Modelagem e otimização da operação de veículos na mineração de cobre subterrâne

Modeling and optimization of vehicle operations in underground copper mining

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Resumo

Na mineração subterrânea, diariamente, uma frota de veículo LHD (load-h dump) deve ser alocada à rede de galerias para extrair o minério segundo um pl estratégico de trabalho. Esse plano é definido pela alta gerência e contém o númer cargas a serem extraídas desde os pontos de remoção de uma dada galeria para c turno de trabalho. Nesse trabalho, formula-se um modelo de programação inteira para minimizar o makespan de uma galeria de trabalho, e propõe-se um algori ótimo em tempo polinomial, para a sua resolução. Em seguida, obtém-se um conju de regras de decisão, a partir do algoritmo proposto, o qual é integrado a um proc de tomada de decisão (PTD). Esse PTD é de simples execução para os operadore LHDs, determina o makespan ótimo e, se existe a possibilidade de conclusão de do turno de trabalho. Finalmente, realiza-se uma análise comparativa entre o P proposto e àquele utilizado na mina chilena subterrânea de cobre *El Teniente.* resultados mostram que a experiência acumulada dos operadores converge para s ções próximas ao makespan ótimo.

Palavras-chave: Pesquisa operacional na mineração, algoritmo ótimo em tempo linomial, processo de tomada de decisão, gestão da frota dos veículos da mineraçã

Abstract

In underground mining, daily a fleet of LHDs must be allocated to a hau network of drifts for extracting ore according to a plan-driven strategy. This pla hierarchically decided by a higher management level and it contains the numbe ore bucketfuls to be extracted from each drawpoint within a drift for each wor shift. In this paper, an integer programming (IP) model for minimizing the makes of drift workload is formulated and a polynomial time optimal algorithm for its r lution is proposed. Next, a set of decision rules obtained from the algorithm abo integrated into the decision-making process (DMP). This DMP is simple to exe for LHD operators, determines the optimal makespan, and whether or not it ca carried out in the working shift. Finally, a comparative analysis between the D proposed and the DMP used in El Teniente copper underground Chilean min studied. The results show that the cumulative operators experience has converge solutions near to optimal makespan.

Keywords: *Operations research in mining, polynomial time optimal algorithm,*

1. Introduction

In block caving underground mining, good production-level performance is the key for all the other levels vertically positioned as reduction and transportation. At this latter level, there is a haulage network containing parallel drifts, turning points and drawpoints, where ore must be extracted by LHDs (load-haul-dump) according to the fleet management established.

LHD fleet management addresses three decision problems: dispatching, routing and scheduling. The aim of dispatching is to choose, for one or many vehicles, a new destination (loading or dumping sites). Routing consists of choosing the best route (road segments) from the origin to the destination. Finally, scheduling consists of deciding the speeds and waiting times of vehicles on road segments of a route to avoid conflicts between vehicles (Gamache et al., 2005).

Ideally, the optimal decision must solve the three problems in a unique model. Nevertheless, its development and later implementation assumes that data on vehicle location are available in a certain period, that waiting and stop areas at the drift are known and fixed, and that decisions of dispatching, routing and scheduling are integrated (Bealieu and Gamache, 2006).

In practice, scheduling and dispatching problems are solved through a plan-driven strategy for a working shift elaborated by high-level management. In this plan, the production goal is computed as a number of ore bucketfuls to extract from a set of drawpoints within a drift, given the average operating speeds and bucket capacity of LHDs (Córdova et al., 2008).

At the beginning of each working shift, the supervisor allocates operators for the LHDs and drifts, taking into account the current resources status. Thereafter, the LHD operator decides by himself the routing. In general, th operator does not have any kind of dec sion support system for the routing an he uses rules of thumb according to h experience (Córdova et. al, 2004).

In this paper, the LHD operatio problem to minimize makespan of dri workload is studied. The work objec tive is to propose a simple and efficien decision making process (DMP) for a LHD operator. The paper is organize as follows: in section two, the intege programming (IP) model is formulatec in section three, the polynomial tim optimal algorithm is presented, its inte gration into a decision making proces (DMP) is made and an illlustrativ example is shown; in section four, th comparative analysis between the DM proposed and the DMP used in the *E Teniente* copper underground Chilea mine is studied; and finally, in sectio five, the conclusions and directions fo further research are given.

2. Integer programming model

The integer programming (IP) model is based on haulage network representation through a digraph, according to drift characteristics and plan-driven strategy data. The model objective is to determine the LHD work path, in order to minimize the makespan subject to the operational constraints. The model is defined as follows.

Let $k \in \{L,R\}$ denote the "k" side of drift, where $k=L$ is left side and $k=R$ is right side. Let $STSD=SSDL \cup SSDR$ be the set of drawpoints selected by the plan -driven strategy, where $SSDL \neq \varnothing$ and $SSDR \neq \varnothing$ are the sets of drawpoints located to left side and right side of drift, respectively. Each drawpoint $i \in STSD$ has a number of ore bucketfuls to be extracted B_i, a transfer time between it and the initial point IT_i; and a the transfer time between it and the dumping site / turning point FT_i. On the other hand, LHD vehicle has a transfer time between the initial point and the dumping site / turning point TL, a turning time TT, and a loading time LT and an unloading time UT for an ore bucketful.

In addition, given the topology of the mine, the characteristics of the drifts and the operational constraints, such as, LHD must enter to drawpoint with its shovel in front position, and it must travel to a turning point and return in opposite direction due to narrow angle; two simplifications are introduced to reduce the number of variables from the model:

a) If LHD is at the dumping site/turning point and is ready to extract ore at the *i-th* drawpoint, then it will work in this *i-t* drawpoint until its workload is complete and.

b) LHD can turn to the opposite sid of the drift, if and only if the workload of side drift is completed.

Thus, the variables of the represen tative digraph define the LHD operatio by four types of movements: 1) from th initial point (with empty bucket) to the firs drawpoint in the path, 2) from a drawpoin to the dumping site and vice versa (unt workload is completed), 3) from turnin point to the first drawpoint on the othe side of the drift, and 4) from the dumpin site back to the initial point. The variable constraints and objective function of LHI operation problem are presented as follow

Variables

$$X_{1k} = \begin{cases} 1 \text{ if the LHD vehicle begins working at the } k \text{ side of the drift, } k \in \{L,R\} \\ 0 \text{ in other case} \end{cases}$$

$$X_{2i} = \begin{cases} 1 \text{ if the LHD vehicle begins working the } i\text{-}th \text{ drift coming from the entrance} \\ 0 \text{ in other case} \end{cases}$$

$$X_{3i} = \begin{cases} 1 \text{ if the LHD vehicle begins working the } i\text{-}th \text{ drawpoint from the dumping site / turn point} \\ 0 \text{ in other case} \end{cases}$$

$$X_{4k} = \begin{cases} 1 \text{ if the LHD vehicle turns back from the } k \text{ side of the drift to the other side } k \in \{L,R\} \end{cases}$$

$$X_{5k} = \begin{cases} 1 \text{ if the LHD vehicle turns back from the } k \text{ side to the initial point of the drift } k \in \{L,R\} \\ 0 \text{ in other case} \end{cases}$$

Constraints

$$X_{1L} + X_{1R} = 1$$

$$\textstyle\sum_{i \in SSDL} X_{2i} = X_{1L}$$

$$\textstyle\sum_{i \in SSDR} X_{2i} = X_{1R}$$

$$X_{2i} + X_{3i} = 1 \qquad \forall i, i \in STSD$$

$$X_{1L} = X_{4L}$$

$$X_{1R} = X_{4R}$$

$$X_{4L} = X_{5R}$$

$$X_{4R} = X_{5L}$$

$$\textstyle\sum_{i \in SSDL} X_{3i} + X_{5L} + X_{4L} = \sum_{i \in SSDL} 1 + X_{4R}$$

$$\textstyle\sum_{i \in SSDR} X_{3i} + X_{5R} + X_{4R} = \sum_{i \in SSDR} 1 + X_{4L}$$

$$X_{5L} + X_{5R} = X_{1L} + X_{1R}$$

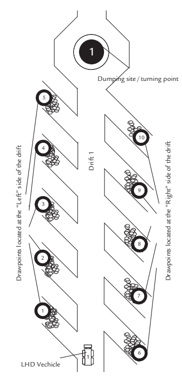
Objective function

$$M = \textstyle\sum_{i \in STSD} (2B_i - 1 + X_{3i})\, FT_i + X_{2i} IT_i + (LT + UT) B_i + \sum_{k \in \{L,R\}} X_{4k} TT + \sum_{k \in \{L,R\}} X_{5k} TL$$

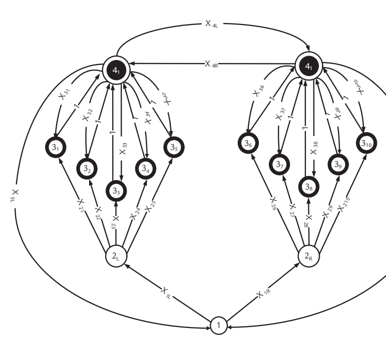
Figure 1 shows an example of a digraph representing a haulage network according to the plan-driven strategy d

Date	1/10/2007				
Drift	1				
Working shift	2				
LHD vehicle number	3				
	Left side				
Drawpoint	**1**	**2**	**3**	**4**	**5**
Bucketfuls	10	13	13	11	9
	Right side				
Drawpoint	**6**	**7**	**8**	**9**	**10**
Bucketfuls	7	10	13	14	10
Dumping site /turning point	1				

(a) Plan-driven strategy data.



(b) Graphical view of drift.



(c) Digraph representing a haulage network

Figure 1
An example of a digraph representing a haulage network according to plan-driven strategy data.

In the digraph, edge one represents the beginning of work at the drift entrance. Edge *2k* is the *k-th* side of the drift, with $k \in \{L,R\}$. Edge *3i* is the *i-th* drawpoint, with $i \in STSD$. The edges number four represent the dumping site / turning point of the drift. This representation of dumping site / turning point by two edges allows to model the turn of the vehicle when changing direction. The arc joining the *i-th* drawpoint with the dumping site / turning point is equal to one and it stands for the last trip. This arc is meant to establish a flow balance at the *i-th* drawpoint; that is, given that a last trip to the dumping site must take place, then an arrival to that drift must oc Constraints (1), (2), (3), (4), (9), (10) (11) can be classified as the equations flow balancing, while constraints (5), (7) and (8) are equations for determi a logical path in such a way that L leaves the drift in a direction opposit its entry.

3. Resolution algorithm and decision making process

Although, the solution of the IP model proposed can be obtained by us- AMPL®, GAMS®, among others), in practice, the use of such tools may be would be needed for the decision m ers (e.g. LHD operator). On the o an easy-to-use interface surely can help in this task, but an investment would be necessary (e.g., optimization engine, customized software, training, etc.).

Therefore, the study and definition of decision rules based on a resolution algorithm is a very interesting subject for approaching the real implementation. Ideally, this should be simple to execute for the workers and the resolution algorithm should lead to optimal solution in polynomial time. Two good examples are Johnson's rule (Johnson, 1954) for minimizing the makespan in flow shops with *n* jobs and tw machines and McNaughton's rule (Mc Naughton, 1959) for minimizing th makespan in a parallel machine with jobs and preemption. In both cases, th optimal result is found in a polynomia time *O(n)*.

Definition of resolution algorithm

In this work, an optimal algorithm for the LHD operation problem is presented. This algorithm yields the variable values of the IP model proposed and the optimal makespan of the drift workloa defined by the plan-driven strategy.

ALGO 1

Input: Plan-driven strategy data, TL, LT, UT, TT, IT_i and $FT_i\ i \in STSD$

Step 1: Select *i*-th* drawpoint such that $(IT_{i^*} - FT_{i^*}) = \underset{i \in STSD}{Min}(IT_i - FT_i)$

Step 2: Define X_{2i^*}:=1; X_{2j} :=0; X_{3i^*}:=0; X_{3j}:=1 such that $j \in STSD \setminus \{i^*\}$

Step 3: *If i*-th* drawpoint is in left side of drift *then*

X_{1L}:=1; X_{4L} :=1; X_{5R}:=1; X_{1R}:=0; X_{4R}:=0; X_{5L}:=0

Else

X_{1L}:=0; X_{4L} :=0; X_{5R}:=0; X_{1R}:=1; X_{4R}:=1; X_{5L}:=1

Step 4: Compute the optimal makespan by equation (12)

Theorem 1. *The algorithm ALGO 1 gives an optimal solution for the LHD vehicle operation problem and polynomial time*.

Proof. Let M be the makespan for the workload assigned to the drift and expressed by equation (12). Then, it can be broken down into three parts: equation (13) represents the total time for the loading and unloading of ore bucketfuls and the trips from / to the drawpoints selected by the plan-driven strategy on both sides o the drift; equation (14) expresses the turn ing time and equation (15) the exit tim that is the travel time between the dumpin site / turning point and the initial point.

$$\sum_{i \in SSDL} (2B_i - 1 + X_{3i})\, FT_i + X_{2i}IT_i + (LT + UT)B_i + \sum_{i \in SSDR} (2B_i - 1 + X_{3i})\, FT_i + X_{2i}IT_i + (LT + UT)B_i \tag{13}$$

$$\sum_{k \in \{L,R\}} X_{4k}TT \tag{14}$$

$$\sum_{k \in \{L,R\}} X_{5k}TL \tag{15}$$

Since B_i is the number of ore bucketfuls to be extracted from the *i-th* drawpoint, then the number of trips from / to the *i-th* drawpoint is $2B_i$. In (13) the value $2B_i$ - 1 is the number of trips from / to the *i-th* drawpoint without considering the initial trip (i.e., the first trip to the *i-th* drawpoint). The variables X_{2i} and X_{3i} are the options for the initial trip to the *i-th* drawpoint where only one must be selected (see constraint (4)). The total time of loading and unloading is $(LT + UT)B_i$ as shown in equation (13). Expressions (14) and (15) represent the time for the turning of the LHD and the time for the final trip, (i.e. the trip between the dumping site / turning point considered by plan-driven strategy to the initial point), respectively. By considering the binary definition of the variables, it can be shown that th value of the objective function given i equation (12) subjected to constraint (1), (2), (3), (4), (5), (6), (7) is equal t (16) and finally reduces to (17).

Therefore, to find *i*-th* drawpoin equal to arg $Min_{i \in STSD}(IT_i - FT_i)$ allow to encounter the variable X_{2i^*}=1 and th values of the other variables by using th constraints of the problem.

$$\underset{i \in STSD}{Min}(IT_i - FT_i) + \sum_{i \in STSD} (2FT_i + LT + UT)B_i + TT + TL \tag{16}$$

$$\underset{i \in STSD}{Min}(IT_i - FT_i) + C \qquad C \text{ constant} \tag{17}$$

Let n be the number of drawpoints

Step 1, Step 2 and Step 4 run in O(n) time

Definition of the decision rules and decision-making process

The definition of the decision rules are obtained from ALGO 1 and they work in conjunction to develop a decision making process (DMP) for the LHD operators. The decision rules are defined as follows:

Decision rule 1 (R1): *"For a given plan-driven strategy, the i*-th chosen drawpoint to start with at the drift is the one for which the difference between the transfer time from the initial (entrance) point and the transfer time towards the dumping site/turning point, is minimum; that is, where* (IT_i - FT_i) *is minimum".*

Decision rule 2 (R2): *"The minimum makespan of the drift workload defined by the plan-riven strategy is equal to equation (16).*

Decision rule 3 (R3): *"The drift workload determined by the plan-driven strategy must be objected (e.g., negotiated, modified, or rejected) if the available time of a working shift TWS is less than the minimum makespan of the drift workload defined by the plan-driven strategy".*

Decision rule 4 (R4): *"The total number of ore bucketfuls assigned to a drawpoint by the plan-driven strategy must be extracted iteratively as many times as defined by the plan".*

Decision rule 5 (R5): *"The workload assigned to a side of the drift must be fully completed before changing to the remaining side, following any order to visit the remaining drawpoints".*

R1 and R2 are obtained from ALGO 1. They show the first drawpoint to visit in the work path and the minimum makespan value for drift workload within a working shift, given the plan-driven strategy data and the average for operation and transfer times(i.e., times for loading, humping, moving, dumping, turning).

R3 gives a feasibility condition for the fulfillment of drift workload withi
working shift.

R4 and R5 are the result of two
plifications introduced into the IP mod
reduce the number of variables. R4 off
feasible way to complete the total work
assigned to the *i-th* drawpoint by minimi
the travel time of the LHD vehicle betw
the drawpoint and the dumping site, w
R5 offers a feasible way to complete
workload assigned to a drift side by mini
ing the LHD vehicle's turning time. In
an important remark is that the remai
drawpoints at the same drift side ca
visited in any order, without affecting
minimum makespan value.

The five decision rules above ar
tegrated into a DMP in a sequential wa
shown in Figure 2.

A complete example of DMP u
given in Figure 3.

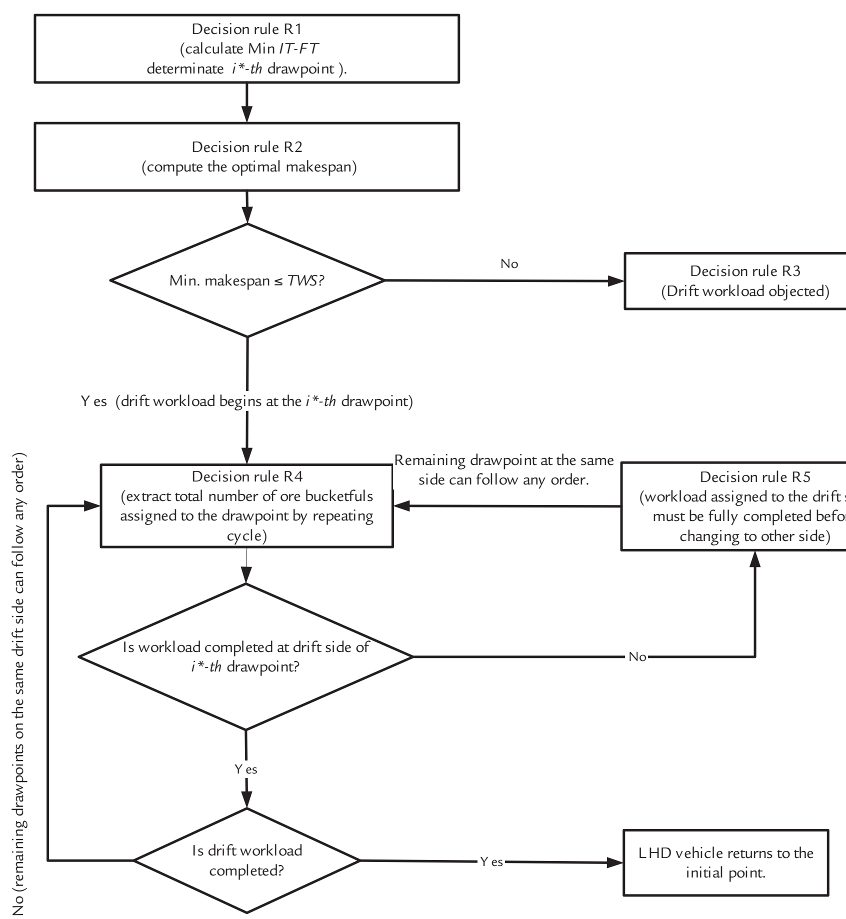


Figure 2
Flow diagram of the DMP proposed for the LHD operation problem.

4. A comparative analysis of DMP: *El Teniente* underground mine case

A comparative analysis between the current and the proposed DMP is realized. The objective is compute the improvement of

the DMP used in *El Teniente* copper Chilean underground mine is described and numerical cases are studied, by using data available from

(plan-driven strategies) are 90 (three mon
which consider different topologies of
and the production goals for each drawp

Date				1/10/2007	
Drift				1	
Working shift				2	
LHD vehicle number				1	
LT (s)	60	*TT (s)*		120	
UT (s)	10	*TL (s)*		360	
	Left side				
Drawpoint	**1**	**2**	**3**	**4**	**5**
Bucketfuls	10	13	13	11	9
IT (s)	240	260	280	305	330
FT (s)	120	100	80	55	30
	Right side				
Drawpoint	**6**	**7**	**8**	**9**	**10**
Bucketfuls	7	10	13	14	10
IT (s)	250	270	295	325	350
FT (s)	110	90	65	35	10
Dumping site /turning point					1
TWS (s)					25,200

(a) The data of plan-driven strategy and average time of operation and transfers are given to the LHD operator.

	Left side				
Drawpoint	**1**	**2**	**3**	**4**	**5**
IT - FT (s)	120	160	200	250	300
	Right side				
Drawpoint	**6**	**7**	**8**	**9**	**10**
IT - FT (s)	140	180	230	290	340
Min IT - FT (s)					120
i-th* drawpoint					1
OPT Makespan (s)					23,340
Feasibility condition (OPT Makespan < *TWS*)					Yes

(b) The LHD operator uses R1 to calculate Min *IT - FT* and to determinate *i*-th* drawpoint (R1 used). Next, he uses R2 to compute to optimal makespan and decides if the fulfillment of drift workload is feasible by using R3.

Drawpoint	**Arrival time(s)**	**Working time (s)**	**Predecessors**
1	240	2,980	-
2	3,320	3,410	1
3	6,810	2,910	2
4	9,775	1,925	3
5	11,730	1,140	4
6	13,100	1,920	5
7	15,110	2,410	6
8	17,585	2,535	7
9	20,155	1,925	8
10	22,090	890	9
Depart time from initial point (s)			0
Arrival time from initial point (s)			23.340

(c) Given, the fulfillment feasibility of the drift workload (by R3) and the selection of number one as first drawpoint to visit from the initial point (by R1), the LHD operator decides the work sequence. Thus, he extracts the total of ore bucketfuls from one number drawpoint (R4 is used). Next, he arrives to another drawpoint on the same side of the drift following any order (R5 is used). In this new drawpoint, he extracts the total of ore bucketfuls before changing to another drawpoint on same side (R4 is used). The two actions above are repeated before changing to the remaining side of drift (Recurrently, R4 and R5 are used). Once completing one drift side, he turns and arrives to one drawpoint at the other side of the drift (R5 is used). On this side, he repeats the two actions used before changing sides, i.e. the total of ore bucketfuls is extracted before changing to another drawpoint on the same side and the visits to the drawpoints on the same side can follow any order (Recurrently, R4 and R5 are used). Finally, once completing the drift workload, the LHD operator travels to the initial point.

Figure 3
An example of DMP use.

DMP in *El Teniente* underground mine. When is it good?

The DMP used by LHD operators is a set of historically-employed, simple-to-execute rules. In this DMP, it is possible to identify two types of decision rules (Sepúlveda et al., 2005): 1) to meet the operational constraints and, 2) to define the work sequence. Surprisingly, the two decision rules of type 1 are equivalent to decision rules R4 and R5, while the decision rule of type 2, denominated HPF (after Highest Production First), defines the visit to drawpoints in decreasing order according to the number of ore bucketfuls that must be extracted. The decision rules above are integrated into a DMP as shown in Figure 4.

It is obvious that the current decision rules of type one are necessary conditions to reach the optimal makespan and that the HPF rule yields the optimal solution, if and only if the *i*-th* drawpoint is the greatest number of assigned ore bucketfuls.

The study of numerical cases shows that DMP leads to satisfactory result since they help to meet the production goa as imposed by the plan-driven strateg Nevertheless, a priori the feasibility of fin ishing the drift workload within a workin shift is not assured. In practice, 62.5% o the cases analyzed matched the optima solution with the HPF rule; the averag improvement of makespan due to DM proposed is 5.6 % and three infeasibilit cases are found.

5. Conclusion

The main results of this work are two: First, a polynomial time optimal algorithm for the LHD operation problem, which gives the variable values for the IP model proposed. Second, a decision making process (DMP), which integrates a set of decision rules obtained from the above algorithm. This DMP is simple to execute determine the optimal makespan and the feasibility of drift workload fulfillment within a working shift.

The comparative analysis between the DMP proposed and the DMP used in the *El Teniente* copper Chilean underground mine showed three interesting results. First, the current DMP has rules which are necessary conditions for find ing the optimal makespan. Second, th HPF rule to define the work sequenc matched the optimal solution in mor than half of the cases studied (62.5% Third, although the current DMP can not assure the drift workload fulfillmen within a working shift, in practice onl results show an important conclusion: cumulative operator experience has converged to solutions near to optimal makespan.

Further research is proposed regarding aspects such as: a) multiple dumping sites / turning points, b) objective functions other than makespan, such as the ore quality, c) on-line L operation problem and d) decis integration with transportation reduction levels.

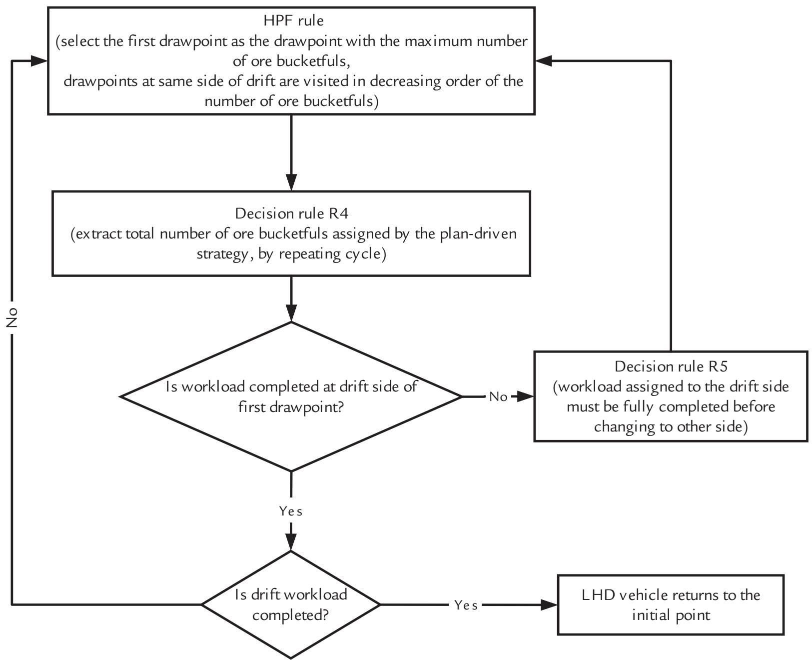


Figure 4
Flow diagram of the DMP used in *El Teniente* copper Chilean underground mine for the LHD operation problem.

6. Acknowledgements

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