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NUCLEAR PHYSICS

Nucleon-Nucleon Scattering Phase Shifts via Supersymmetry and the Phase Function Method

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Abstract By adapting a suitable ground state interaction for the nucleon-nucleon system, the higher partial wave potentials are derived via a supersymmetry inspired factorization method. The merits of our generated interactions are examined by computing the scattering phase shifts through the judicious use of the phase function method and are compared with the standard results.

Keywords Nucleon-nucleon scattering · Supersymmetric quantum mechanics · Phase function method · Hulthen potential

1 Introduction

The theoretical description of nucleon-nucleon (NN) scattering can be considered as a basic ingredient to understand the nuclear structure and scattering of few and many body systems [1, 2]. The low-energy NN scattering is described in terms of some degrees of freedom in which spin and isospin symmetries play an important role. However, at the medium energy, the production processes and inelasticities are more significant. The NN scattering, being a long-standing problem, is considerably reviewed with the help of the database present in refs. [3–19]. The low-energy data is analyzed by the VIP/GWU group [20] for $T_{\mathrm{Lab}} \leq 450$ MeV and the Nijmegen group [19] with NN phase shift results PWA98 for $T_{\mathrm{Lab}} \leq 350$ MeV. Till now, a number of groups [3–29] have already worked both experimentally and theoretically to interpret the NN scattering data. Although the methods adapted by them differ to some extent, they have obtained nearly the same results. Therefore, one can easily rely on these data. These data are rather accurate for proton-proton (pp) system while minor uncertainties arise for dealing with neutron-proton (np) system [21].

In supersymmetric quantum mechanics (SQM) [30–33], one often deals with the hierarchy problem. Within the framework of SQM, one is able to generate the Hamiltonian hierarchy, the adjacent members of which are supersymmetric partners in which they share the same eigenvalue spectrum except the missing ground state. Using the formalism of the SQM-inspired factorization method, we have constructed the higher partial wave potentials from the ground state interaction and studied the related physical observables associated with them in a number of earlier publications [34–38]. In the present text, by considering a simple potential model of Hulthen type without spin orbit coupling as a ground state interaction through the formalism of SQM, we have generated the higher partial wave potentials for nucleon-nucleon systems and computed the scattering phase shifts via the phase function method (PFM) [39]. In Section 2, we briefly discuss the algebra of SQM and generate the higher partial wave interaction via SQM inspired factorization method. Section 3 is devoted to the computation of scattering phase shifts via PFM and related discussion. Finally in Section 4, we put some concluding remarks.

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2 Higher Partial Wave Potentials via SQM

Arnold and MacKeller [40] have parameterized the nuclear Hulthen potential for the nucleon-nucleon system which reads as

$$V(r) = -\left(\beta^2 - \alpha^2\right) \frac{e^{-(\beta-\alpha)r}}{\left(1-e^{-(\beta-\alpha)r}\right)}. \quad (1)$$

Here, $(\beta-\alpha)$ and $(\beta^2-\alpha^2)$ stand for the range and depth of the potential. This potential can well account the very low-energy phase shifts (up to 25 MeV) but are unable to produce higher energy experimental values [3–8, 41]. As a one-term potential has not the ability to produce sign change in the phase shifts, we propose a two-term Hulthen-like interaction to take into account the change of sign in the S-wave scattering phase shifts. The S-wave potential for the nucleon-nucleon system is expressed as

$$V_0(r) = V(r) + \frac{(\beta-\alpha)^2 e^{-(\beta-\alpha)r}}{\left(1-e^{-(\beta-\alpha)r}\right)^2}. \quad (2)$$

In SQM, the supersymmetric partner potential $V_1(r)$, which misses the first bound state of the ground state interaction $V_0(r)$ are related by

$$V_1(r) = V_0(r) - \frac{d^2}{dr^2}\ln\phi_0^{(0)}(r), \quad (3)$$

where $\phi_0^{(0)}(r)$ is the near the origin behavior of the ground state wave function. Thus, one may write the general relation between two consecutive partial wave potentials as

$$V_{\ell+1}(r) = V_\ell(r) - \frac{d^2}{dr^2}\ln\phi_\ell^{(0)}(r);\ \ell = 0, 1, 2\ldots. \quad (4)$$

For the continuous energy eigenvalue spectrum $E=k^2\rangle 0$, the regular solution for the ground state interaction defined in Eq. (2) is obtained in the form [32]

$$\phi_0(k,r) = \frac{e^{ikr}}{(\beta-\alpha)^2}\left(1-e^{-(\beta-\alpha)r}\right)^2 {}_2F^1\left(2+A, 2+B; 4; 1-e^{-(\beta-\alpha)r}\right) \quad (5)$$

with

$$A = -\frac{ik}{(\beta-\alpha)} + \frac{i}{(\beta-\alpha)}\left[k^2 - \left(\beta^2-\alpha^2\right)\right]^{1/2} \quad (6)$$

and

$$B = -\frac{ik}{(\beta-\alpha)} - \frac{i}{(\beta-\alpha)}\left[k^2 - \left(\beta^2-\alpha^2\right)\right]^{1/2}. \quad (7)$$

Combination of Eqs. (2), (3), and (5) leads to

$$V_1(r) = V_0(r) + \frac{3(\beta-\alpha)^2 e^{-(\beta-\alpha)r}}{\left(1-e^{-(\beta-\alpha)r}\right)^2}. \quad (8)$$

The above interaction is termed as the P-wave potential. Similarly using the P-wave solution

$$\phi_1(k,r) = \frac{e^{ikr}}{(\beta-\alpha)^3}\left(1-e^{-(\beta-\alpha)r}\right)^3 {}_2F_1\left(3+A, 3+B; 6; 1-e^{-(\beta-\alpha)r}\right) \quad (9)$$

one gets the D-wave interaction as

$$V_2(r) = V_0(r) + \frac{6(\beta-\alpha)^2 e^{-(\beta-\alpha)r}}{\left(1-e^{-(\beta-\alpha)r}\right)^2}. \quad (10)$$

However, for charged hadron scattering, one has to add the electromagnetic interaction with the nuclear part. For electromagnetic interaction, we use the atomic Hulthen potential. Therefore, for pp scattering, the corresponding interactions for various partial waves are defined as

$$V^P_{\ell+1}(r) = V_H(r) + V_{\ell+1}(r);\ell = 0, 1, 2.... \quad (11)$$

with $V_H(r) = \omega_0 \frac{e^{-\delta r}}{1-e^{-\delta r}}$; δ, the inverse of screening radius and $\omega_0$is the strength parameter of the interaction.

3 Results and Discussion

The phase function method [39] is an efficient approach to evaluate the scattering phase shifts for quantum mechanical problems. The method deals with the separation of the radial wave function of the Schrödinger equation into an amplitude part $\alpha(k,r)$ and an oscillating part with variable phase $\delta(k,r)$. At every point, the function $\delta(k,r)$, called as the phase function, represents the phase shift of the wave function for scattering by potential truncated at a distance r. For a local potential $\delta(k,r)$ satisfies a first-order non-linear differential equation given by

$$\delta'_\ell(k,r) = -k^{-1} V(r)\left[\hat{j}_\ell(kr)\cos\delta_\ell(k,r) - \hat{\eta}_\ell(kr)\sin\delta_\ell(k,r)\right]^2, \quad (12)$$

where $\hat{j}_\ell(kr)$ and $\hat{\eta}_\ell(kr)$ are the Riccati-Bessel functions. Here, we shall follow the phase convention of Calogero [39] with the Hankel function of first kind written as $\hat{h}_\ell(x) = -\hat{\eta}_\ell(kr) + i\hat{j}_\ell(x)$.

Using the parameters $\beta = 1.4054\,\mathrm{fm}^{-1}$ and $\alpha = -0.0404\,\mathrm{fm}^{-1}$ for $^1S_0$state and $\beta = 1.4054\,\mathrm{fm}^{-1}$ and $\alpha = 0.232\,\mathrm{fm}^{-1}$ for $^3S_1$state, we have computed and portrayed the scattering phase shifts for various partial wave states up to $\ell = 2$ in Figs. 1, 2, 3, and 4. Our S-wave phase shifts for both singlet and triplet states produce correct nature of the phase shift. For

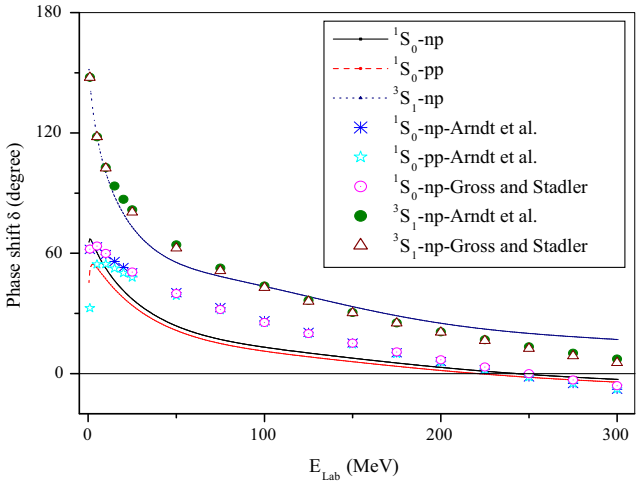

Fig. 1 S-wave phase shift as a function of E_{Lab}

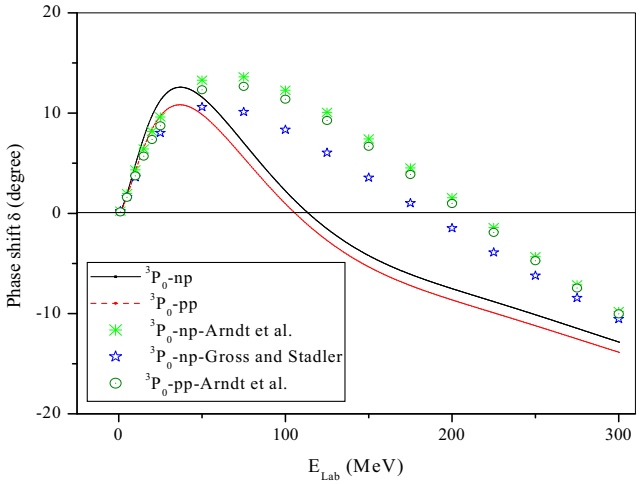

Fig. 3 $^3P_0$-phase shifts as a function of E_{Lab}

$^1S_0$state, it is observed that the numerical values of our phase shifts differ slightly from those of Arndt et al. [3–8] and Gross and Stadler [41] in the energy range 25 MeV $< E_{\mathrm{Lab}} <$ 175 MeV. However, beyond 175 MeV, we obtain good agreement with the experimental result. For $^3S_1$state, we observe fairly good agreement with those of refs. [3–8, 41] over the entire range of energy under consideration.

Our supersymmetry generated P-wave potentials for singlet and triplet states produce phase shifts that correspond to $^1P_1$and $^3P_0$states, respectively. The $^1P_1$ phase shifts shown in Fig. 2 generated from $^1S_0$ parameters are in exact agreement with those of Gross and Stadler [41] but differ slightly from ref. [3–8] on either side of the energy E_{Lab} = 175 MeV. The other P-wave phase shifts generated from $^3S_1$parameters correspond to $^3P_0$state. In case of both np and pp scattering, phase shifts produce correct peak values but differ from experimental data [3–8, 41] beyond 75 MeV. Our phase shift values for np and pp scattering changes their sign at E_{Lab} = 114 MeV and E_{Lab} = 106 MeV, respectively, whereas experimental values change their sign around E_{Lab} = 200 MeV for ref. [3–8] and E_{Lab} = 190 MeV in case of np scattering for ref. [41].

In Fig. 4, we have plotted $^3D_1$ phase shifts for np scattering. These phase shifts are generated from $^3S_1$ parameters and agree quite well with the values of Gross and Stadler [41] up to 75 MeV, and beyond that, they started deviating from the standard result by few degrees up to 150 MeV, and beyond 150 MeV, these differences become constant by 7° approximately. However, our SUSY-generated D-wave potential is unable to produce $^1D_2$ phase shifts with $^1S_0$ parameters. Thus, our supersymmetry generated higher partial wave potentials are quite capable of producing the qualitative nature of the phase shifts especially for triplet scattering.

4 Conclusion

The higher partial wave potentials developed through the formalism of SQM belong to Eckart class of potentials. It is well

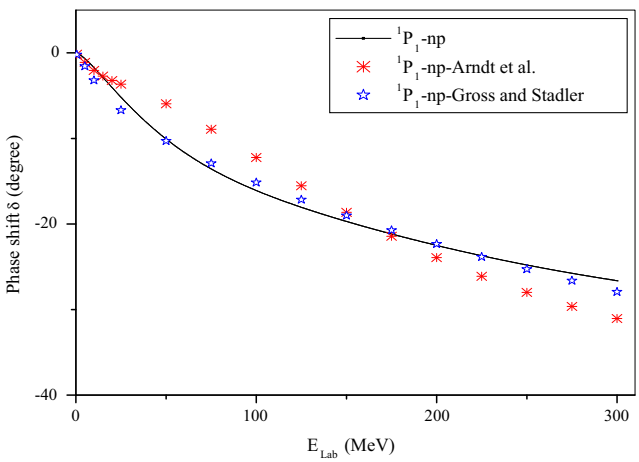

Fig. 2 $^1P_1$-phase shifts as a function of E_{Lab}

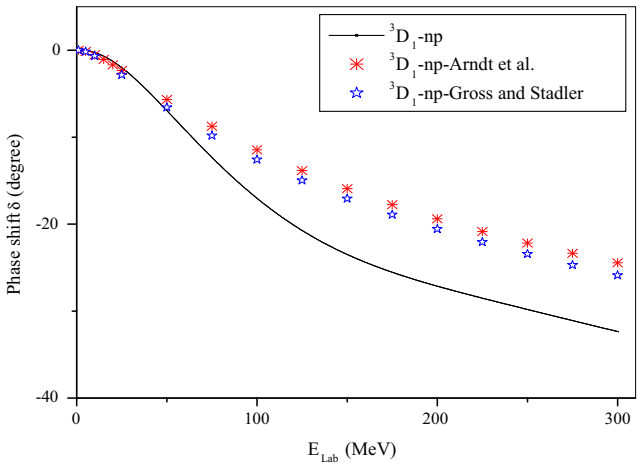

Fig. 4 $^3D_1$-phase shifts as a function of E_{Lab}

known that the nuclear potentials are highly state dependent [3–17, 22] and are generally parameterized by several parameters. In our earlier publications [34–38], we have adapted the supersymmetry inspired factorization method for construction of higher partial wave potentials by considering a one-term nuclear Hulthen interaction as the ground state potential and observed that these potentials are able to produce the nature of the corresponding phase shifts but differ in their numerical values from the standard data [3–8, 41]. To achieve better agreement with the experimental results, we have considered here a two-term nuclear Hulthen potential as the ground state interaction which takes into account the proper behavior of the $^1S_0$ and $^3S_1$ phase shifts. It is also noticed that we achieve better numerical agreement in the higher partial wave phase shifts than our previous approaches to the problem under consideration. Thus, one may conclude by noting that the two-term nuclear Hulthen potential is superior to its one-term counterpart for generation of the higher partial wave potentials and phase shifts. Our simple-minded model can also be extended for other hadronic systems like $\alpha-\alpha$ scattering. The $\alpha-\alpha$ scattering is in our active consideration and will be communicated in a future correspondence.

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