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Nonlinear Recursive Design for the Underactuated IWP System

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ABSTRACT
The nonlinear feedback cascade model of the underactuated IWP is obtained through a collocated partial feedback linearization and a global change of coordinates. A nonlinear controller is designed with the nonlinear recursive technology. The system stability is proved with Lyapunov theory. The simulation results show the system is globally asymptotically stable to the origin.

Keywords: Underactuated mechanical system, IWP, recursive design, nonlinear control.

1. Introduction

Many researchers focus on the inertia wheel pendulum (IWP) to look it as a test bed for the effectiveness of control algorithms [1-5]. There are two control problems in this system: one is to control the pendulum swinging up from the hanging position to the upright vertical position; the other is to stabilize the IWP around its unstable equilibrium point. Much remarkable work is done: a control energy approach based on the passivity [1] is used to solve the balance problem of the IWP. The interconnection and damping assignment passivity based control [2] is used for the asymptotic stabilization of the IWP around its top position while two necessary matching conditions have to be satisfied in order to obtain a stabilizing controller. A nested saturation function [3] is used to stabilize the IWP. To reduce the dependence upon the Lyapunov functions, a backstepping approach [4] is proposed and a complex controller is obtained. A recursive design algorithm is designed for the inertia wheel pendulum, but a sigmoid function is needed [5].

In this paper the asymptotic stabilization is considered for the underactuated and strongly damping IWP around its unstable top position. Our main contribution is to utilize a suitable set of transformations that allows us to accomplish a nonlinear control design with the recursive technology to bring the system to the unstable top position. This paper is organized as follows. In Section 2 we present the IWP model and the model transformation to obtain the strict feedback cascade model. In Section 3 we develop the control strategy based on the recursive technology. In Section 4 some simulation results are given and Section 5 is the conclusions.

2. The IWP system model

The inertia wheel pendulum is shown in Figure 1, which consists of a physical pendulum with the equivalent mass m_1 and a revolving wheel with the equivalent mass m_2 at the end. The motor torque produces an angular acceleration of the revolving wheel which generates a coupling torque at the pendulum. The task is to stabilize the pendulum in its upright equilibrium point while the wheel stops rotating. The specific angle of the rotation of the wheel is not important. The revolving wheel is actuated and the joint of the pendulum at the base is unactuated. That is to say, it is a benchmark example of the underactuated mechanical system [6, 7], which has one control input τ and two configuration variables (q_1, q_2), and its Euler-Lagrange equations of motion can be obtained as

$$\begin{cases} m_{11}\ddot{q}_1 + m_{12}\ddot{q}_2 - m_0\sin(q_1) = 0 \\ m_{21}\ddot{q}_1 + m_{22}\ddot{q}_2 = \tau \end{cases} \tag{1}$$

where,

$$m_{11} = m_1 l_1^2 + m_2 L_1^2 + I_1 + I_2$$

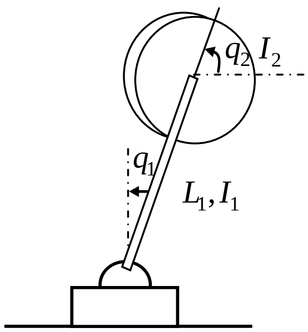

Figure 1. IWP system configuration.

$m_{12} = m_{21} = m_{22} = I_2$,

$m_0 = (m_1 l_1 + m_2 L_1)g$.

In order to simplify the system dynamics. The following collocated partial feedback linearization [8] is used

$$\tau = \left(m_{22} - \frac{m_{21} m_{12}}{m_{11}} \right) u + \frac{m_{21} m_0}{m_{11}} \sin(q_1) \qquad (2)$$

The dynamics of the shape variable q_2 is simplified to

$$\ddot{q}_2 = u$$

The following global change of coordinates [9] is designed

$$\begin{cases} z_1 = m_{11}\dot{q}_1 + m_{12}\dot{q}_2 \\ z_2 = q_1 \\ z_3 = \dot{q}_2 \end{cases} \qquad (3)$$

to transform the system dynamics into a nonlinear system as

$$\begin{cases} \dot{z}_1 = m_0 \sin z_2 \\ \dot{z}_2 = (z_1 - m_{12} z_3) / m_{11} \\ \dot{z}_3 = u \end{cases} \qquad (4)$$

Since that q_2 does not play any important role in the dynamics of the IWP, it is ignored as a state variable. From Eq. 4, it can be seen that the system model of IWP is a nonlinear feedback cascade model.

3. The nonlinear controller design through recursive technology

Since the model of IWP can be transformed into a cascade nonlinear system with a collocated partial feedback linearization Eq. 2 and a global change of coordinates Eq. 3, the controller can be designed with the recursive technology. The design process is:

Step 1.

From the dynamic equation of state x_1 in the IWP system model Eq. 4

$$\dot{z}_1 = m_0 \sin z_2 \qquad (5)$$

Firstly look z_2 as the virtual control input and define a reference trajectory z_{2r} for z_2 to follow as

$$z_{2r} = -m_0 \sin z_2 + z_2 - k_1 z_1$$

which leads to an error e_2 defined as

$$\begin{aligned} e_2 &= z_2 - z_{2r} \\ &= m_0 \sin z_2 + k_1 z_1 \end{aligned} \qquad (6)$$

where, k_1 is a positive constant.

Consider a scalar positive definite Lyapunov function given by

$$V_1 = \frac{1}{2}\left(z_1^{\,2} + e_2^{\,2} \right) \geq 0$$

The time derivative $\dot{V}_1$ is given by

$$\begin{aligned} \dot{V}_1 &= z_1 \dot{z}_1 + e_2 \dot{e}_2 \\ &= z_1 (e_2 - k_1 z_1) + e_2 \dot{e}_2 \\ &= -k_1 z_1^{\,2} + e_2 (\dot{e}_2 + z_1) \\ &= -k_1 z_1^{\,2} + e_2 (m_0 \cos z_2 (z_1 - m_{12} z_3) / m_{11} \\ &\quad + k_1 \dot{z}_1 + z_1) \end{aligned} \qquad (7)$$

We note that the variable z_3 enters the right hand side of Eq. 7. We now proceed to look z_3 as the control variable and design a reference trajectory z_{3r} for it to make the second term of right hand in Eq. 7 be non-positive.

Step 2.

In step 1, the time derivative of the Lyapunov function V_1 is obtained in Eq. 7. In order to make the $\dot{V}_1$ be a negative definite function, state z_3 is looked as the virtual control input in Eq. 7. A reference trajectory z_{3r} is defined as

$$z_{3r} = -m_0 \cos z_2 \frac{z_1 - m_{12} z_3}{m_{11}} + z_3 - k_1 \dot{z}_1 - z_1 - k_2 e_2$$

The tracking error e_3 defined as

$$\begin{aligned} e_3 &= z_3 - z_{3r} \\ &= m_0 \cos z_2 \frac{z_1 - m_{12} z_3}{m_{11}} + k_1 \dot{z}_1 + z_1 + k_2 e_2 \end{aligned} \tag{8}$$

So

$$\dot{V}_1 = -k_1 z_1^2 - k_2 e_2{}^2 + e_2 e_3$$

We modify the scalar positive Lyapunov function $V_1 \geq 0$ as

$$\begin{aligned} V_2 &= V_1 + \frac{1}{2} e_3{}^2 \\ &= \frac{1}{2} z_1{}^2 + \frac{1}{2} e_2{}^2 + \frac{1}{2} e_3{}^2 \geq 0 \end{aligned} \tag{9}$$

Differentiating V_2

$$\begin{aligned} \dot{V}_2 &= -\dot{V}_1 + e_3 \dot{e}_3 \\ &= -k_1 z_1{}^2 - k_2 e_2{}^2 + e_3(\dot{e}_3 + e_2) \end{aligned} \tag{10}$$

From Eq. 8,

$$\begin{aligned} \dot{e}_3 = &-m_0 \sin z_2 \dot{z}_2 \frac{z_1 - m_{12} z_3}{m_{11}} + m_0 \cos z_2 \frac{\dot{z}_1 - m_{12} \dot{z}_3}{m_{11}} \\ &+ k_1 \ddot{z}_1 + \dot{z}_1 + k_2 \dot{e}_2 \end{aligned} \tag{11}$$

The system control variable $u = \dot{z}_3$ enters in the right hand of the Eq. 11.

Step 3.

In order to make the $\dot{V}_2$ be a negative definite function, we can make the following equation hold since the control variable arises.

$$\dot{e}_3 + e_2 = -k_3 e_3 \tag{12}$$

Such that

$$\begin{aligned} &-m_0 \sin z_2 \dot{z}_2 \frac{z_1 - m_{12} z_3}{m_{11}} + m_0 \cos z_2 \frac{\dot{z}_1 - m_{12} \dot{z}_3}{m_{11}} \\ &+ k_1 \ddot{z}_1 + \dot{z}_1 + k_2 \dot{e}_2 + e_2 = -k_3 e_3 \end{aligned} \tag{13}$$

Therefore, the control law can be obtained from Eq. 13 as

$$\begin{aligned} u = \frac{1}{m_0 m_{12} \cos z_2} \{ &-m_0 \sin z_2 \dot{z}_2 (z_1 - m_{12} z_3) + m_0 \cos z_2 \dot{z}_1 \\ &+ k_1 m_{11} \ddot{z}_1 + m_{11} \dot{z}_1 + k_2 m_{11} \dot{e}_2 + m_{11} e_2 + k_3 m_{11} e_3 \} \end{aligned} \tag{14}$$

Theorem 1: The feedback cascade model Eq. 4, which is transformed from the IWP system described by Eq. 1 through the collocated partial feedback linearization Eq. 2 and the global change of coordinates Eq. 3, is asymptotically stable under the control input Eq. 14.

Proof:

The recursive design process has proved: the time derivative of the chosen positive definite Lyapunov function V_2 is negative definite. That is to say, the three terms of the right hand in Eq. 9 is asymptotically approach to 0. Since that the first term $z_1{}^2/2$ approaches to 0, z_1 must asymptotically approach to 0. From the second term $e_2{}^2/2$ approaches to 0, e_2 must asymptotically approach to 0 and it is known from Eq. 6 that z_2 must asymptotically approach to 0. The third term $e_3{}^2/2$ approaches to 0 implies that z_3 asymptotically approach to 0 from Eq. 8.

Therefore, the system states (z_1, z_2, z_3) of the IWP described by Eq. 4 asymptotically approach to $(0,0,0)$.

Remark 1: Both the collocated partial feedback linearization Eq. 2 and the global change of coordinates Eq. 3 in the second section are invertible transformation, which is

$$\begin{cases} q_1 = z_2 \\ \dot{q}_1 = \dfrac{1}{m_{11}}(z_1 - m_{12} z_3) \\ \dot{q}_2 = z_3 \end{cases} \tag{15}$$

It can be seen from Eq. 15 that (z_1, z_2, z_3) asymptotically approach to $(0,0,0)$ implies that $(q_1, \dot{q}_1, \dot{q}_2)$ approach to $(0,0,0,0)$. The control input τ can be calculated with Eqs. 2, 14 and 15.

Remark 2: There is a singularity when $z_2 = \pm\pi/2$ in the controller Eq. 14. The method to deal with the singularity in the simulations is: the $\cos z_2$ in Eq. 14 is represented by a positive number (φ) for $z_2 \in (\pi/2 - \Delta, \pi/2)$ or $z_2 \in (-\pi/2, -\pi/2 + \Delta)$, and by a negative number ($-\varphi$) for $z_2 \in (\pi/2, \pi/2 + \Delta)$ or $z_2 \in (-\pi/2 - \Delta, -\pi/2)$. The value of φ can be decided by the output limit of the actual controller.

Remark 3: The design method is proposed for the nonlinear feedback cascade system Eq. 4, so it can be used for all the underactuated mechanical systems that can be transformed to the cascade system Eq. 4, such as the TORA and the Acrobot. Compared with other recursive controllers, the proposed algorithm is simple and easy to be implemented as the implementation of the neural control systems [10].

4. Simulation studies

In order to test the proposed control algorithm, the following system parameters [11] are used:

$m_{11} = 4.83 \times 10^{-3}$, $m_{12} = m_{21} = m_{22} = 32 \times 10^{-6}$, $m_0 = 38.7 \times 10^{-3} \times 9.8$.

The parameters of the nonlinear controller are chosen as

$k_1 = 4$, $k_2 = 4$, $k_3 = 4$, $\varphi = 0.001$, $\Delta = 0.057°$.

The simulation results are shown in Figures 2-5.

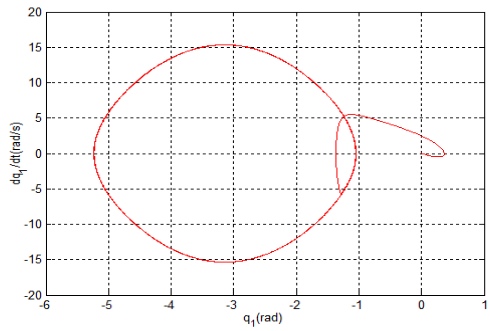

Figure 2. The phase plane of q_1 in the system simulation.

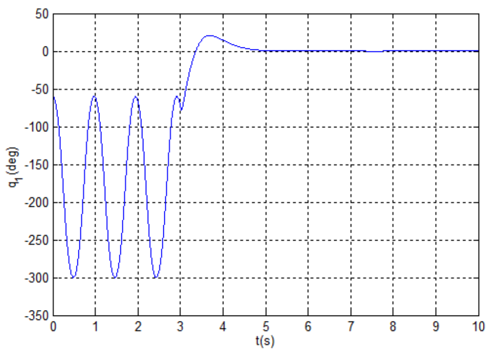

Figure 3. The time response of q_1 in the system simulation.

The simulation results in Figures 2-5 are obtained under the initial state $(z_1, z_2, z_3) = (0, -60^\circ, 0)$ i.e. $(q_1, \dot{q}_1, q_2) = (60^\circ, 0, 0)$ and the proposed control algorithm is added at 3rd second. Figure 2 is the phase plane of the $(q_1, \dot{q}_1)$, Figure 3 is the time response of q_1, Figure 4 is the time response of $\dot{q}_2$ and Figure 5 is the control torque τ of the IWP. It can be seen from the simulation results that: the IWP system is freely swinging before the control algorithm is added and the IWP system is asymptotically stable under any initial states with the proposed control algorithm. On the other hand, the control performance can be improved through adjusting the parameters of the proposed controller. Lots of simulation experiments show that the parameters k_2, k_3 respectively correspond to the system state $q_1, \dot{q}_2$, therefore it is easy to adjust the parameters for an improved system performance.

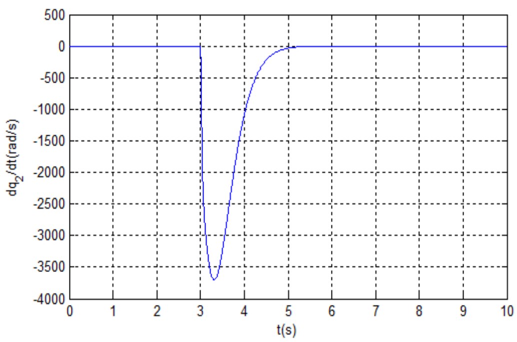

Figure 4. The time response of $\dot{q}_2$ in the system simulation.

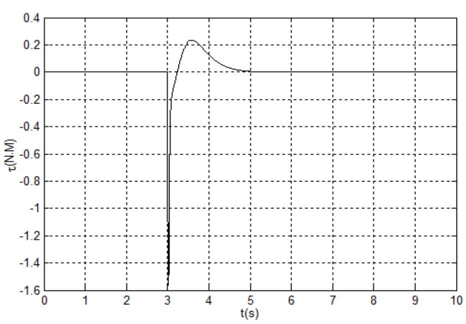

Figure 5. The control torque τ in the system simulation.

5. Conclusions

A collocated partial feedback linearization and a global change of coordinates are used to transform the underactuated IWP to a nonlinear feedback cascade system. A nonlinear control algorithm is proposed with the recursive technology. A Lyapunov function is found step by step in the design procedure and illustrates the system stability. The design method is proposed for the nonlinear feedback cascade system, so it can be used for all the underactuated mechanical system that can be transformed to the cascade system.

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