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## Simulation of the mineral breakage using a fractal approach

*Simulação computacional da quebra mineral usando uma abordagem fractal*

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### Resumo

A Lei de Hukki é uma lei empírica que não leva em conta vários tipos de perda de energia durante o processo de fragmentação mineral. Uma vez que resultados experimentais são muito difíceis de obter para uma ampla faixa de tamanho de fragmentos, a verificação desta lei se torna muito difícil. A relação entre as fraturas e o processo de fragmentação com geometria fractal foi proposta algumas décadas atrás. Leis empíricas conhecidas neste contexto mostram características básicas da geometria fractal, principalmente auto afinidade e comportamento de lei de potência. Desta forma, neste artigo é apresentado um modelo para simular o processo de fragmentação e verificar a relação entre o consumo de energia e o tamanho dos fragmentos gerados. O modelo é representado por uma rede regular onde os arcos da rede representam os caminhos gerados no processo da geração da fratura. A energia dos eventos de fragmentação foi modelada por uma função de distribuição de probabilidades. No modelo proposto não existe perda de massa e a propagação das fraturas ocorre seguindo uma caminhada aleatória auto evitável em uma rede regular.

**Palavras-chave:** Simulação, quebra mineral, fractal.

### Abstract

*Hukki's law is an empirical law, which does not take into account several events of energy loss during the mineral fragmentation processes. Since experimental results are very difficult to obtain for a large range of fragment sizes, the verification of the law is very difficult. The relation between the fracture and fragmentation processes with fractal geometry has been proposed some decades ago. Empirical laws along this context show basic features of fractal geometry, mainly self-affinity and the power law behavior. Thus, in this paper, a model to simulate the fragmentation process and to check the relationship between energy consumption and fragment sizes was developed. The model is represented on a regular lattice where links represent pathways for fracture processes. The energy of fragmentation events was modeled by a probability distribution function. In the proposed model there is no mass loss and the fracture propagation occurs as self-avoiding random walks on the regular lattice.*

**Keywords:** Simulation, mineral breakage, fractal.

## 1. Introduction

Mineral comminution represents a significant portion of energy consumption around the world. So, a precise knowledge of the structural properties of the products is necessary for the control of energy expenditure. It is understood by structural properties, the quantitative description of morphological and architectural features, primarily the system of discontinuities genuine mineral particle before the comminution and, secondly, the product of comminution, as shown by Thomas and Filippov (1999).

Several authors have published

papers on the propagation of fractures in metal alloys, but only a few have worked with the same problem applied to minerals and always using metallurgical methods and hypotheses. In this context, many of the approximations made are consistent, but as the hypotheses are greatly simplified they almost always end up neglecting a somewhat obvious observation: the mineral breakage process involves several different processes occurring simultaneously or successively.

So, the energy balance, commonly used in analyses of this kind, refers to

a single mechanical process arbitrarily selected, without the estimation of energy expenditure with thermic energy loss from the comminution system to the environment around them. Therefore the determination of models relating the energy spent with the surface created are not available.

The aim of this paper is to present a model to simulate the process of mineral comminution, as well as to verify the relationship between energy consumption and the size of fragments generated in the process of breaking.

## 2. Material and methods

### Global experimental model

The more general expression, and also the simplest, relating the energy consump-

tion per unit mass ( $E$ ) to reduce the average size of a particle  $x$  of the broken material is

$$dE = -K \cdot x^{-n} \cdot dx \quad (1)$$

The expressions of Kick, Rittinger and Bond used today are only integrations of the

Kapur equation for different values of the exponent  $n$  (see Figure 1). The more general

$$dE = -K \cdot x^{-f(x)} \cdot dx \quad (2)$$

Hukki's Law is an empirical law which does not take into account the various types of energy losses in the comminution process. Even if it were

theoretically possible to estimate the energy dissipation (mechanical, kinetic, thermal and ultrasonic) through sophisticated experiments this would be ex-

pression is Hukki's Law, which is a generalization of the expression of Kapur, given by:

pensive. All energy losses depend on the comminution devices and the products themselves, whose identity is not exact by Hukki's Law.

### Calculation of the constants of Hukki's Law

According to Thomas and Filippov (1999) Hukki did not propose an analytical expression for the function

$f(x)$  of his expression. However, it is possible to consider an approximate graphic for the Hukki's Law which

according to Lynch (1977) has a slope given by:

$$d(E_p) = d(\log E_p) / d(\log x) \quad (3)$$

Figure 1 (Thomas and Filippov, 1999) shows the experimental curves

found by Lynch for Hukki's Law.

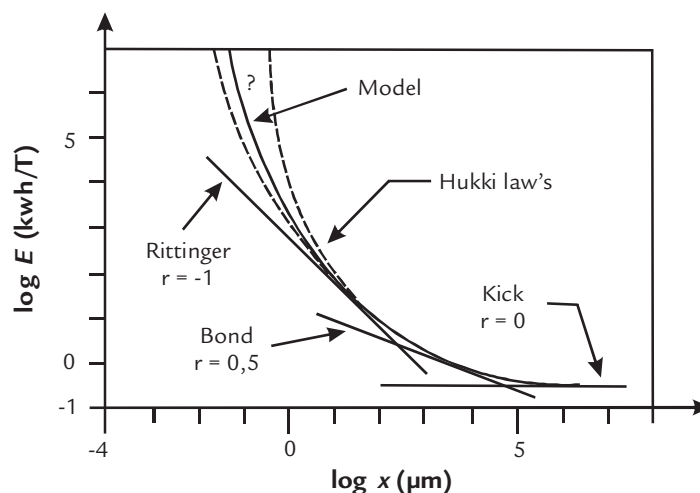


Figure 1  
Experimental curves for  
Hukki's Law found by Lynch  
(cited in Thomas and Filippov, 1999).

Thomas and Filippov (1999) demonstrated that a linear regression of the

Hukki's Law in a logarithmic coordinate system for particle size between 1 and  $10^4$

$\mu\text{m}$  has a strong correlation with the curve and is given by:

$$d(E_h) = C \log x + B \quad (4)$$

Where B and C are constants from

the linear regression. The integration of

equation (4) results in:

$$\begin{aligned} \log E_h &= \frac{1}{2}C (\log x)^2 + B \log x + \log A \\ E_h &= A \cdot x^{B + \frac{1}{2}C \cdot \log x} \end{aligned} \quad (5)$$

Where A is an integration constant. The three constants (A, B and C) define all

properties of Hukki's Law. To introduce such constants in Hukki's Law (equation 2) Silva

and Luz (2007) proposed to derive equation (5) depending on the variable x. Thus:

$$dE_h / dx = E(x) = A \cdot (B + C \log x) \cdot x^{(B-1) + \frac{1}{2}C \cdot \log x} \quad (6)$$

Comparing the equation (6) with the

equation (2) it follows that:

$$K = -A(B + C \log x) \text{ and } f(x) = (1 - B) - \frac{1}{2}C \cdot \log x$$

## Modelling the comminution process

To model the comminution process of a mineral particle a simple model was developed based on computer simulation technique called Monte Carlo. The model consists of the following steps:

1. A regular lattice is created to represent one of the faces of a mineral particle (or a mineral grain). The lattice sites are the points where the impact energy can be transferred to the lattice.
2. All external sites over the lattice are interconnected to form the outer surface of the mineral face (or border of a mineral grain).
3. A lattice site and the amount of energy to be transferred by the impact are randomly selected. The energy to be transferred is given by a Weibull distribution. The energy transferred to the particle is stored in a variable.
4. If the energy is high enough the fracture percolates to the adjacent sites of the lattice. Each time a fracture is established between two lattice sites the total impact energy is decreased. This step repeats until the total energy becomes equal to zero or the fracture cannot find any adjacent intact sites to

continue the breaking process.

5. The steps 3 and 4 repeat while there are sites in the lattice to be broken.

The fourth step of the algorithm characterizes a self-avoidable random walk, i.e. a path that can only be walked once. For the model in question this is equivalent to saying that once the fracture has been established a new border is created, preventing the creation of another fracture. The result of running the algorithm is the quantification of the energy applied to the lattice to break it. In the proposed model the total mass of the system is conserved, since the mineral particle is only subdivided into smaller ones. Figure 2 shows a graphical representation of the operation of the proposed algorithm on a square lattice with four sites. Figure 2A shows a newly created lattice, without any fracture. Figure 2B shows the final result of the first fracture, where the site shown in grey was hit in the impact and, from this site, the fracture percolated to the adjacent sites. The end of the algorithm occurs when the entire lattice was fractured (Figure 2C).

To simulate the fact that large particles require smaller amounts of energy

to fragment in all simulations the same square lattice with 1600 sites was used, where the particle that is desired to break was located within the lattice. Therefore, large particles almost always receive the impact directly, but small particles tend to escape the impact, consuming more energy in their process of comminution.

For the operation of the proposed model it is not necessary that the lattice be square, but may have any configuration. As the shape of the lattice will impose in the form of breakage products, is possible to understand the lattice as the projection of the cleavage planes of the mineral. To model the breakage of minerals with preferential planes of breakage can be established a higher probability of the fracture percolation in this planes, so that the breakage is favored in certain crystallographic planes. This assumption is similar to the hypothesis adopted by Turcotte (1986). The author consider the mineral natural discontinuities that first induce the size distribution of fragments, may be cut out in sets of quasi-planar elements that present a scaling architecture defined by a self-similar fractal law.

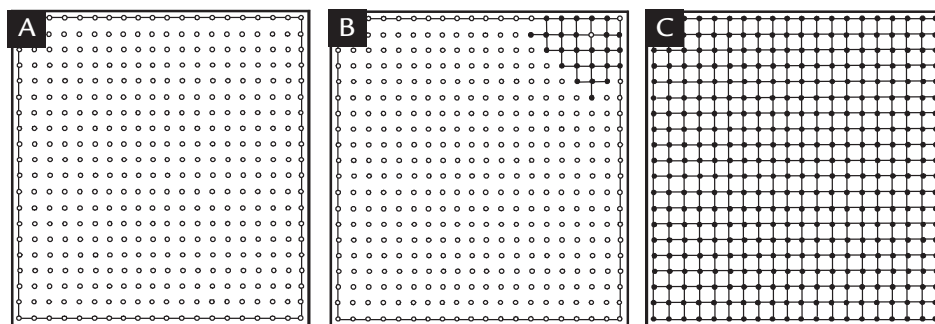


Figure 2  
Graphical representation  
of the proposed algorithm:  
(A) setting up the lattice.  
(B) end of the first fracture process.  
(C) end of the algorithm with the  
lattice fully fractured.

### 3. Results

Simulations were conducted varying the particle size, always on a lattice with total size of 1,600 sites. For each particle size 1.000 simulations were performed and the average energy

spent for the comminution was considered. Figure 3 shows the results of the proposed model and the comparison with the fractal equation proposed by Silva and Luz (2007). As can be seen

in Figure 3 the equation of Silva and Luz fits with approximately 98 per cent of adherence to the data generated by the simulation, a result considered satisfactory.

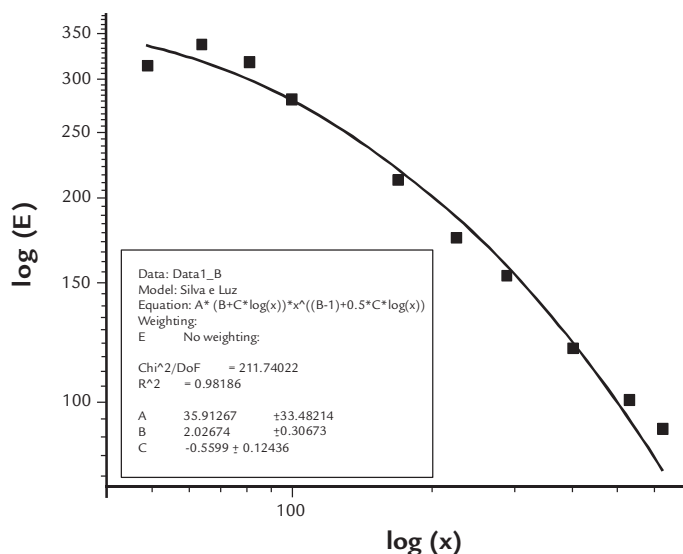


Figure 3  
Results of data generated by the simulation in relation to the equation proposed by Silva and Luz (2007).

### 4. Conclusions

Two conclusions can be obtained from the data generated by the simulation: the mineral breakage, as expected, follows a power law, and the same hypothesis applies to the proposed model. Secondly that the simulation data fit well to Silva

and Luz equation (2007) ( $R^2 \approx 0.98$ ). The Monte Carlo technique was tested to simulate, in a simple model, the mineral comminution process. Although the process itself is very complex, the model agrees with the proposed equation test.

However, experiments and simulations are needed to more extensive validation of the proposed model and to better understand the possibility of practical uses of the equation proposed by Silva and Luz (2007).

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