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GROWTH AND YIELD PREDICTION USING THE MODIFIED BUCKMAN MODEL[1]

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ABSTRACT – A model to manage even-aged stands was developed using a modification of the Buckman model. Data from *Eucalyptus urophylla* and *Eucalyptus cloeziana* stands located in the Northern region of Minas Gerais State, Brazil were used in the formulation of the system. The proposed model generated precise and unbiased estimates in non-thinned stands.

Keywords: Buckman model, growth and yield, basal area growth.

PREDIÇÃO DE CRESCIMENTO E PRODUÇÃO USANDO O MODELO BUCKMAN MODIFICADO

RESUMO – Desenvolveu-se um modelo para o manejo de florestas equiâneas usando-se uma modificação do modelo de Buckman. Foram utilizados na formulação do sistema dados provenientes de povoamentos de ***Eucalyptus urophylla*** *e* ***Eucalyptus cloeziana*** *localizados na região Norte do Estado de Minas Gerais, Brasil. O modelo proposto gerou estimativas precisas e não tendenciosas para povoamentos não desbastados.*

Palavras chaves: modelo Buckman, crescimento e produção, crescimento de área basal.

1. INTRODUCTION

One of the three essential elements of forest management is prognosis, that is, growth and yield prediction. Forests cannot be managed without proper information on the future availability of wood and/or sub-products. Such information is obtained by growth and yield models.

Buckman (1962) and Clutter (1963) were the first researchers to explain the mathematical relationships between growth and yield. Clutter derived compatible models for growth and yield in cubic volume of *Pinus taeda*, ensuring that the algebraic form of the yield model could be derived by the mathematical integration of the growth model. Later, Sullivan and Clutter (1972) perfected this model, resulting simultaneously in yield and accumulated growth estimates, as a function of an initial age, projected age, site index and basal area.

A model to estimate the yield, using age, site index, Reineke index and a competition index as independent variables was fitted to natural *Pinus taeda* plantations by MacKinney and Chaiken (1939). According to Avery and Burkhart (1994) many researchers have since used multiple regression techniques to predict growth and yield for total plantations or for some commercial portion of the plantation.

Buckman (1962) worked with periodic growth data from *Pinus* and used the following regression model to estimate the basal area growth rate in function of age, basal area and the site index: $dV=\beta_0+\beta_1 B+\beta_2 B^2+\beta_3 I+\beta_4 I^2+\beta_5 S$, where $dV=$ net annual periodic increment

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of the basal area, B = basal area, I = age and, and S = site index.

The author then substituted the basal area growth estimated by the equation derived from the previous model, and the height growth, in a differential form from a plantation volume equation, represented by $V = f(k,B,Hd)$ where V is the volume, k a constant representing the mean form of the trees in the plantation, and Hd is the dominant height in the plantation. Summing the results, the author obtained estimates of net accumulated yield for several combinations of independent variables.

Later, Clutter (1963) formalized the relationships necessary between growth and yield observed in the Buckman model and developed equations to express net volume and growth rate of the basal area as function of age, from the site index and the plantation density, by differentiation of the yield function. Sullivan and Clutter (1972) refined this technique, estimating the cumulative yield and the growth as a function of the initial age of the plantation, the initial basal area of the site index and future age.

Several studies were carried out after 1972, including those by Beck and Della-Bianca (1972), Burkhart and Sprinz (1984), Trevizol Jr. (1985), Resende (1991) Campos et al. (1996) among many others, showing the flexibility of the Buckman model. In spite of this, the model has not been used in Brazil. In this country, the growth and yield modeling have always been conducted using the Clutter model (CAMPOS e LEITE, 2002).

This study was carried out with the objective of proving the efficiency of this model in predicting growth and yield in eucalyptus plantations in Brazil. For greater efficiency, a modification of the model is presented and fitted to *Eucalyptus urophylla* and *Eucalyptus cloeziana* plantations located in the northern region of Minas Gerais State, Brazil.

2. MATERIAL AND METHODS

2.1. Data used in the growth and yield study.

The data used in this study were provided from *Eucalyptus urophylla* and *Eucalyptus cloeziana* plantation located in the northern region of Minas Gerais State. Two data sets were used from a research project implemented in 1988 by the Forest Research Society - SIF (Sociedade de Investigações Florestais) of the Federal University of Viçosa, Minas Gerais, Brazil. Each set consists of two groups of 30 permanent 500 m^2 plots measured at ages ranging from 28 to 96 months. When the project was established, the variations in site quality were considered and the initial spacing was 3.0 x 1.5 m. On each occasion the following parameters were measured: the diameter at 1.30 m (*dbh*) of all the trees, the total height of the trees of the first row of each plot and the total height of ten dominant trees.

Data from 225 sample trees (130 *Eucalyptus urophylla* and 95 *Eucalyptus cloeziana* trees) were used to fit the volumetric model. These trees were felled and cubed using the Smalian formula, with 1.0 m long sections and included diameters between 4 and 18 cm and total heights between 5 and 24 m.

2.2. Site quality, height and volume per tree

The site quality was determined using the guide-curve method (CAMPOS e LEITE, 2006) and Chapman-Richards model, employing the following expression:

$$S = Hd\left(1-e^{-\beta_1 A_i}\right)^{\beta_2}\left(1-e^{-\beta_1 A}\right)^{-\beta_2}$$

where Hd = heights of dominant trees, m, A = age in months and β_0, β_1, β_2 = parameters, A_i = index age = 66 months.

The following models were fitted to estimate the total heights per tree:

$$Ht = \beta_0 + \beta_1 A + \beta_2 S + \beta_3 Lndbh + \beta_4 Lndbh.A + \beta_5 LnA + \varepsilon,$$

where Ht = total height, S= site index, and β_i, *dbh, A* and ε as previously defined.

The volumetric model proposed by Leite et al. (1995) was fitted to quantify the volume per tree. The coefficient of determination and the graphic analyses of the residues were the statistics used to assess the estimates generated by all the equations fitted in this study.

2.3. Growth and Yield model

After obtaining the totalized plots, the model

$$LndB = \beta_0 + \beta_1 A + \beta_2 S + \beta_3 A^{-1} + \beta_4 LnB_1$$

was fitted, where dB = net periodic increase of the basal area, m^2ha^{-1}, B_1= basal area at initial age, m^2ha^{-1}, A ,S, Ln and β_i as previously defined.

The previous functional relationship is different from that proposed by Buckman (1962) and is a variation of the Schumacher model. Here the variable was included in the model based on results from some preliminary adjustments.

The yield in basal area (B) was obtained by the sum of the successive increases, that is, for an interval of ages A_1 and A_2,

$$B = \int_{A_1}^{A_2} e^{c+\beta_1+\beta_3 A^{-1}} dA$$, where B = basal area, m^2ha^{-1}, A_1 and A_2 = age 1 and age 2, in months, $C=\beta_0+\beta_2 S+\beta_4 Ln\beta_1$.

After obtaining the estimates of basal area increase and the height and yield capacity, the model suggested by Schumacher (1939) was used to predict the volume obtained $LnV = \beta_0 + \beta_1 S + \beta_2 A^{-1} + \beta_3 LnB + \varepsilon$, where V = volume, in m^3ha^{-1}, β_0, β_1, β_2, β_3 = parameters to be estimated, and A, B and S as previously defined.

3. RESULTS AND DISCUSSION

The site quality was classified using the independent equations for the two species, using an age index (A_i) equal to 66 months, and the following equations were obtained, respectively, for *Eucalyptus cloeziana* and *Eucalyptus urophylla:*

$$Hd = 26.51124\left(1-e^{-0.013944.A}\right)^{(1-0.112217)^{-1}},$$

with $r_{Y\hat{Y}} = 0.905$, $Y = Hd$,

$$Hd = 20.62326\left(1-e^{-0.018998.A}\right)^{(1-0.059762)^{-1}},$$

with $r_{Y\hat{Y}} = 0.759$.

The model fitted to estimate the total height, resulted in the following equations:

$$Ht = -24.0243 - 0.1773A + 0.5294S + 1.0789Lndbh + $$
$$+ 0.0714Lndbh.A + 6.6414LnA,$$

with R^2 = 0.944 and S_{YX} = 0.95 m, for *Eucalyptus cloeziana,* and

$$Ht = -26.0664 - 0.1934A + 0.4913S + 1.3943Lndbh +$$
$$+ 0.0641Lndbh.A + 7.7363LnA,$$

with R^2 = 0.910 and S_{YX} = 0.97 m, for *Eucalyptus urophylla*.

These equations do not ensure meeting of the biological phenomenon involved, but they were efficient quantitatively for the height estimates and are useful when the mechanistic models do not apply.

The multiple volume equations obtained from the two species did not reveal bias in the estimates, proving their efficiency:

Eucalyptus cloeziana and $dbh \le 10$ cm:

$$V = 0.000116 dbh^{1.298786} Ht^{1.213757} e^{-2.06221.k / dbh\left(1-\left(\frac{d_u}{dbh}\right)^{1+0.51801 d_u}\right)}, \; dbh \le 10 \text{ cm}, \; r_{V\hat{V}} = 0.980,$$

Eucalyptus cloeziana and $dbh > 10$ cm:

$$V = 0.000071 dbh^{1.59879} Ht^{1.177664} e^{-3.90959.k / dbh\left(1-\left(\frac{d_u}{dbh}\right)^{1+0.35880 d_u}\right)}, \; dbh > 10 \text{ cm}, \; r_{V\hat{V}} = 0.976,$$

Eucalyptus urophylla and $dbh \le 12$ cm:

$$V = 0.000086 dbh^{1.329838} Ht^{1.322434} e^{-2.29267.k / dbh\left(1-\left(\frac{d_u}{dbh}\right)^{1+0.448761 d_u}\right)}, \; dbh \le 12 \text{ cm}, \; R_{V\hat{V}} = 0.971,$$

Eucalyptus urophylla and $dbh > 12$ cm:

$$V = 0.000054 dbh^{1.837853} Ht^{1.044908} e^{-3.18198.k / dbh\left(1-\left(\frac{d_u}{dbh}\right)^{1+0.30719 d_u}\right)}, \; dbh > 12 \text{ cm}, \; R_{V\hat{V}} = 0.984,$$

where V = volume in m^3, dbh = diameter outside bark in cm measured at 1.30 m, k = dummy variable, where k = 0 volume outside bark and k = 1 volume inside bark, and d_u = commercial superior diameter, outside bark, in cm.

The growth and yield estimates were obtained from the annual measurements of the permanent plots. However, in this study, the annual growth was transformed in mean monthly growth, which corresponds to the mean point of the measurement interval of one year. For this purpose the following equations were generated:

$$LndB = 0.1046 - 0.0279A + 0.0668S - 66.9749A^{-1} + 0.0651LnB_1 \quad (1),$$

with $R^2 = 0.450$, for *Eucalyptus urophylla*, and

$$LndB = 2.516 - 0.04328A + 0.083S - 105.6486A^{-1} - 0.518LnB_1 \quad (2),$$

with $R^2 = 0.710$, for *Eucalyptus cloeziana*.

This model fitted best the basal area increase data. All the independent variables were significant at 1% probability. The inclusion of the age variable twice in the model eliminated an undesirable tendency ascertained when using the model in its original form, and despite a "low" R^2, the distribution of the percentage residues can be considered acceptable (Figure 1).

The basal area yield can be obtained from the sum of the basal area increases by differential equations, or by approximations of a table of basal area growth, or by repeated solution of the basal area growth function. It can be written that $dB = \frac{dB}{dA} = e^{c - 0.0279A - 66.9749A^{-1}}$ (3) and $dB = \frac{dB}{dA} = e^{c - 0.04328A - 105.64286A^{-1}}$ (4) for, respectively, *Eucalyptus urophylla* and *Eucalyptus cloeziana*, where *dB* is the basal area growth, *B* the yield in basal area, *dB/dA* the change in the basal area with age, and *C* a constant that contains the terms *S* (site index), LnB_1 (logarithm of the initial basal area) and the β_0 value (intercept). Integration of the growth equations generates the yield function.

If a function *f(x)* is continuous in an interval [a,b] its primitive F(x) can be known, corresponding to the integral defined of this function, in this interval. Thus:

$$\int_a^b f(x)dx = F(b) - F(a) \quad (5)$$

In the case of this study, the primitive value of F(x) is not known or easily obtained. Thus, to calculate the value of the defined *f(x)* integral, a numerical method had to be used. The Newton-Cortes formulas were used that employed *f(x)* values, where the *x* values were equally spaced (monthly values). Specifically among the Newton-Cortes formulas the compound formula of the 1st Simpson rule was used (BARROSO et al., 1982). This rule is obtained by approximating the function *f(x)* by a 2nd degree interpoled polynomial.

Thus $f(x) = P_2(x) = y_0 + z\Delta y_0 + \frac{z(z-1)}{2!}\Delta^2 y_0$ and

$$A = \int_a^b f(x)dx = \int_a^b P_2(x)dx = \int_a^b [y_0 + z\Delta y^0 + \frac{z(z-1)}{2!}\Delta^2 y_0]dx$$

Developing this integral, we obtain $A = \frac{h}{3}(y_0 + 4y_1 + y_2)$ (6), which is the 1st Simpson rule. To obtain the compound formula, the integration interval [a,b] should be divided into *n* equal subintervals of *h* amplitude and the 1st Simpson rule should be applied to each pair of subintervals. Thus, it follows that

$$n = \frac{b-a}{h}, \; x_i; \; i = 0, 1, 2, \ldots, n, \; I = \int_a^b f(x)dx$$

$$A = \frac{h}{3}[y_0 + 4y_1 + y_2] + \frac{h}{3}[y_2 + 4y_3 + y_4] + \ldots + \frac{h}{3}[y_{n-2} + 4y_{n-1} + y_n]$$

$$A = \frac{h}{3}[y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + \ldots + 2y_{n-2} + 4y_{n-1} + y_n] \quad (7)$$

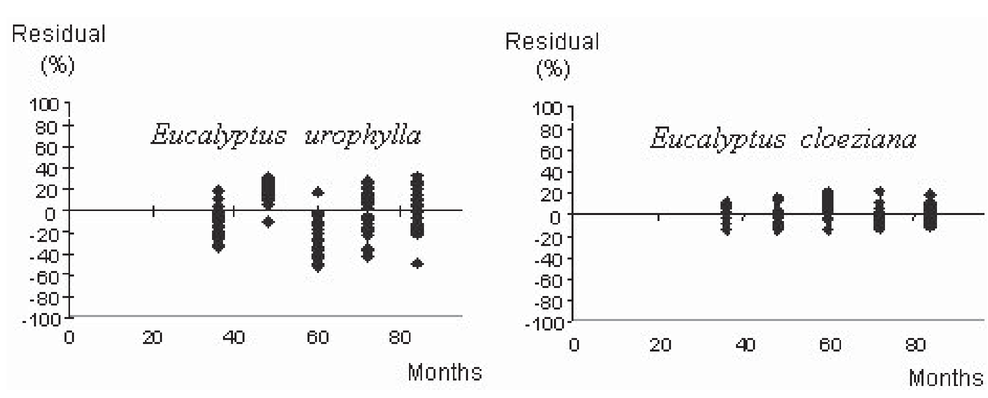

Figure 1 – Distribution of percent residues of the basal area increase estimates.
Figura 1 *– Distribuição dos resíduos porcentuais das estimativas de incremento em área basal.*

Applying the Simpson rule to the growth function in *Eucalyptus urophylla* basal area,

$B = \int_a^b e^{(C-0,0279\,A-66,9749\,/\,A)}dA$ was obtained and B is the yield in basal area (m²ha⁻¹) in the interval [a,b], and C is the constant that contains $\beta_0+\beta_2 S+\beta_4 LnB_1$. For example, when considering the interval a = 30 months and b = 42 months, a site index of 13 and an initial basal area (at 30 months) of 4.52331, we have

$$B = \int_{30}^{42} e^{(1.07125-0,0279A-66.9749A^{-1})}dI$$

The first subinterval y_0 for age 30 months is

$$y_0 = e^{(1.07125-0.0279(30)-66.9749/30)} = 0.13557$$

The second subinterval y_1 for 31 months is

$$y_1 = e^{(1.07125-0.0279(31)-66.9749/31)} = 0.14169$$

The last subinterval y_n for age 42 months is

$$y_1 = e^{(1.07125-0.0279(42)-66.9749/42)} = 0.18356$$

Applying the subinterval values (12 months) in equation 7, we obtain $B = \int_{30}^{42} e^{(C-0.0279I-66.9749/A)}dA = 1.96903$ (α_1), which is a yield in basal area at the interval 30 to 42 months.

The possibility of integrating the growth functions allows proof of the compatibility between growth and yield. The sum of the basal area increases obtained at various subintervals of the interval [a,b] is equal to the integral of the growth function at this same interval.

The second method of sum of the basal area increases results in the growth of the mean point of a determined interval (Figure 2).

The yield in basal area at the interval [30,42] is obtained by multiplying the number of subintervals (12 months) by the growth obtained from the growth curve (equation 1) for example, for site index S = 13 and initial basal area B_1 = 4.5233 at the mean point of the interval. The growth in basal area at 36 months is dB=0.166 and the yield in basal area in the period [30, 42] is 1.99 m²ha⁻¹ (α_2).

The third method to obtain yield in basal area consists of repeated solutions of the growth in basal area function. This method involves the solution of the growth equation for a determined site index, age and plantation density. The growth is then added to the initial density of the plantation, one month is added to the age and the equation is solved again. For example, for the same site index, 13, considering equation 1 of basal area growth of 4.5233 and age ranging from 30 to 42 months, yield is obtained by repeated solutions. In this case, the yield in basal area in the 12 month period is 6.4640 – 4.5233 – 1.94 m²ha⁻¹ (α_3).

The comparison of the results of the three methods, for the site index 13 and initial basal area of 4.5233 m²ha⁻¹ (α_1, α_2 and α_3) shows that the difference among them was small. Although not presented, the difference was even less for higher ages, within the interval studied. Therefore, any one of these methods can be used to estimate the basal area growth.

After obtaining the yield in basal area, the yield and growth in volume with bark can be estimated to build the yield table. The equations of plantation volume fitted with yield data as a function of the site index (S) age (I) and basal area (B) for *Eucalyptus urophylla* and *Eucalyptus cloeziana* were, respectively

$$Ln(V_{CC}) = 1.1463 + 0.0479.S - 25.0645 / A + 1.1678.Ln(B) \quad (8)$$

$$Ln(V_{CC}) = 1.1300 + 0.0664.S - 31.2344 / A + 1.1215.Ln(B) \quad (9)$$

With determination coefficients 0.998 and 0.998.

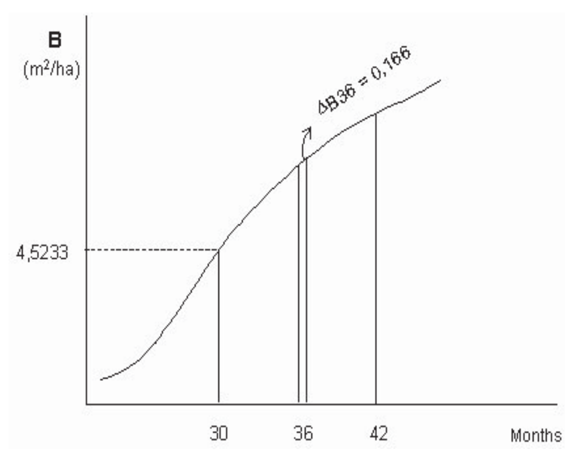

Figure 2 – Growth in basal area at the mean point of the 30 to 42 month interval (dB$_{36}$) for *Eucalyptus urophylla,* site index 13 and initial basal area of 4.5233m²ha⁻¹.

Figura 2 – *Crescimento em área basal no ponto médio do intervalo de 30 a 42 meses (dB$_{36}$) para Eucalyptus urophylla, índice de local 13 e área basal inicial de 4,5233 m²ha⁻¹.*

The increase in volume is obtained by deriving equation 8, for age, that is

$$dV / dA = V[-b_2 / A^2 + b_3 / B * dB / dA] \quad (10)$$

$$dV / dA = V[25.0645 / A^2 + 1.1678 / B * dB / dA] \quad (11)$$

The yield tables or curves are obtained by the calculation of the increase and yield in basal area and the growth and yield in volume, as shown in Table 1, for some ages. In this table, the increase in basal area obtained from equation 1 is shown in column 2. The method of obtaining yield in basal area by repeated solutions results in the total basal area (column 3). Replacing the total basal area in equation 8, yield in volume is obtained (column 4). The felling age, from the point of view of mean yield, occurs when the mean monthly increase (column 6) is maximum and equal to the monthly periodic increase (column 5) in the example, 96 months. Finally increase in volume can be obtained, using equation 11 (column 7) and summing the increases in volume, the yield in volume is obtained (column 8). No type of tendentiousness was observed in the growth and yield estimates and they can be considered precise (Figure 3).

Table 1 – Table of variable density yield for *Eucalyptus urophylla* in the Northern Region of Minas Gerais State, Brazil, with initial basal area of 4.5233 m^2ha^{-1} and site index 13
Tabela 1 – *Tabela de produção de densidade variável de Eucalyptus urophylla, na região de Montes Claros, Minas Gerais, com área basal inicial de 4,523 m^2ha^{-1} e índice de local 13*

Age (months)	Increase Área Basal (m^2ha^{-1})	Basal area (m^2ha^{-1})	Volume (m^3ha^{-1})	Monthly periodic increase (m^3ha^{-1})	Monthly mean increase (m^3ha^{-1})	$\frac{dV}{dA}$ (m^3ha^{-1})	VOL+ $\frac{dV}{dA}$ (m^3ha^{-1})
30	0,1356	4,6589	15,3405		0,5114		
31	0,1417	4,8006	16,3207	0,9802	0,5265	0,9882	16,3287
32	0,1474	4,9480	17,3401	1,0194	0,5419	1,0277	17,3484
36	0,1664	5,5870	21,7996	1,1706	0,6055	1,1797	21,8087
...	...	...	...	...	...	...	...
84	0,1262	13,6759	92,3101	1,3219	1,0989	1,3229	92,3111
96	0,0998	15,0155	106,8667	1,1199	1,1132	1,1199	106,8668
97	0,0977	15,1133	107,9696	1,1030	1,1131	1,1030	107,9697
98	0,0957	15,2090	109,0558	1,0861	1,1128	1,0861	109,0557

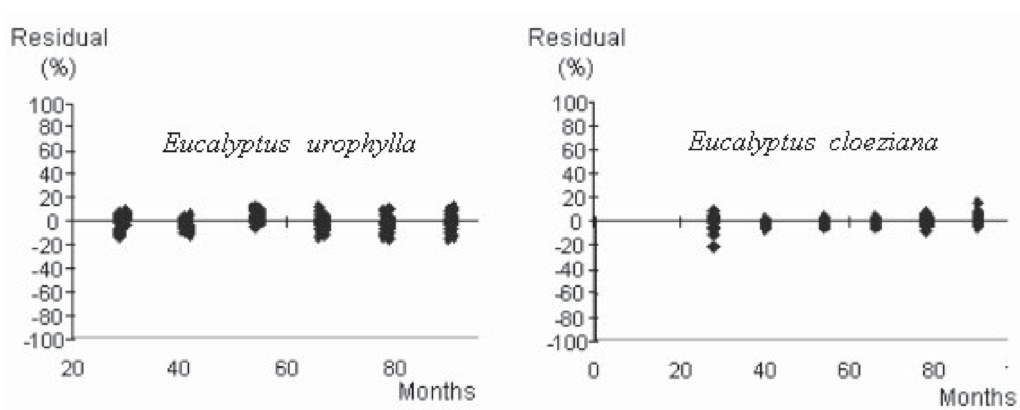

Figure 3 – Distribution of observed and estimated volume residues for *Eucalyptus urophylla* and *Eucalyptus cloeziana* projected by the growth and yield model.
Figura 3 – *Distribuição de resíduos dos volumes observados e estimados pelo modelo de crescimento e produção de Eucalyptus urophylla e Eucalyptus cloeziana.*

4. CONCLUSIONS

The Buckman model, modified in this study, results in unbiased estimates and is indicated to estimate the present and future growth and yield of eucalypt plantations. Three alternatives for estimating the growth in basal area are presented, all of which are efficient.

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