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GENERAL AND APPLIED PHYSICS

Analysis of All-Optical State Generator for "Encoding a Qubit in an Oscillator"

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Abstract The fault-tolerant quantum computation scheme proposed by Gottesman (Phys. Rev. A **64**, 012310 (2001)) can be performed using relatively simple linear optical resources and provides a natural protection against arbitrary small errors. On the other hand, preparing the initial GKP states is a difficult task. A few proposals to generate GKP states have been done over the last years. Our objective here is to analyze the performance of a particular GKP generator that uses cat states, linear optical devices, squeezing, and homodyne detection. We use numerical simulations to study the behavior of the fidelity between the generated and the ideal states and show that the proposal in consideration is indeed a promising scheme.

Keywords Cat states · GKP states · Fidelity

1 Introduction

Few years ago, it was believed that optical scalable quantum computation would only be possible through the use of non-linear elements, such as Kerr medium. Many logical gates based on non-linear optics were proposed and studied [3–6]. The major difficulty with these mentioned schemes where the fact that all of them needed a non-linear medium with greater non-linearity than available nowadays. This scenario changed radically when Knill, Laflamme, and Milburn showed in 2001 that linear optics alone would suffice to implement efficient quantum computing [7]. Since then, other linear optical schemes were proposed, some based on the discrete variable encoding of [7], and latter some based on the continuous variable encoding of [8].

The linear optical quantum computer proposed by Gottesman, Kitaev, and Preskill (GKP) [1] encodes the qubit in the infinite-dimensional space of the position and momentum variables of an oscillator. The two-dimensional Hilbert space of a qubit embedded in the infinite-dimensional Hilbert space of a system described by x and p, such as in this case, is described by the following stabilizer generators:

$$S_x = e^{2i\sqrt{\pi}x}, \tag{1}$$
$$S_p = e^{-2i\sqrt{\pi}p}. \tag{2}$$

The ideal logical $|0\rangle$ GKP state is then defined as a state having an infinite series of delta-function peaks in its x-quadrature wave function. The logical $|1\rangle$ state, in turn, would also be an infinite series of delta-function peaks, but displaced by a distance of $\sqrt{\pi}$ from the logical $|0\rangle$. These defined states are non-normalizable states, in other words, they are unphysical. GKP overcame this problem by adopting an approximate state whose x-quadrature wave function is a sum of Gaussian peaks with width Δ, weighted by a larger Gaussian envelope function of width $1/k$. The wave function of this approximated logical $|0\rangle$ is given by

$$\psi_{GKP}(x) = N \sum_{s=-\infty}^{\infty} e^{-\frac{1}{2}(2sk\sqrt{\pi})^2} e^{-\frac{1}{2}(\frac{x-2s\sqrt{\pi}}{\Delta})^2}, \tag{3}$$

where N is a normalization factor.

GKP quantum computing scheme is fault tolerant, provides a natural protection of the information against small arbitrary shifts in the quadrature variables, and can be

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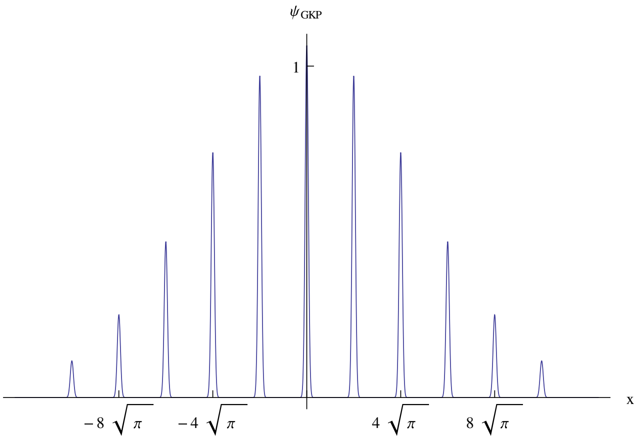

Fig. 1 GKP logical $|0\rangle$ state's wave function. In here, $\Delta = k = 0.12$

performed using relatively simple linear optics devices, squeezing, and homodyne detection. Recently, a device-independent quantum cryptography protocol for continuous variables based on the GKP encoding has also been proposed [9]. On the other hand, preparing the initial GKP state is challenging. Some different proposals to generate GKP states have been done over the years [1, 2, 10–13].

The generation of specific quantum states at a high fidelity plays a crucial role in the fields of quantum computation and information. That could not be different when discussing the generation of GKP states. That is why it is important to evaluate if a generator scheme is able to perform at high fidelity rates. Our goal here is to assess the scheme described in [2], that uses superpositions of optical coherent states (also known as "cat states"), linear optical devices, squeezing, and homodyne detection. Figure 2 shows a basic schematic of the proposal.

As seen in Fig. 2, two previously squeezed cat states are sent to interfere in a beam splitter. Each cat state contains two Gaussian peaks in its x-quadrature wave function, whose width will be reduced after the squeezing. Homodyne detection is then performed on one of the output ports of the beam splitter, and depending on the result of the measurement, an approximate GKP state is obtained on the other port. The resulting state will have three Gaussian peaks in

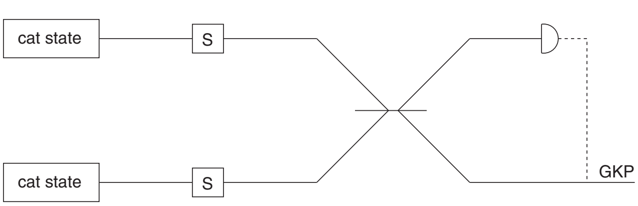

Fig. 2 Basic schematic of the proposal described in [2]. Each mode has a cat state that will be squeezed by an amount of s. The two modes are then sent to the inputs of a beam splitter. A homodyne detection is performed in mode 1 and depending on the result of the measurement, an approximate GKP state is obtained in mode 2

its x-quadrature wave function. But, as shown in Fig. 1, the state we desire to produce has more than three Gaussian peaks. This is not a problem, since we may repeat the procedure described to produce states with larger numbers of Gaussian peaks.

Despite the fact that this scheme is built of simple optical devices and operations, many obstacles are expected in a real experiment. Creating cat states is not a trivial quest. There are many different proposals for creating such states [14, 15], and several experiments were successful in generating cat states (or squeezed cat states) with $|\alpha|$ up to 1.75 and fidelities varying from 0.6 to 0.7 [16–24]. Matching the transverse and longitudinal shapes of the optical modes and photon loss during squeezing may also be concerning [25, 26].

A cat state is defined as a superposition of two coherent states with opposite phases:

$$|\psi_{cat}\rangle = \frac{|-\alpha\rangle + |\alpha\rangle}{\sqrt{2(1+e^{-2\alpha^2})}}, \tag{4}$$

where a coherent state is the eigenstate of the annihilation operator: $\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$. A cat state given by (4) can be written in the x-quadrature basis as

$$\psi_{cat}(x,\alpha) = G(x,1,-\sqrt{2}\alpha) + G(x,1,\sqrt{2}\alpha), \tag{5}$$

where $G(x,V,\mu) = e^{-(x-\mu)^2/(2V)}$ denotes a Gaussian with width $\sqrt{V}$ and centered at μ. If two cat states with a chosen value of α given by $\alpha = \sqrt{2}^{m-1}\sqrt{\pi}\,e^{\zeta}$, where ζ is the amount of squeezing, are sent though the scheme described above, and the p-quadrature of mode two is measured to be $p_2 = 0$, the resulting unnormalized state is [2]:

$$\tilde{\beta}(x,\zeta,m) = \sum_{n=0}^{2^m} \binom{2^m}{n} G\left[x, e^{-2\zeta}, 2\sqrt{\pi}(n-2^{m-1})\right]. \tag{6}$$

This scheme can be used iteratively to generate binomial states of higher orders. For example, if we take two copies of the order m binomial state (6), with spacing between the Gaussian peaks of $2\sqrt{2}\sqrt{\pi}$, we can show that a binomial state of order $m+1$,

$$\beta(x,\zeta,m+1) = \sum_{q=0}^{2^{m+1}} \binom{2^{m+1}}{q} G\left[x, e^{-2\zeta}, 2\sqrt{\pi}(q-2^m)\right], \tag{7}$$

can be obtained [2].

We wish to generate a state as close to (3) as possible. Ideally, it means we require infinite squeezing and a large value of m, that is a large number of cat states. Realistically, infinite squeezing is impossible to generate in a laboratory and we have limited cat resources, since producing such states is a challenge itself.

Our goal here is to study how to combine all these restrictions in a way to get the best match possible. In order to work on that, we will analyze the behavior of the fidelity between the ideal and the generated states. This kind of analysis is necessary to determine if a state generation scheme is promising, an essential characteristic considered by anyone who wishes to implement the scheme in a laboratory.

2 Fidelity

The fidelity of two pure states is defined as

$$F = |\langle\phi|\psi\rangle|. \tag{8}$$

In our case, since we have a continuous system, the fidelity will be given by

$$F = \int_{-\infty}^{\infty} \psi_{GKP}(x) \times \beta(x, \zeta, m)\, dx. \tag{9}$$

We have two reasons to choose fidelity in here: it is a quite intuitive concept and is a mathematically simple measure often used in the literature.

Let us explore the fidelity between ψ_{GKP}, given by (3), and the binomial state $\beta(x, \zeta, m)$, for realistic values of squeezing and number of iteration m. Choosing a reasonable m reflects directly on the requested resources to create a specific binomial state, since generating an order m binomial state requires a minimum of 2^m cat states.

Once we define the binomial state through m, the resulting fidelity calculated in (9) will depend on the amount of squeezing ζ, Δ, and k. Let us choose $m = 3$ and analyze the fidelity as a function of Δ and k. This will help us to determine the amount of squeezing ζ.

As shown in [27], GKP states with $\Delta \leq 0.15$ and $k \leq 0.15$ has probability equal or higher than 0.99 of being free of shift errors larger than $\sqrt{\pi}/6$, on both quadratures. We will keep this in mind and try to determine the best fidelity respecting these limits on Δ and k.

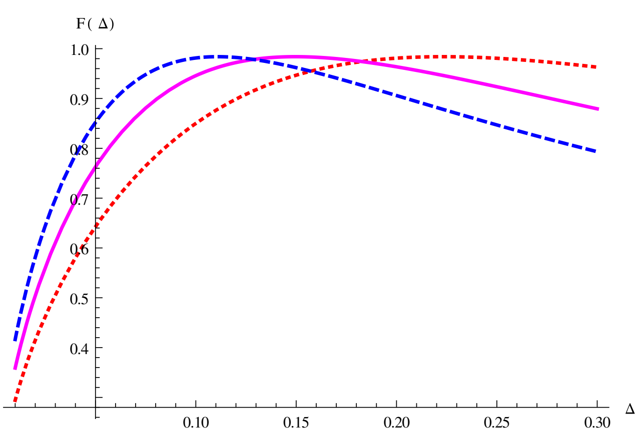

Fig. 3 Graphs show the fidelity between ψ_{GKP} and $\beta(x, \zeta, m)$, for **a** $\zeta = 1.5$ (*dotted curve*), **b** $\zeta = 1.9$ (*solid curve*), and **c** $\zeta = 2.2$ (*dashed curve*). In all cases, the fidelity is a function of Δ, and k is fixed and equal to 0.15

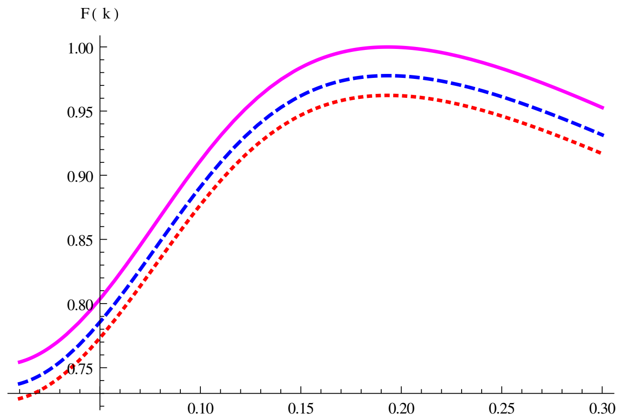

Fig. 4 Graphs show the fidelity between ψ_{GKP} and $\beta(x, \zeta, m)$, for **a** $\zeta = 1.5$ (*dotted curve*), **b** $\zeta = 1.9$ (*solid curve*), and **c** $\zeta = 2.2$ (*dashed curve*). In all cases, the fidelity is a function of k, and Δ is fixed and equal to 0.15

Plots of the fidelity as a function of Δ, with k fixed and equal to 0.15, for three different values of ζ appear in Fig. 3. On the other hand, Fig. 4 shows the fidelity as a function of k, for a Δ fixed and equal to 0.15 and for the same values of ζ. Graph Fig. 3 shows that if we choose a larger value for ζ, we get away from the desired interval of $\Delta \leq 0.15$, keeping k fixed. That leaves us with $\zeta = 1.9$ and $\zeta = 1.5$ as good candidates.

Now, it is time to take a look at Fig. 4. Within the two values of ζ, we see that the case where $\zeta = 1.9$ gives us a higher fidelity than the case where $\zeta = 1.5$. The $\beta(x, 1.9, 3)$ is the state that presents the best fidelity values.

In Fig. 5, we plot the fidelity, F, as a function of both Δ and k. Here, the binomial state considered is $\beta(x, 1.9, 3)$. This plot shows a very large loss of fidelity for lower values of Δ and k. The fidelity has higher values for a Δ about 0.15 and a $0.15 \leq k \leq 0.20$. The fidelity $F(\Delta, k) =$

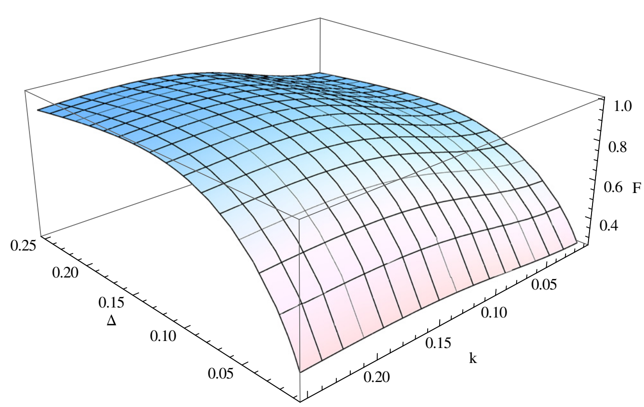

Fig. 5 Graph of fidelity F between ψ_{GKP} and $\beta(x, 1.9, 3)$ as a function of k and Δ

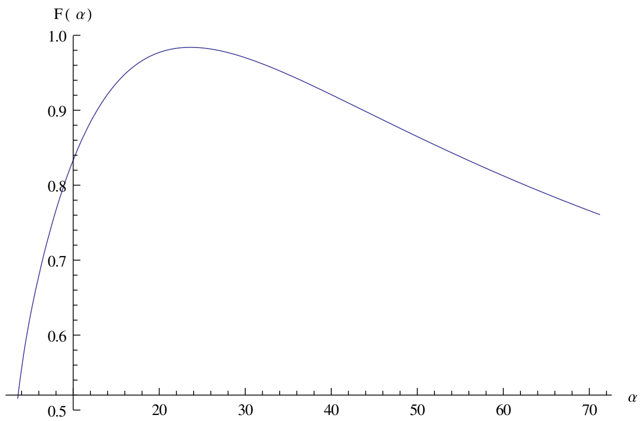

Fig. 6 Graph of fidelity F between ψ_{GKP} and $\beta(x, 1.9, 3)$ as a function of α

$F(0.15, 0.15) = 0.9839$, while $F(0.15, 0.20) = 0.9996$. But, as discussed before, our interest lies for values of $\Delta, k \leq 0.15$ to guarantee a GKP quantum computing scheme free of shift errors larger than $\sqrt{\pi}/6$. We can see that the scheme is initially able to generate a state that has fidelity above 0.9 with a ideal state that is essential to have a quantum computer with errors under the desired limit.

Our last plot is shown in Fig. 6, where we can see how the fidelity varies as a function of the coherent state amplitude, α, of the cat state described in (4). This can give us an idea of how any kind of photon loss would affect the fidelity of the generated GKP state, $\beta(x, 1.9, 3)$. The maximum value of the fidelity ($F \approx 0.9839$) takes place for $\alpha \approx 24$. This maximum value decreases to $F = 0.9480$ for $\alpha = 16$, showing that the state generation scheme in consideration has a tolerance to photon loss.

Let us compare the results we found and results from experimental implementation of cat states, since optical generation of cat states face similar constrains that we expect to face in the scheme here analyzed. Theoretical calculations for different schemes to generate cat states find fidelity as high as 99 %. But as expected, the fidelity found on experimental realizations of some of these schemes is lower, varying from 0.6 to 0.7, for $|\alpha|$ up to 1.75, as mentioned before. Our results predict an optimal fidelity of about 98 %. Considering that implementing this scheme would face many obstacles reported in [16–24], we would expect to have experimental results similar to those found for cat states generation.

3 Conclusions

In summary, we have used numerical simulation to investigate the performance of the GKP state generator described in [2]. We were able to certify that the closest state to a ψ_{GKP} is a generated binomial state $\beta(x, 1.9, 3)$. We analyzed the behavior of the fidelity F between the generated state $\beta(x, 1.9, 3)$ and ψ_{GKP} as a function of ψ_{GKP} parameters Δ and k. The results show that the highest values of the fidelity are for $\Delta \approx 0.15$ and $0.15 \leq k \leq 0.20$. Despite the fact that the highest values for F lies for k greater than 0.15, we choose $k \approx 0.15$ to keep the quantum computing scheme free of shift errors larger than $\sqrt{\pi}/6$. Lastly, we verified that the generation scheme under consideration is tolerant to photon loss.

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