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luizno.bjp@gmail.com
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M. Alsaif, Norah A.; Radiman, Shahidan; Yahaya, Redzuwan; Saleh Ahmed, Saad M.
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NUCLEAR PHYSICS

Alpha Decay Preformation Factors for Even–Even $^{280–316}$116 Superheavy Isotopes

Norah A. M. Alsaif[1,2] · **Shahidan Radiman**[1] · **Redzuwan Yahaya**[1] · **Saad M. Saleh Ahmed**[3]

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Abstract The success of the cluster formation model (CFM) in deriving an energy-dependent formula for the preformation factors of heavy nuclei has motivated us to expand this approach to the superheavy isotopes (SHI). In this paper, the alpha-cluster formation (preformation factor) behavior inside the parent nuclei of SHI with atomic number $Z = 116$ and neutron numbers $164 \leq N \leq 200$ is determined using the alpha preformation formula contained within the CFM. The cluster formation energy of the alpha particles and the total energy of the parent nuclei are calculated on the basis of the various binding energies. Our results clearly show that the CFM remains valid for superheavy nuclei (SHN). In addition, our calculations reveal that the alpha clustering mechanism and formation probability in $^{280–316}$116 even–even SHI are similar to those of even–even heavy nuclei in a general sense.

✉ Norah A. M. Alsaif
azam142428@gmail.com

Shahidan Radiman
shahidan.ukm@gmail.com

Redzuwan Yahaya
redzuwan@ukm.my

Saad M. Saleh Ahmed
saadtm2000@gmail.com

[1] School of Applied Physics, Faculty of Science and Technology, Universiti Kebangsaan Malaysia, 43600 Bangi, Selangor, Malaysia

[2] Department of Physics, Faculty of Science, Princess Nourah bint Abdul Rahman University, Riyadh, Saudi Arabia

[3] Department of Physics, Faculty of Science, University of Malaya, Kuala Lumpur 50603, Malaysia

Keywords Alpha preformation factor · Cluster formation model · Even–even nuclei · Alpha formation probability

1 Introduction

The synthesis of superheavy nuclei (SHN) is one of the most important topics in modern nuclear physics research. Several experimental investigations conducted in the past decade focused on exploring the "island of stability" of superheavy elements and investigating more neutron-rich isotopes closer to the region of spherical SHN [1–7]. Synthesis of the neutron-rich isotopes of elements 112, 114, 116, and 118 has been realized using hot fusion and fusion–evaporation reactions for irradiation of ^{233,238}U, ^{242}Pu, ^{248}Cm, ^{249}Cf, and Pb targets with a ^{48}Ca beam [1, 6–9]. Further, many decay chains of the parent isotopes of elements 112, 114, 116, and 118 have been observed as daughter products [3, 5–7]. For instance, the isotopes ^{289}114 or ^{288}114 have been observed as daughter products of the decay of the parent isotope of element 116 [3]. In addition, the isotopes of element 116 are the decay daughters of element 118 isotopes, which are produced via the ^{249}CF + ^{48}Ca reaction [2, 7]. The results of these studies reveal that, for superheavy elements, alpha decay rather than fission is the dominant decay mode [1, 5, 6, 10].

As regards theoretical studies, recent extensive investigations of the alpha decay properties of SHN isotopes have been conducted using the Wentzel–Kramers–Brillouin (WKB) method [11–20] and various models for the nuclear potential, such as the preformed cluster model (PCM) [11–13], the dynamical cluster-decay model (DCM) [13], the Coulomb and proximity potential model (CPPM) [17], the generalized liquid drop model (GLDM) [18], the cluster model using the Hamiltonian energy density approach in terms of the SLy4 Skyrme-like effective interaction [19, 20], and the systematic alpha-nucleus folding potentials [21]. The basic quantities of the alpha decays, which include the half-lives, Q-values, penetrabilities, preformation factors (P_α), and branchings, have been calculated.

As the most important dominant process in heavy nuclei and SHN is the alpha decay mode, the concept of P_α has been introduced to alpha decay theories. Further, the discovery of the cluster property has provided much insight on the clusterization effect in nuclear structure. The P_α of an alpha cluster in a nucleus is defined as the probability of finding the alpha cluster inside the parent nucleus, and it therefore refers to the clusterization of an alpha particle from four nucleons before the alpha emission [22]. This factor can provide additional information on nuclear structure [12–15, 22] and also allow information on the nuclear deformation of SHN to be extracted [13, 23, 24]. Technically, determining the exact value of the P_α for a given substance is a nontrivial problem that requires theoretical calculations with considerable accuracy, especially when the alpha clusterization effect is incorporated [18, 24–27]. For many years, the majority of theoretical studies on SHN has focused on the calculation of alpha decay half-lives/widths, but a small number have aimed to obtain a consistent estimation of P_α values. In both the phenomenological formula and the cluster model, the P_α is dependent on the mass number A of the emitted particle [17], the parent nucleus type (even–even, even–odd, odd–even, or odd–odd) [23, 28, 29], and an adjustable constant. As the discrepancies between the calculated and experimental values are considerable, this factor may be used to help determine the alpha-cluster preformation probability (represented by P_α, as explained above) [18].

In addition, little development has occurred in relation to the establishment of microscopic models of P_α; this is due to the complexity of the nuclear many-body problem. Varga, Lovas, and Liotta [30] have calculated P_α values for some nuclei using the R-matrix method, the cluster model, and the shell model for the many-body system with an effective nucleon–nucleon (NN) interaction. In that study, the ground-state wavefunction was well constructed and could successfully reproduce the experimentally measured energy of the ^{212}Po alpha decay. This wavefunction was composed of a large number of high-configuration mixing bases of single-particle states and merged with the cluster wavefunction. However, this model appears to be too complicated for application to SHN. Note that theoretical approaches have shown that P_α varies smoothly in the open-shell region and has a value smaller than 1.0 [31]. In the SHN region, Zhang and co-workers calculated the P_α, in different shell regions near $N=162$ and $N=184$ within the WKB approximation, whereas the nuclear potential between an alpha cluster and the daughter nucleus is obtained in the frame of a double-folding model with a density-dependent NN interaction [14]. In another approach, Seif defined the spectroscopic factor as P_α using semi-microscopic calculations based on the SLy4 Skyrme-like NN interaction and the WKB approximation, taking into account the nuclear deformations. These calculations were performed for even–even, odd–even, even–odd, and odd–odd heavy and superheavy parents [19, 20]. Furthermore, Gupta and collaborators have calculated P_α values using the PCM by solving the Schrodinger equation for the dynamical flow of mass and charge for the alpha decay chain of ^{293}118 [11]. In addition, it has been shown that P_α values can be obtained from the ratio of the experimental and theoretical half-lives obtained from the self-consistent models of the alpha clustering and resonance scattering [16]. From another perspective, P_α can be investigated using a deformed potential barrier for alpha decay, which is constructed using the GLDM [32] obtained by taking the mass and charge asymmetry and the proximity energy into account [33].

The differences between the P_α values obtained in the various studies remain large. For instance, the reported P_α for the alpha clusters inside SHN parent nuclei vary across a wide range, from 0.003 to 0.83 [14, 16–19, 28, 34]. Therefore, more accurate calculations of this factor must be performed using microscopic models particular to the superheavy region [19]. Recently, Ahmed et al. presented the clusterization theory and proposed the cluster formation model (CFM) [22, 25]. In CFM, the P_α is determined using a formula that is dependent on the alpha-cluster formation energy (E_α) and the total energy of the surface nucleons in the parent nucleus (E) [22, 25, 35]. In these studies, realistic P_α values were determined for a wide range of even–even heavy nuclei. Further, Deng et al. [24] have extended the CFM to evaluate the P_α of odd–A and odd–odd heavy nuclei by modifying the E_α extraction following consideration of the effect of the unpaired nucleon.

In the present work, we conduct P_α calculations for even–even superheavy isotopes (SHI) with $A=280$–316, based on the CFM proposed by Ahmed et al. [22, 25]. The P_α values depend on the E_α and E calculations; these energies are determined from the differences in the binding energies.

2 Theoretical Background

Since heavier clusters than the alpha particle were emitted from the heavy and superheavy nuclei, the nuclear system is considered to have more than one clusterization state. The formation of the alpha cluster inside the parent nuclei is one of the most dominant clusterization states, and its Hamiltonian (see Appendix A) is different from any other. When the clusterization of alpha occurs among the surface nucleons inside the alpha decay nuclei, the interaction responsible for this state is divided into two: one for binding the four nucleons tightly to each other, called cluster formation, and another for binding the formed cluster as one particle to the daughter, called the separation. The derivation of the preformation probability for this clusterization state is not presented in the work because it is not our target and it is already given in [22, 25].

In accordance with the clusterization state representation and the CFM, the probability of alpha preformation is given by [22, 25]

$$P_\alpha = \frac{\langle \phi_f | H_f | \phi_f \rangle}{\langle \phi_f | H_f | \phi_f \rangle + \langle \phi_r | H_r | \phi_r \rangle} \tag{1}$$

ϕ_f is the quantum-mechanical cluster formation that describes the clusters formed in the clusterization state. It has recently been found that the daughter cluster can be considered inert and the clusterization state depends on the nucleons of the nuclear surface [25], so, the alpha cluster formation was only included. The Hamiltonian H_f that represents this formation state is for all internal interactions among the four nucleons and responsible for gathering them and forming the alpha cluster. Also, ϕ_r is the quantum-mechanical dynamic state of the alpha cluster (its center of mass) that describes only its relative motion with respect to the core or daughter inside the parent nuclei. These two wavefunctions that describe the formation and the separation mechanisms can be obtained from the solution of the time-independent Schrödinger equations (TISEs)

$$H_f \phi_f = E_f \phi_f, \quad H_r \phi_r = E_r \phi_r \tag{2}$$

where E_f and E_r are the eigenvalues of the cluster formation energy and the relative motion of the cluster, respectively. The clusterization state has a Hamiltonian expressed as a sum of these two Hamiltonians, $H_f + H_r$, and total energy

$$E = E_f + E_r \tag{3}$$

Equation (1) can also be expressed in terms of the energies as

$$P_a = \frac{E_f}{E} \tag{4}$$

Instead of solving TISEs and dealing with the complexity of the many-body problem to calculate the eigenvalues of energy, the differences of the binding energies (mass defect) were used to derive the formula presented by Ahmed et al. [22]. Thus,

$$E_f = E_a = 3B(A,Z) + B(A-4,Z-2) - 2B(A-1,Z-1) - 2B(A-1,Z) \tag{5}$$

$$E = B(A,Z) - B(A-4,Z-2) \tag{6}$$

Substituting Eqs. (5) and (6) in (4), the preformation factor is

$$P_a = \frac{3B(A,Z) + B(A-4,Z-2) - 2B(A-1,Z-1) - 2B(A-1,Z)}{B(A,Z) - B(A-4,Z-2)} \tag{7}$$

The clustering amount CA is defined as the amount of alpha clustering occurring in the nucleus with respect to the clustering of the free alpha particle. It can be calculated from

$$CA = \frac{3B(A,Z) + B(A-4,Z-2) - 2B(A-1,Z-1) - 2B(A-1,Z)}{B(4,2)} \tag{8}$$

The B values of the mass regions for A = 280–316 are calculated using the excess mass taken from the Koura, Tachibana, Uno, and Yamada (KTUY) mass formula [36].

3 Results and Discussion

In this study, the P_α values of an alpha cluster inside a parent nucleus were determined using the CFM for even–even SHI with $Z = 116$ and $164 \leq N \leq 200$ (N is the neutron number). The calculation of P_α using Eq. (7) requires the determination of two energy values: E_α and the E (of the surface nucleon system) in the parent nucleus. These two values were determined using Eqs. (5) and (6). The results for CA, P_α, E_α, and E are shown in Table 1.

From Table 1, the minimum and maximum values of E_α are approximately 3400 and 4400 (keV), respectively. These values do not differ from those obtained for the majority of even–even nuclei (3000, 4500 (keV)) using the CFM [22]. This indicates the extent of the similarity of the clustering of the last four nucleons in both even–even heavy nuclei and SHN. As the total energy of the free alpha is approximately 28 MeV, the formation of alpha particles inside the parent nuclei is a preliminary action. The calculated E (the total clusterization energy) has a similar range (approximately 15000–21000 (keV)) to that of even–even heavy nuclei, which is approximately 16000–21000 (keV). The resemblance between these two energies for the superheavy and heavy nuclei indicates the validity of the CFM for the SHN.

In Figs. 1 and 2, E_α and E are depicted as functions of N. Hence, it can be seen that E_α decreases and E increases with increasing N. The reduction in E_α indicates that a nucleus with a greater number of neutrons has an alpha cluster with tighter structure, whereas the increase in E means that the surface nucleons have more degrees of freedom. The three rapid increases in E_α at $N = 180$, 186, and 192 (see Fig. 1) indicate that these alpha-cluster nucleons are more tightly bound than others. Slight fluctuations are apparent for the E values of the nuclei with $N = 178$,192, and in particular, 186 (see

Table 1 Surface energies (E), formation energies (E_α), preformation factors (P_α), and clustering amounts (CA) calculated from Eqs. (6), (5), (7), and (8), respectively, for superheavy $^{280-316}116$ isotopes

Isotope	N	E (keV)	E_α(keV)	P_α	CA
$^{280}116$	164	15340	4380	0.29	0.18
$^{282}116$	166	15870	4270	0.27	0.17
$^{284}116$	168	16480	4220	0.26	0.17
$^{286}116$	170	17120	3760	0.22	0.15
$^{288}116$	172	17460	3740	0.21	0.15
$^{290}116$	174	17720	3600	0.2	0.15
$^{292}116$	176	17970	3490	0.19	0.14
$^{294}116$	178	18140	3460	0.19	0.14
$^{296}116$	180	17800	3780	0.21	0.15
$^{298}116$	182	18020	3460	0.19	0.14
$^{300}116$	184	18150	3410	0.19	0.14
$^{302}116$	186	16570	3790	0.23	0.15
$^{304}116$	188	17420	3700	0.21	0.15
$^{306}116$	190	18171	3389	0.19	0.14
$^{308}116$	192	18940	3760	0.2	0.15
$^{310}116$	194	19880	3440	0.17	0.14
$^{312}116$	196	20140	3440	0.17	0.14
$^{314}116$	198	20380	3420	0.17	0.14
$^{316}116$	200	20620	3360	0.16	0.14

Fig. 2). These rapid changes are similar to that for the nuclei with $N = 128$ in the case of the even–even heavy nuclei [35].

It is important to remember that methods successfully applied in the study of even–even heavy nuclei are generally used as a basis for further research, in which they are expanded to encompass the investigation of unobserved SHN. As a result, comparison of the properties of these two nuclei groups provides further insight into the nuclear structure and allows greater understanding of this topic. Therefore, an assessment of the preformation factor values obtained in this study was conducted via comparison of these results with those obtained from other investigations. This comparison was made for both

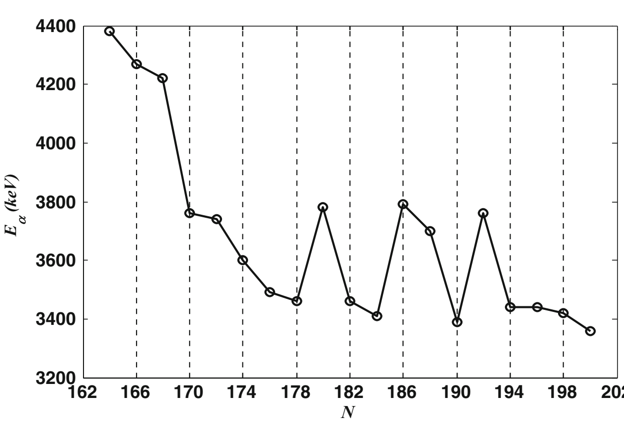

Fig. 1 Formation energy (E_α) calculated from Eq. (5) as a function of neutron number (N) for $^{280-316}116$ superheavy isotopes

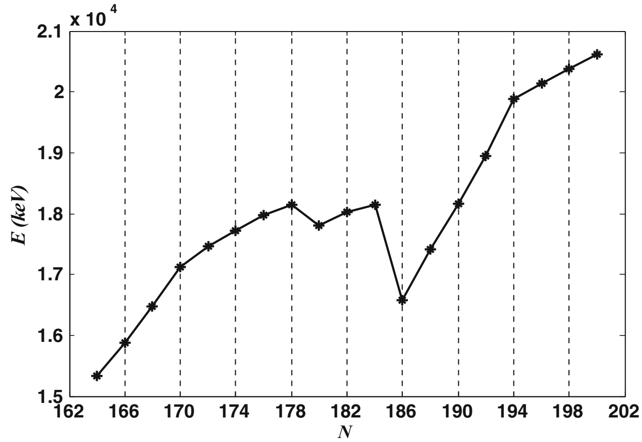

Fig. 2 Total energy (E) calculated from Eq. (6) as a function of neutron number (N) for superheavy $^{280-316}116$ isotopes

the superheavy and the heavy nuclei. However, note that the determination of P_α is significantly influenced by the chosen method (CFM, in this case). Therefore, some degree of discrepancy is expected. To determine realistic P_α for even–even heavy nuclei, a comparison of results obtained using all recently proposed methods is given in Table 2.

As can be seen from Tables 1 and 2, the range of P_α values for the $^{280-316}116$ SHI obtained in this study is 0.16–0.29. This range does not differ from the common range (0.06–0.18) reported in [14], which was obtained for a wide range of even–even SHN (Table 2). It is also similar to the range (0.05–0.2) obtained in [17] for $^{290-314}116$ SHI. Further, for SHN with $Z = 102$–120 and $N = 150$–180, values of P_α of 0.096–0.196 have been obtained [16]. The P_α for certain SHN have also been reported as 0.005–0.05 [34] and 0.003–0.172 [19]. All of the above results and the methods used to obtain them are presented in Table 2.

For the even–even heavy nuclei, the P_α range has been reported as 0.01–0.19, whereas the reported range for the isobaric even–even nuclei is 0.01–0.18, according to Seif and Seif, Shalaby, and Alrakshy, respectively [19, 20]. For the Po, Rn, and Ra isotopes, values of 0.02–0.9 have been obtained by Zhang, Le, and Zhang [15]. Based on experimental data for heavy nuclei, the range of P_α is closer to 0.01–0.3, according to Zhang et al. [26]. From the CFM, however, Ahmed et al. [22, 25] have reported that the realistic preformation factor values range between 0.1 and 0.27. This range is close to our results (0.16–0.29). Such calculations reveal that the alpha-clustering mechanism and P_α in even–even SHN are similar to those of heavy nuclei. The slightly higher probabilities of the alpha formation for the SHN compared to the heavy nuclei are related to greater degrees of freedom exhibited by the surface nucleons in the SHN.

The determined P_α of the $^{280-316}116$ isotopes are plotted in Fig. 3, as functions of the N values of the parent nuclei. In general, P_α decreases steadily with increasing N. A rapid

Table 2 Comparison of reported preformation factor value (P_α) ranges for different even–even $^{280–316}116$ superheavy nuclei

Reference	Nuclei	P_α range	Method
Our work	$^{280–316}116$	0.16–0.29	CFM
Zhang, Le, and Zhang (2009) [14]	Near $N = 162$ and 184	0.06–0.18	Penetration probabilities calculated using the WKB method and nuclear potential obtained using the double-folding potential of density dependence
Santhosh and Priyanka (2014) [17]	$^{290–314}116$ superheavy isotopes	0.05–0.2	Using the empirical relation that depends only on the mass number of the emitted particle
Silisteanu et al. (2011) [16]	For SHN with $Z = 102–120$ and $N = 150–180$	0.096–0.196	From the ratio of the calculated and measured half-lives obtained using self-consistent models for the alpha clustering and resonance scattering
Zhang and Royer (2008) [34]	$^{294}118$, $^{292}116$ $^{290}116$, $^{288}114$ $^{286}114$	0.005–0.05	Extracted from experimental alpha decay energies and half-lives using the GLDM
Seif (2013) [19]	Some superheavy nuclei with $A = 286–294$	0.003–0.172	Cluster model using the Hamiltonian energy density approach in terms of the SLy4 Skyrme-like effective interaction with the WKB approximation

increase occurs between $N = 184$ and 190; the $N = 184$ parent nucleus is of closed shell type with a minimum P_α, whereas the $N = 190$ isotope contains four neutrons more than the shell closure; thus, P_α is at the maximum value. A similar decrease in alpha clustering in open-shell nuclei up to shell closure has also been reported in [14, 15, 32, 35]. Several studies have confirmed that shell effects play an important role in alpha preformation and that the lowest P_α values are found in heavy nuclei with magic shells. This reduction confirms the obstacle to the occurrence of alpha clustering in parent nuclei at the magic number [11, 17, 19, 32, 35]. Further, this behavior has been reported in [14] for a wide range of even–even SHI and also for $^{290–314}116$ SHI [17]. Similarly, this behavior has been found in even–even heavy nuclei under application of the CFM [35] and in Po, Rn, Ra, Th, and U isotopes [15]. Thus, results similar to those shown in Fig. 3 have been obtained in the above cases.

The *CA* of the alpha particles was calculated from Eq. (8), and the results are given in Table 1. The range of values (0.14–0.18) is narrower than the P_α range (0.16–0.29). The examination of these two quantities within the CFM yields two insights: (1) P_α is associated with the internal interaction of nucleons in the parent nucleus, which causes the formation of a primary alpha particle in a clusterization process prior to emission, and (2) the *CA* is correlated with some of the internal interactions that cause the transition from the primary alpha to the free alpha during the alpha decay process [22]. From the relationship between CA and E_a (see Eq. (8)), the CA behavior is the same as that of E_a in Fig. 1 for increasing *N*. The consistency and the similarity between them have also been found elsewhere for heavy nuclei [22].

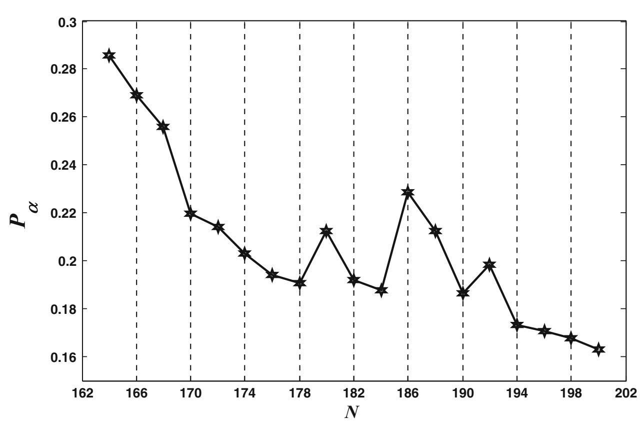

Fig. 3 Preformation factor (P_α) deduced from Eq. (7) as a function of neutron number (*N*) for superheavy $^{280–316}116$ isotopes

4 Conclusion

A more realistic preformation factor associated with the preformation probability of an alpha cluster within an even–even SHI ($^{280–316}116$) has been determined using the proposed CFM. The results were obtained by taking the energy of the surface nucleons as being from the last four nucleons only, as in the case of heavy nuclei. The preformation probability results indicate that the alpha clustering behavior in even–even SHI and heavy nuclei is similar. The initial alpha particle is formed inside the parent nucleus with a formation probability of approximately 0.17–0.29. Therefore, the inner alpha particle differs from the free particle. The probability values listed in Table 1 can be used as a preformation factor estimation to be incorporated in calculations of the alpha width. The clustering amount results listed in Table 1 may be applied in calculations of alpha decay constants in certain theoretical investigations.

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Appendix A. Hamiltonian of Clusterization States

In the quantum description of the many-body system, the total Hamiltonian operator of a system of A nucleons in the laboratory system can be written as [22, 25]

$$H=\sum_{i=1}^{A}\frac{\hat{P}_i^2}{2m_i}+\sum_{i<j=1}^{A}V_{ij}\,, \tag{A.1}$$

where $\hat{P}_i$ and m_i are the momentum operator and the mass of the i[th] nucleon that interacts with each j[th] nucleon in the system by a two-body potential V_{ij}. The total wavefunction of the system Ψ can be obtained from the solution of the TISE using the symmetrised Hamiltonian of the system of total energy E as

$$H\Psi=E\Psi. \tag{A.2}$$

In some nuclear reactions and decays, the system exhibits a behavior of two clusters that is called the clusterization effect. If the nucleons of the system are considered as two clusters with A_d nucleons and A_c nucleons, so $A=A_d+A_c$, then the Hamiltonian can then be written as

$$H=\left(\sum_{1\le i\le A_d}\frac{\hat{P}_i^2}{2m_i}+\sum_{i\le A_d,i<j\le A}V_{ij}\right)+\left(\sum_{A_d\le i\le A}\frac{\hat{P}_i^2}{2m_i}+\sum_{A_d\le i\le A,i<j\le A}V_{ij}\right), \tag{A.3}$$

where A_d are the nucleons of $i=1$ to A_d and A_c are the nucleons of $i=(A-A_d)$ to A. Each of the two terms inside each parenthesis in Eq. (A.3) is for a cluster of nucleons, but the potential energy sum is expanded to all nucleons. To separate this interaction, it requires that the kinetic energy of any nucleon is represented as a sum of two parts, $K_i=K_{oi}+K_{1i}$; the first term is due to the interaction of the i[th] nucleon with the other nucleons in its cluster (internal motion), and the second term is the interaction with the nucleons in the other cluster (for the center-of-mass motion). In terms of operators,

$$\hat{P}_i^2=\hat{P}_{oi}^2+\hat{P}_{1i}^2 \tag{A.4}$$

Substituting Eq. (A.4) in (A.3), merging the $\hat{P}_i^2$ terms in the first and second parentheses into one summation, splitting the range of the potential-energy sum in the first parenthesis into $i\le A_d, i<j\le A_d$ and $i\le A_d, A_d<j\le A$, and rearranging these terms, we obtain

$$H=\left(\sum{}_{1\le i\le A_d}\frac{\hat{P}_{oi}^2}{2m_i}+\sum{}_{i\le A_d,i<j\le A}V_{ij}\right)+\left(\sum{}_{A_d\le i\le A}\frac{\hat{P}_{oi}^2}{2m_i}+\sum{}_{A_d\le i\le A,i<j\le A}V_{ij}\right)+\left(\sum{}_{1\le i\le A}\frac{\hat{P}_{1i}^2}{2m_i}+\sum{}_{i\le A_d,i<j\le A}V_{ij}\right) \tag{A.5}$$

This equation contains three sets of parentheses. In each one, there are momentum operators and potential energies that could be separately used in the TISE to obtain the quantum-mechanical states and the wavefunction that describe the nucleons; therefore, these three terms can be written in terms of different Hamiltonians as

$$H=H_{fd}+H_{fc}+H_{dc} \tag{A.6}$$

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