\phi_2, 0)$ enclosing the center at $E_0(\phi_0, 0)$ (see Fig. 1).

(ii) When $c < 0, ac > 0, bc > 0, b^2 - 4ac > 0$ and $2b^2 - 9ac < 0$, then the system (7) has three equilibrium points at $E_0(\phi_0, 0)$, $E_1(\phi_1, 0)$ and $E_2(\phi_2, 0)$, with $\phi_1 < \phi_2 < 0$, where $E_0(\phi_0, 0)$ and $E_1(\phi_1, 0)$ are centers, and $E_2(\phi_2, 0)$ is a saddle point. There is a pair of homoclinic orbits at $E_2(\phi_2, 0)$ enclosing the centers at $E_0(\phi_0, 0)$ and $E_1(\phi_1, 0)$ (see Fig. 2).

(iii) When $c < 0, ac < 0, bc > 0$, and $b^2 - 4ac > 0$, then the system (7) has three equilibrium points at $E_0(\phi_0, 0)$, $E_1(\phi_1, 0)$ and $E_2(\phi_2, 0)$, with $\phi_1 < 0 < \phi_2$, where $E_1(\phi_1, 0)$ and $E_2(\phi_2, 0)$ are centers, and $E_0(\phi_0, 0)$ is a saddle point. There is a pair of homoclinic orbits at $E_0(\phi_0, 0)$ enclosing the centers at $E_1(\phi_1, 0)$ and $E_2(\phi_2, 0)$ (see Fig. 3).

(iv) When $c < 0, ac < 0, bc < 0$, and $b^2 - 4ac > 0$, then the system (7) has three equilibrium points at $E_0(\phi_0, 0)$, $E_1(\phi_1, 0)$ and $E_2(\phi_2, 0)$, with $\phi_1 < 0 < \phi_2$, where $E_1(\phi_1, 0)$ and $E_2(\phi_2, 0)$ are centers, and $E_0(\phi_0, 0)$ is a saddle point. There is a pair of homoclinic orbits at $E_0(\phi_0, 0)$ enclosing the centers at $E_1(\phi_1, 0)$ and $E_2(\phi_2, 0)$ (see Fig. 4).

4 Analytical solutions

In this section, we present solitary wave solutions and periodic wave solutions with the help of the dynamical system (7) and the Hamiltonian function (8). It should be noted that $sn(\Omega\xi, k)$ is the Jacobian elliptic function [56] with the modulo k.

(i) Corresponding to family of periodic orbits about $E_0(\phi_0, 0)$ in Fig. 1, the system (6) has a family of periodic wave solutions:

$$\phi(\xi) = \frac{\delta_1(\beta_1 - \gamma_1)sn^2(\Omega\xi, k) - \gamma_1(\beta_1 - \delta_1)}{(\beta_1 - \gamma_1)sn^2(\Omega\xi, k) - (\beta_1 - \delta_1)}, \quad (10)$$

where $\Omega = \sqrt{-\frac{c}{8}(\beta_1 - \delta_1)(\gamma_1 - \alpha_1)}$, $k = \sqrt{\frac{(\alpha_1-\delta_1)(\beta_1-\gamma_1)}{(\alpha_1-\gamma_1)(\beta_1-\delta_1)}}$, and, α_1, β_1, γ_1 and δ_1 are roots of the equation $h + \frac{c}{4}\phi^4 + \frac{b}{3}\phi^3 + \frac{a}{2}\phi^2 = 0$, with $\alpha_1 > \beta_1 > \gamma_1 > \delta_1$, $h \in (h_2, 0)$.

(ii) Corresponding to pair of homoclinic orbits at $E_0(\phi_0, 0)$ in Fig. 3, the system (6) has both compressive and rarefactive solitary wave solutions:

$$\phi(\xi) = \pm \frac{1}{\sqrt{2(1 - \frac{b^2}{9ac})\, sin(2\sqrt{\frac{a}{c}}\xi) + \frac{b}{6a}}}. \quad (11)$$

In the work [53], the authors derived compressive solitary wave solution involving $sech^2\xi$ corresponding to the homoclinic orbit at the saddle point (see Fig. 4 in [53]) and periodic wave solutions involving $sec^2\xi$ corresponding to the periodic orbits about the center (see Fig. 2 in [53]) of the dynamical system. But, in this work, we derive both compressive and rarefactive solitary wave solutions (11) corresponding to the pair of homoclinic orbits at the saddle point $E_0(\phi_0, 0)$ in Fig. 3 and a family of periodic wave solutions involving Jacobian elliptic function $sn^2(\Omega\xi, k)$ corresponding to the family of periodic orbits about the center $E_0(\phi_0, 0)$ in Fig. 1.

It should be noted that corresponding to the homoclinic orbit at $E_2(\phi_2, 0)$ in Fig. 1, one can obtain solitary wave solution of the system. Corresponding to the pair of homoclinic orbits at $E_0(\phi_0, 0)$ in Fig. 4, one can obtain a pair of solitary wave solutions of the system, same as the solutions (11). Similarly, one can obtain solitary and periodic

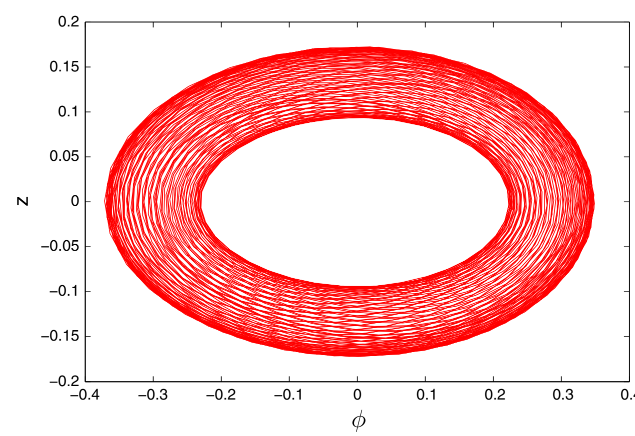

Fig. 5 Phase portrait of the perturbed system (12) for $q = -0.8$, $v = 1.8$, $f_0 = 0.01$, and $\omega = 0.6$, with initial condition $\phi = 0.11$, $z = 0.13$

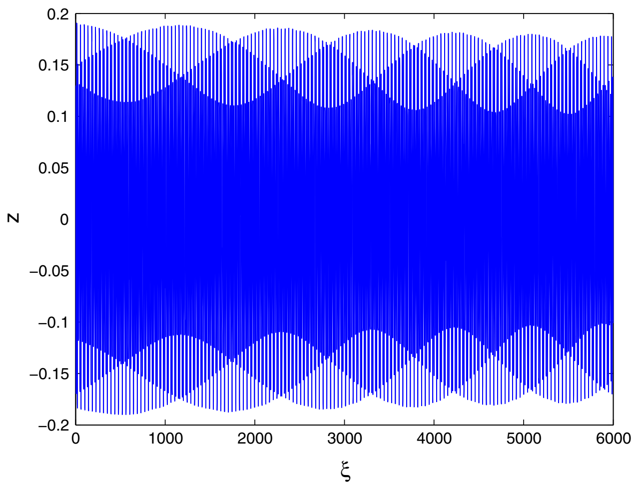

Fig. 6 Plot of z vs. ξ for same set of values of parameters as Figure 5

wave solutions of the system corresponding to homoclinic and periodic orbits of the system (7).

5 Quasiperiodic and Chaotic motions

In this section, we will present the quasiperiodic and chaotic motions of the perturbed dynamical system:

$$\begin{cases} \frac{d\phi}{d\xi} = z, \\ \frac{dz}{d\xi} = a\phi + b\phi^2 + c\phi^3 + f_0\, cos(\omega\xi), \end{cases} \tag{12}$$

where $f_0\, cos(\omega\xi)$ is an external periodic perturbation, f_0 is the strength of the external perturbation, and ω is the frequency. The difference between the system (12) and the system (7) is that a external periodic perturbation is added to the system (7).

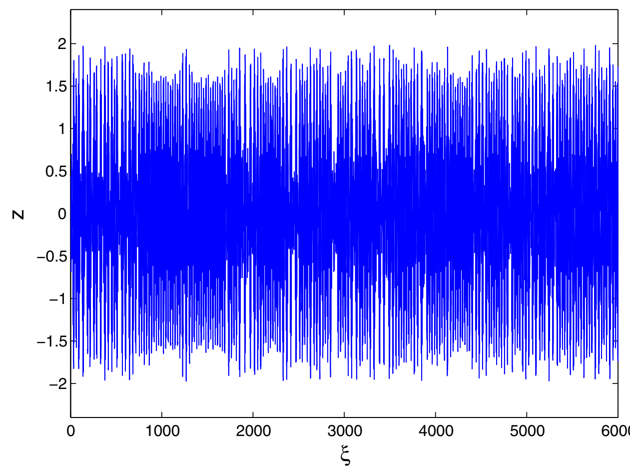

Fig. 8 Plot of z vs. ξ for same set of values of parameters as Fig. 7

In Fig. 5, we have presented the phase portrait of the perturbed dynamical system (12) for $q = -0.8$, $v = 1.8$, $f_0 = 0.01$ and $\omega = 0.6$. In Fig. 6, we have plotted z vs. ξ for the perturbed system (12) with same values of parameters as Fig. 5. It is clear that the perturbed system (12) shows quasiperiodic behavior for special value of q in the range $-1 < q < 0$. In Fig. 7, we have presented the phase portrait of the perturbed dynamical system (12) for $q = 0.8$, $v = 1.2$, $f_0 = 0.24$, and $\omega = 0.81$. In Fig. 8, we have plotted z vs. ξ for the perturbed system (12) with same values of parameters as Fig. 7. In Fig. 9, we have presented the phase portrait of the perturbed dynamical system (12) for $q = 1.2$, $v = 1.4$, $f_0 = 1.17$, and $\omega = 1.97$. In Fig. 10, we have plotted z vs. ξ for the perturbed system (12) with same values of parameters as Fig. 9. It is found that the perturbed system (12) shows chaotic motions for some special values of q in the ranges $0 < q < 1$ and $q > 1$. Furthermore, the developed chaotic motions occur (see Figs. 7–10), and

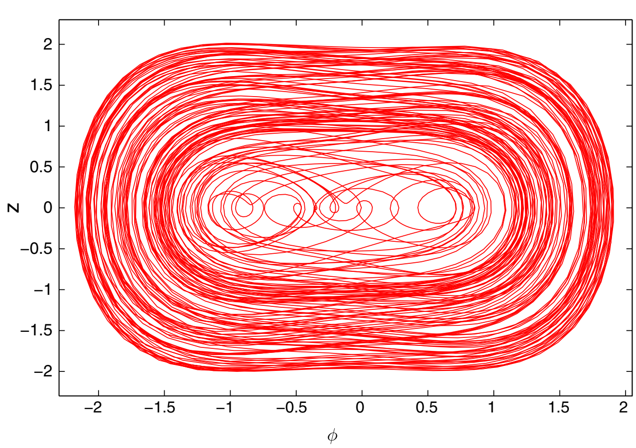

Fig. 7 Phase portrait of the perturbed system (12) for $q = 0.8$, $v = 1.2$, $f_0 = 0.24$, and $\omega = 0.81$, with initial condition $\phi = -0.17$, $z = 0.13$

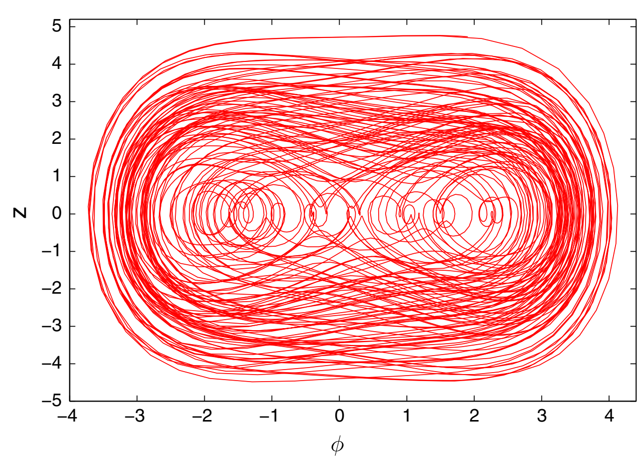

Fig. 9 Phase portrait of the perturbed system (12) for $q = 1.2$, $v = 1.4$, $f_0 = 1.17$ and $\omega = 1.97$, with initial condition $\phi = 1$, $z = 0.18$

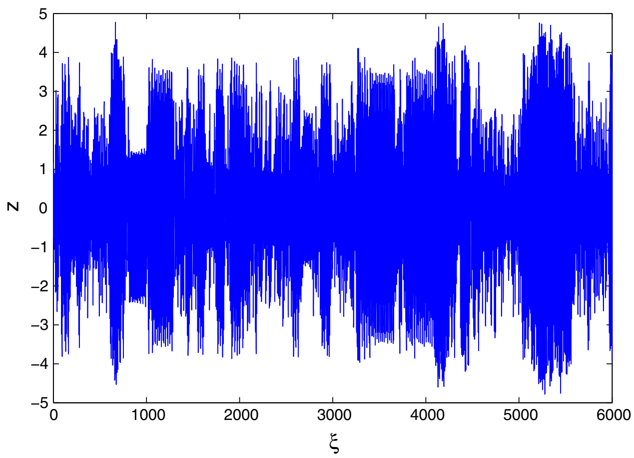

Fig. 10 Plot of z vs. ξ for same set of values of parameters as Fig. 9

the solutions ignore the periodic motions and represent random sequences of uncorrelated oscillations. It is easily seen that the quasiperiodic and chaotic behaviors of ion acoustic waves are visible in the perturbed system (12) for some special values of q in different regimes.

6 Conclusions

We have addressed the dynamic motions of ion-acoustic waves in an unmagnetized plasma with q-nonextensive electrons and cold ions through direct approach. We have presented the solitonic, periodic solutions for the unperturbed dynamical system and quasiperiodic and chaotic motions for the perturbed dynamical system. By applying the bifurcation theory of planar dynamical systems, we have derived two new analytical forms of solitary and periodic wave solutions depending on parameters q and v. It has been observed that the system has both compressive and rarefactive ion-acoustic solitary wave solutions. Considering an external periodic perturbation, the quasiperiodic and chaotic motions of ion acoustic waves have been presented. Depending upon different regimes of the nonextensive parameter q, we have shown the effect of q on the quasiperiodic and chaotic motions of ion acoustic waves. It is found that the perturbed system has the quasiperiodic behavior for some special values of q in the range $-1 < q < 0$, and chaotic behaviors in the ranges $0 < q < 1$ and $q > 1$. The results of this study may be helpful to explain the solitonic, periodic, quasiperiodic, and chaotic features of the nonlinear collective processes associated with highly energetic q-nonextensive particles in space plasmas as well as in laboratory plasmas.

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