Brazilian Journal of Physics
ISSN: 0103-9733
luizno.bjp@gmail.com
Sociedade Brasileira de Física
Brasil

Zhou, Li
On Inverting the Heat Flow with Engineering Materials
Brazilian Journal of Physics, vol. 46, núm. 1, febrero, 2016, pp. 35-40
Sociedade Brasileira de Física
Sâo Paulo, Brasil

Available in: http://www.redalyc.org/articulo.oa?id=46443233005

How to cite
Complete issue
More information about this article
Journal's homepage in redalyc.org

Scientific Information System
Network of Scientific Journals from Latin America, the Caribbean, Spain and Portugal
Non-profit academic project, developed under the open access initiative

Braz J Phys (2016) 46:35–40
DOI 10.1007/s13538-015-0378-z

CONDENSED MATTER

On Inverting the Heat Flow with Engineering Materials

Li Zhou [1,2]

Received: 3 August 2015 / Published online: 5 November 2015
© Sociedade Brasileira de Física 2015

Abstract Transformation thermodynamics enriches our understanding of heat flow and makes it possible to manipulate the heat flow at will, like shielding, concentrating and inverting. The inverting of heat flow is the extreme one, which has not been studied specifically yet. In this study we firstly inverted the heat flow by transformation thermodynamics and provided the formula for the transformed thermal conductivity. Finite element simulations were conducted to realize the steady and non-steady inverting of heat flow, based on the eccentric-semi-ring structures with natural materials. To do the inverting of heat flow, a simple "L"-shape conductive structure was proposed and verified with an infrared camera. It is concluded that inverting heat flow can be done by both complex engineering materials and some simple structures.

Keywords Transformation thermodynamics · Thermal cloak · Heat flow · Metamaterials

1 Introduction

With the pioneering work of Pendry et al. and Leonhardt in 2006, the transformation optics grows a lot and has turned into a subject of great interest since it not only provides a general method in coordinate transformation but also provides an effective design tool for manipulating electromagnetic (EM) wave [1–4]. In the past few years, transformation optics has been well developed beyond the domain of EM wave to many extensions in other fields, including acoustic wave [5, 6], liquid wave [7], matter wave [8], and elastic wave [9], etc.

However, the manipulation/control of heat flow, not like the EM wave, is still in its infancy since heat propagates are in a diffusive way. The ability to accurately manipulate/control the heat flow has important implications beyond scientific curiosity, and can potentially lead to the development of thermal analogues of electronic transistors, rectifiers, and diodes. In 2012, the concept of transformation thermodynamics [10] that origins from the transformation optics, has attracted attention and interests since then. The differences between the transformation thermodynamics and transformation optics are obvious. From a theoretical point of view, heat is not a wave in contrast to EM waves. Heat conduction is governed by parabolic differential equations, underlying that heat transfer is distinctly different from the hyperbolic differential equations of wave propagation. For instance, the basic phenomena such as reflection, scattering, polarization, or interference are common to waves but do not occur in thermodynamics at all. In contrast to the optical invisible cloak, the corresponding outcomes of transformation thermodynamics mainly include thermal cloak [10–16] and thermal concentrator [10]. In optical invisible cloak, any objects that are placed in the hidden region are undetectable from the outside. While heat still diffuses into the hidden region of the thermal cloak, only the temperature gradient is eliminated. In the thermal concentrator, heat is tuned to converge in the hidden region so as to do the local heating or energy harvesting. Both the thermal cloak and thermal concentrator can be done by pre-engineering metamaterials or even by homogeneous

✉ Li Zhou
zhouli_hust@126.com

1 Department of Mechatronics Engineering, Wuhan Business University, Wuhan 430056, China

2 School of Civil Engineering and Mechanics, Huazhong University of Science and Technology, Wuhan 430074, China

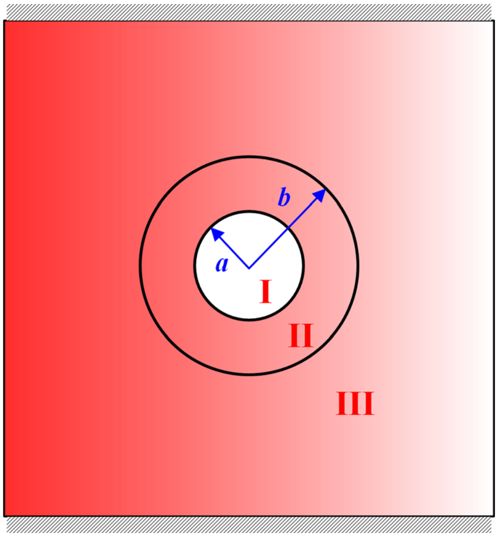

Fig. 1 Schematic of cylindrical coordinate transformation. The *left* and *right boundaries* denote the high and low temperature

natural available materials with special-designed structures [11, 12]. Experimental achievements of such thermal cloak or thermal concentrators have been reported recently. With transformation thermodynamics we can artificially control the heat flow at will. Some new heat transfer phenomena, like shielding, concentrating, or even inverting the heat flow, become possible, which were hard to imagine in the past [17]. Among these new phenomena, inverting the heat flow is the extreme one, which may invoke the concept of apparent thermal conductivity. The current studies on this phenomenon are limited on theory or not deep enough to understand/control the inverting heat flow. How we can do and control this phenomenon remains a task to be further studied.

Here, we first discuss the phenomenon of inverting heat flow with mathematic description and then describe the phenomenon by finite element simulations with engineering materials. The transient experiments with an infrared camera were conducted by a simple conductive structure. Detailed discussions and explanations are presented.

2 Transformation Thermodynamics

In the absence of additional heat sources, heat flow diffuses from a region of high temperature to a region of low temperature spontaneously. Without heat source, the transient heat flow is governed by Fourier's law as

$$\nabla\cdot(-\kappa\nabla T) + \rho c\frac{\partial T}{\partial t} = 0, \tag{1}$$

where κ is the thermal conductivity, T the temperature, ρ the density, and c the specific heat. Similar to the invariance of Maxwell equations in the transformation optics, the heat conduction equations also possess the form-invariant characteristic under coordinate transformations by varying the thermal conductivity and specific heat:

$$\nabla'\cdot\left(-\kappa'\nabla'T'\right) + \rho'c'\frac{\partial T'}{\partial t} = 0, \tag{2}$$

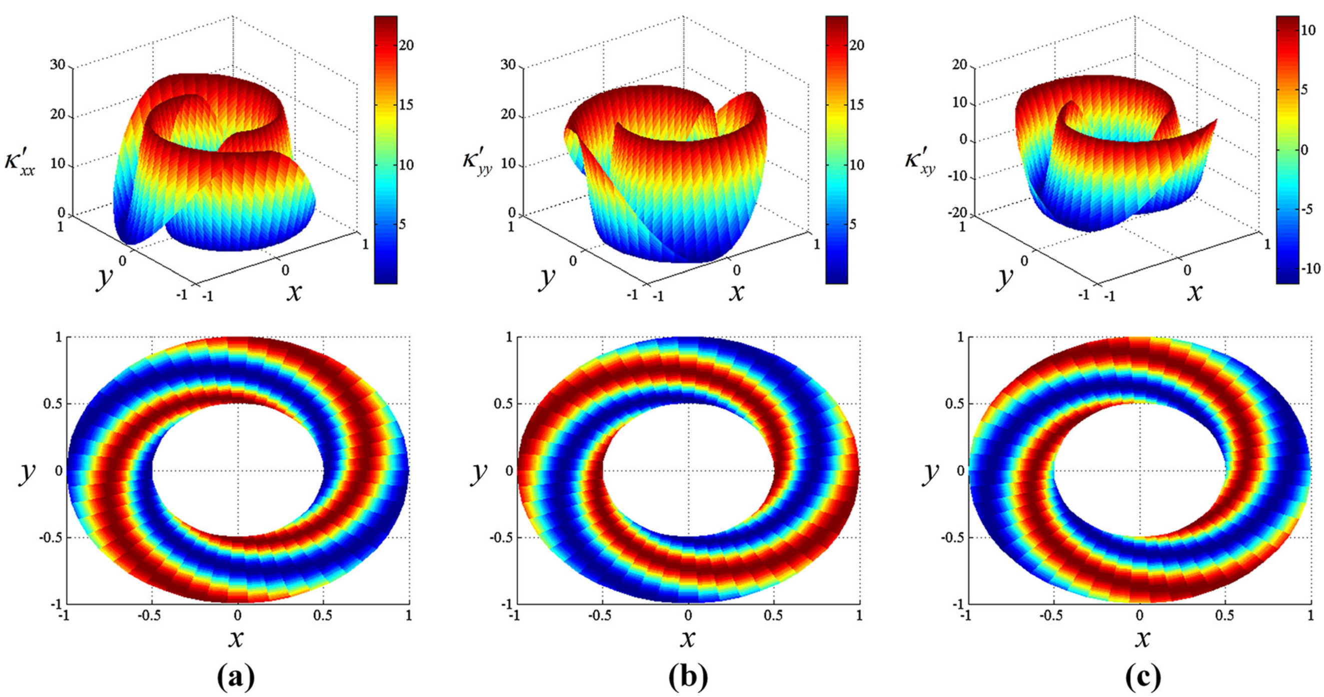

Fig. 2 3D distribution and the corresponding 2D contour of the thermal conductivity in the transformed space: **a** κ'_{xx}, **b** κ'_{yy}, and **c** κ'_{xy} (= κ'_{yx})

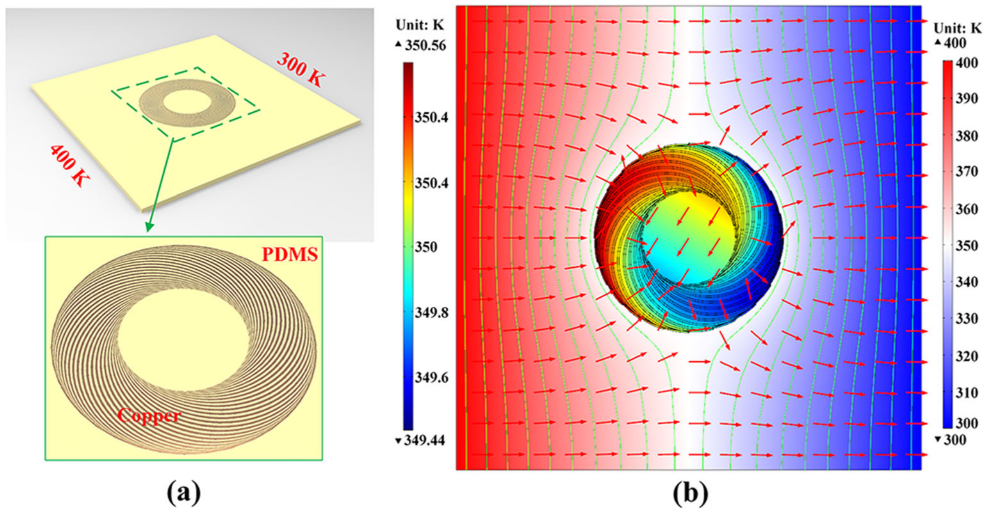

Fig. 3 **a** 3D model of an eccentric-semi-ring-structure thermal cloak. Two kinds of isotropic materials, i.e., copper and PDMS, are used to fabricate the composite plate. **b** Its temperature profile. The left and right boundaries are kept at 400 K and 300 K respectively. Heat is partly inverted by this structure in the inner region. The *left legend* denotes the temperature distribution of the inner region; the *right legend* denotes the temperature field outside. *Red arrows* denote the local direction of heat flux. *Green lines* denote the isotherm in panel

with

$$\kappa' = \frac{\boldsymbol{\Lambda}\kappa\boldsymbol{\Lambda}^{\mathrm{T}}}{\det|\boldsymbol{\Lambda}|}, \tag{3}$$

and

$$\rho'c' = \frac{\rho c}{\det|\boldsymbol{\Lambda}|}, \tag{4}$$

where $\boldsymbol{\Lambda}$ $(=\partial(x',y',z')/\partial(x,y,z))$ is the Jacobian matrix. This lays the foundation of the transformation thermodynamics and provides the guideline for the design parameters to achieve extreme heat flux manipulation by an arbitrary coordinate transformation. To invert the heat flow, as shown in Fig. 1, we define the following coordinate mapping [18–20]:

$$\begin{cases} r' = r, \quad z' = z, \quad \text{and} \quad \theta' = \theta + \pi \quad \in \text{Region I} \\ r' = r, \quad z' = z, \quad \text{and} \quad \theta' = \theta + \pi\dfrac{f(b)-f(r)}{f(b)-f(a)} \quad \in \text{Region II}, \\ r' = r, \quad z' = z, \quad \text{and} \quad \theta' = \theta \quad \in \text{Region III} \end{cases} \tag{5}$$

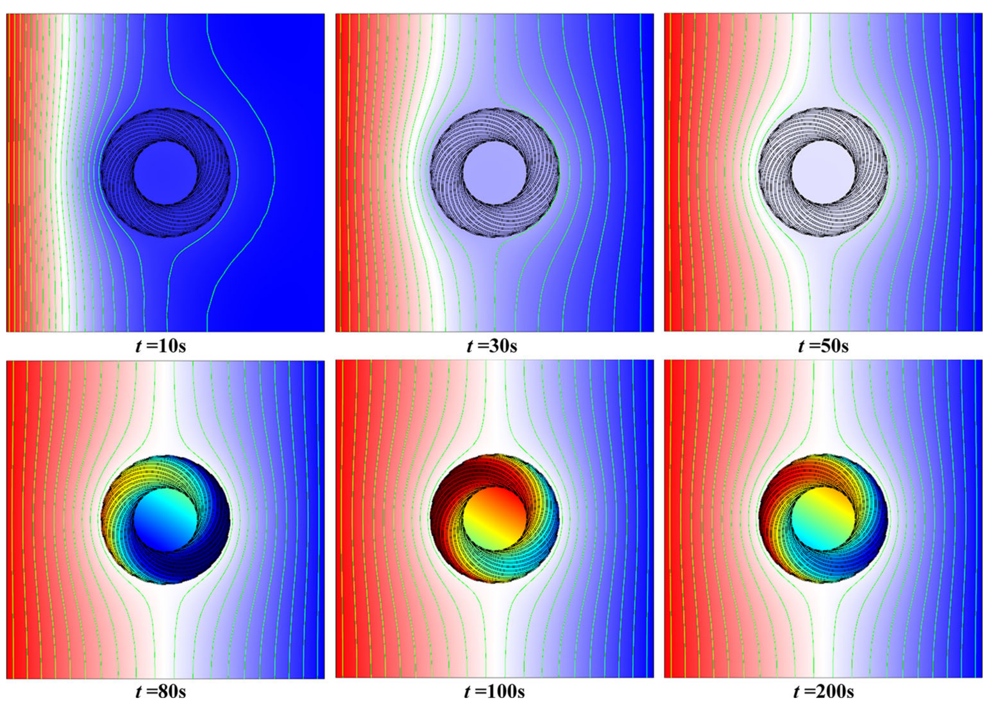

Fig. 4 Transient behaviors of the eccentric-semi-ring-structure thermal cloak

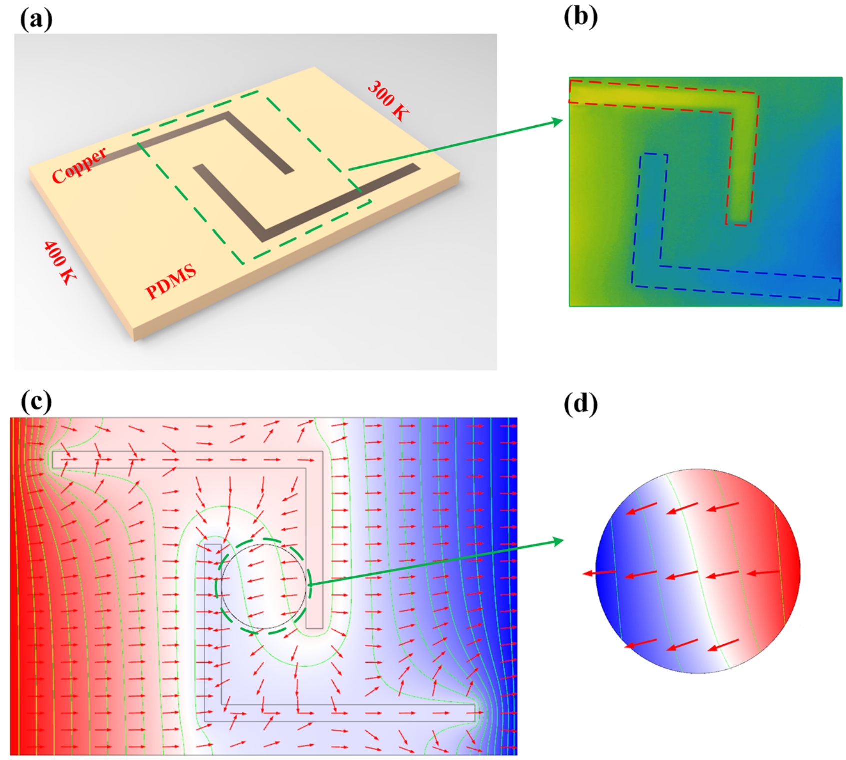

Fig. 5 Experimental observation of the inverting heat flow phenomenon through a simple structure. **a** 3D model of the fabricated structure. Two copper "L"-shape structures are the special conductive material, and the rest part is made by PDMS. **b** Temperature snapshot of the inner region in the experiment taken by an infrared thermal camera. **c** Simulated temperature field of the simple structure. **d** Temperature field of the inner region in the simulation. *Red arrows* denote the local direction of heat flux. *Green lines* denote the isotherm in panel

where a and b are the inner and outer radii of the annular thermal cloak, respectively. $f(r)$ could be any continuous function of radius r and here we let it be $\ln(r)$. According to the coordinate transformation in Eq. (5), for any given

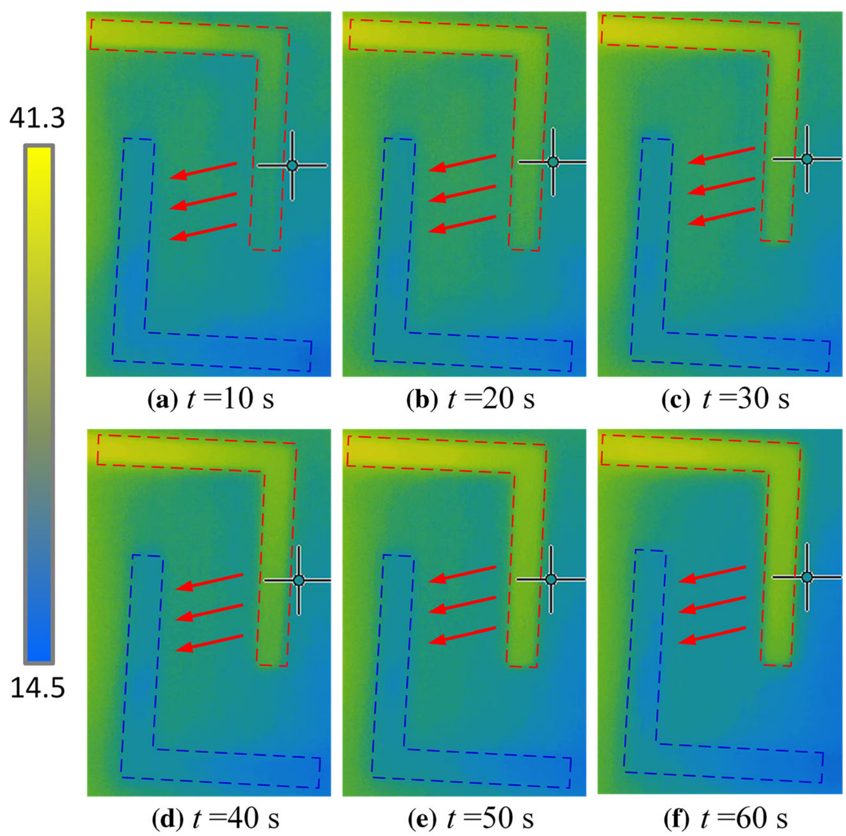

Fig. 6 Measured temperature distributions at different time t: **a** 10 s, **b** 20 s, **c** 30 s, **d** 40 s, **e** 50 s, and **f** 60 s. Left boundary is heated while the right boundary is cooled. "L"-shape structure is made of copper, while the rest part is made of PDMS

azimuth angle θ, the transformed azimuth angle θ' is rotated. Substituting Eq. (5) into Eq. (3), the thermal conductivity tensor of the annular metamaterial between $r=a$ and $r=b$ should be

$$\kappa' = \begin{bmatrix} \kappa'_{xx} & \kappa'_{xy} & \kappa'_{xz} \\ \kappa'_{yx} & \kappa'_{yy} & \kappa'_{yz} \\ \kappa'_{zx} & \kappa'_{zy} & \kappa'_{zz} \end{bmatrix} = \begin{bmatrix} 1+2\mathfrak{R}\cos\theta'\sin\theta' + \mathfrak{R}^2\sin^2\theta' & -\mathfrak{R}^2\cos\theta'\sin\theta' - \mathfrak{R}\left(\cos^2\theta' - \sin^2\theta'\right) & 0 \\ -\mathfrak{R}^2\cos\theta'\sin\theta' - \mathfrak{R}\left(\cos^2\theta' - \sin^2\theta'\right) & 1-2\mathfrak{R}\cos\theta'\sin\theta' + \mathfrak{R}^2\cos^2\theta' & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (6)$$

where $\mathfrak{R}$ is a constant as $\mathfrak{R} = \frac{\pi r f'(r)}{f(b)-f(a)} = \frac{\pi}{\ln(b/a)}$ and the initial thermal conductivity tensor is $\kappa = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. In such transformation, the determinate of $\mathbf{\Lambda}$ is unit, thus $\rho' c' = \frac{\rho c}{\det|\mathbf{\Lambda}|} = \rho c$. With this transformation, we can invert the heat flow, but it is seen from Eq. (6) that the thermal conductivity is angle-dependent, which is quite hard to realize with natural materials. That is why we have to seek the alternative methods to realize the present phenomenon.

3 Results and Discussions

With the thermodynamics transformation techniques, we could derive the desired distribution of the thermal conductivity according to Eq. (6), in order to invert the heat flow. In our design, the inner and outer radii of thermal cloak are 0.5 m and 1 m, respectively. The thermal conductivity of the ambient material is 1 W/(mK), implying that the initial thermal conductivity tensor is a unit matrix. The calculated 3D distribution and the corresponding 2D contour of the thermal conductivity distribution in the transformed space are shown in Fig. 2. It is seen that the thermal conductivity distribution is rather complicated that the diagonal components of the thermal conductivity tensor (κ'_{xx} and κ'_{yy}), the off-diagonal components (κ'_{xy} or κ'_{yx}) are angle-dependent. As for the magnitude, it is seen in Fig. 2 that the diagonal components are positive in the whole angle; while for the off-diagonal component, things are different. For some angle θ, κ'_{xy} is positive, while for some other angle θ, κ'_{xy} is negative. As we known, negative thermal conductivity cannot exist in nature according to the second law of thermodynamics. Comparing the three subfigures in Fig. 2, one may find something in common from the 2D contours of thermal conductivity distribution. A rotated distribution of thermal conductivity could be observed in the three subfigures. And for the point whose κ'_{xx} is large, the corresponding κ'_{yy} is small with κ'_{xy} in between. It is because of such unique distribution of thermal conductivity that the inverted heat flow phenomenon could be realized. However, two limitations hinder the realization. One is the angle-dependent characteristics that we cannot realize such unique distribution with natural materials. The other is the negative thermal conductivity that we cannot find such "negative" thermal materials.

A feasible alternative to realize such inverted heat flow phenomenon could employ the alternating layered natural materials. In such configurations, two kinds of materials are needed throughout with thermal conductivities in different magnitudes. Inspired by the concentric-ring-structure thermal cloak [10], we used the eccentric-semi-ring-structure thermal cloak to invert the heat flux here. As shown in Fig. 3a, the high conductive material is copper (400 W/(m·K)) and the relatively low conductive material is polydimethylsiloxane (PDMS) (0.15 W/(m·K)). The left boundary was kept at 400 K while the right boundary was at 300 K. The interfacial thermal resistance between copper and PDMS was neglected. The software package COMSOL Multiphysics was used to simulate the corresponding temperature distribution of the structures in Fig. 3a by solving the transient Fourier's equation through finite element methods (FEM).

As shown in Fig. 3b, the temperature field inside the inner region is kind of rotated clockwise. Heat is conducted from the left to the right outside the thermal cloak; but in the inner region, heat is conducted from the right part to the left part (the heat flux has component along the—x axis). This implies that the thermal gradient changes its sign in the inner region. According to Fourier's law, $q_x = -\kappa_x A \frac{dT}{dx}$ where the subscript x denotes the components along x axis. The heat flux component q_x is generally considered as positive, but the temperature gradient dT/dx changes its sign from negative to positive in the inner region. As a result, the thermal conductivity κ_x is negative! In fact, a thermal term called as apparent thermal conductivity is invoked here [21]. The paradox lies in the sign of heat flux. In this case, the heat flux changes its sign as well, but people from the outside still consider that heat is conducted from left to right and the sign of heat flux is considered as positive from a macroscopic view. The thermal conductivity is positive according to the second law of thermodynamics, but the apparent thermal conductivity can be negative. By such structure, the inverting heat flow comes from the rotated temperature field. The transient behaviors of such structure were shown in Fig. 4. It is seen that with time elapses, heat diffuses from left to right and the isotherms are bended away from the thermal cloak structures. The inner temperature seems uniform and the rotated heat flux is not obvious before t=80 s. After t=80 s, the inner temperature field becomes non-uniform and the rotated heat flux was observed gradually. At t=200 s, the temperature field becomes steady, similar as Fig. 3b.

The essence of thermal cloak lies in the anisotropic dispersion of thermal conductivity, and the different heat conduction ability in different conductive materials/structures. Understanding this, we may realize the inverting of heat flow by some simple structures rather than complicated eccentric-semi-ring structures. As shown in Fig. 5a, we design and fabricate a simple structure, in which two copper "L"-shape structures act as the conductive material, and the rest part is made by PDMS. The dimensions are 300 mm long, 200 mm wide, and 2 mm thick. The thickness of the plate is not significant for the heat conduction problem of interest, but the thickness influences the strengths of artifacts. When the thickness is large, the volume-to-surface ratio is big enough so that the heat conduction or convection to air can be neglected. However, too large thickness may lead to a big thermal capacity which becomes difficult to quantify the heat source and cold source in the experiment. Here, two water baths are chosen as the heat source in the experiment: one is boiling water and the other is ice water. Thickness of 2 mm is found to provide a good trade-off. To further remove the influence of heat conduction or convection to air, both the surfaces of the composite plate are insulated by a thin layer of PDMS. To observe the overall temperature profiles, it is better to use an infrared (IR) thermal camera (FLIR SC620) rather than thermocouples since the latter just can obtain point temperature. The principle of the infrared thermal camera is based on Kirchhoff's law, so setting an accurate emissivity of the material surface is of great importance. Here, we used thermocouples to check the emissivity to make sure that the overall thermal profiles are correct (=0.84 in this study). The IR camera is connected to a computer and the temperature distributions are recorded in real time.

The temperature field observed by the infrared thermal camera is shown in Fig. 5b. Heat is conducted from the left to the right, but due to the high thermal conductivity of copper (400 W/(m·K)), the temperature on the right "L"-shape structure is higher than that on the left near the inner region. Heat is conducted from the right to the left in the inner region consequently. As shown in Fig. 5c, heat is conducted from the left boundary to the right boundary and the red arrows denote the direction of heat flow. However, in the inner region, the heat flow is inverted and conducts from right to left, which can be observed clearly in the Fig. 5d. The measured temperature distributions at different time t as indicated are shown in Fig. 6. It is seen that heat is conducted faster in "L"-shape structure, thus the left "L"-shape structure has higher temperature than the right one. With time elapses, the temperature difference between the two "L"-shape structures increases. From the experimental results, the inverting heat flow is realized by this simple structure, which means heat flow can be controlled even by simple structures rather than complicated eccentric-semi-ring structures.

4 Conclusions

In this study we focused on the phenomenon of inverting heat flow. Mathematic description, finite element simulations, and experiments were conducted to validate the phenomenon. It is found that although the inverting of heat flow could be done by transformation thermodynamics, the local thermal conductivity turned out to be negative and the distribution of thermal conductivity is rather complicated and hard to perform in practice. As an alternative, the inverting phenomenon could be made by the eccentric-semi-ring thermal cloak. The essence of inverting heat flow is to control the anisotropic dispersion of thermal conductivity. Based on this understanding, we designed an "L"-shape simple structure and experimentally observed the phenomenon of inverting heat flow. It is concluded that the phenomenon of inverting heat flow can be performed not only by thermal metamaterials, engineering materials with complex structures, but also by some simple structures.

Acknowledgments The financial support by The Innovation Funds from HUST is acknowledged. The discussions on the experimental setup and measurement with Dr. Yang Liu are appreciated.

References

1. J.B. Pendry, D. Schurig, D.R. Smith, Science **312**, 1780–1782 (2006)
2. U. Leonhardt, Science **312**, 1777–1780 (2006)
3. J.B. Pendry, A. Aubry, D.R. Smith et al., Science **337**, 549–552 (2012)
4. F. Chen, D.Y. Lei, Sci. Rep. **5**, 11552 (2015)
5. S. Zhang, C.G. Xia, N. Fang, Phys. Rev. Lett. **106**, 024301 (2011)
6. J. Zhu, Y.Y. Chen, X.F. Zhu et al., Sci. Rep. **3**, 1728 (2013)
7. M. Farhat, S. Enoch, S. Guenneau, A.B. Movchan, Phys. Rev. Lett. **101**, 134501 (2008)
8. S. Zhang, D.A. Genov, C. Sun, X. Zhang, Phys. Rev. Lett. **100**, 123002 (2008)
9. M. Brun, S. Guenneau, A.B. Movchan, Appl. Phys. Lett. **94**, 061903 (2009)
10. S. Guenneau, C. Amra, D. Veynante, Opt. Express **20**, 8207 (2012)
11. T.C. Han, T. Yuan, B.W. Li, C.W. Qiu, Sci. Rep. **3**, 1593 (2013)
12. R. Schittny, M. Kadic, S. Guenneau, M. Wegener, Phys. Rev. Lett. **110**, 195901 (2013)
13. U. Leonhardt, Nature **498**, 440 (2013)
14. H.Y. Xu, X.H. Shi, F. Gao, H.D. Sun, B.L. Zhang, Phys. Rev. Lett. **112**, 054301 (2014)
15. T.C. Han et al., Phys. Rev. Lett. **112**, 054302 (2014)
16. A. Alu, Physics **7**, 12 (2014)
17. S. Narayana, Y. Sato, Phys. Rev. Lett. **108**, 214303 (2012)
18. H.Y. Chen, C.T. Chan, Appl. Phys. Lett. **90**, 241105 (2007)
19. H.Y. Chen et al., Phys. Rev. Lett. **102**, 183903 (2009)
20. S. Guenneau, C. Amra, Opt. Express **21**(5), 6578 (2013)
21. C.Z. Fan, Y. Gao, J.P. Huang, Appl. Phys. Lett. **92**, 251907 (2008)