Application of the Wigner-Ville distribution to interpret ground - penetrating radar anomalies

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Resumen

Con base en la Distribución de Wigner–Ville (WVD) se realizó un análisis en tiempo y frecuencia de datos obtenidos con el Radar de Penetración Terrestre (GPR), basado en el estudio de la descomposición de la señal espectral. Se calcula una correlación entre la señal original y las componentes de tiempo-frecuencia para obtener anomalías estructurales de la información contenida en el radargrama relacionándola con la geología disponible. En primer lugar se describe la aplicación de un ejemplo teórico constituido por lo que representaría un túnel (tubería). Se obtuvieron las firmas correspondientes en el dominio del tiempo y en el dominio de la frecuencia. Finalmente se analiza esta metodología en un sitio de prueba en la detección de un tambo enterrado donde son conocidas la geometría y su profundidad. Este especial sitio fue facilitado por la Universidad Nacional Autónoma de México, en los terrenos del Observatorio Magnético de Teoloyucan, Estado de México. Los resultados obtenidos son bastante alentadores, ya que la WVD es capaz de definir los rasgos morfológicos relacionados con el tambo y abre la posibilidad de localizar este tipo de estructuras.

Palabras clave: Radar de Penetración Terrestre (GPR), análisis tiempo-frecuencia, distribución de Wigner-Ville, proceso de señales.

Abstract

Time- frequency analysis, based on signal spectral analysis decomposition, is performed by the Wigner-Ville distribution (WVD) for Ground Penetrating Radar (GPR) data. The cross correlation between the original signal and the time-frequency components is obtained to get the structural anomalies available in the information provided by the GPR, related to the geology. We describe the application to a theoretical example representing a tunnel (pipe). Corresponding signatures are obtained in the time–frequency domain. An actual application at a test site is presented, where a drum of known geometry has been buried. This is a special test site at the National Autonomous University of México, Magnetic Observatory at Teoloyucan, State of Mexico. Depth and dimensions are well controlled, and the results are encouraging, since the WVD is capable to define morphological features related to the drum and opens the possibility to locate these structures.

Key words: Ground Penetration Radar (GPR); time-frequency analysis; Wigner-Ville distribution; signal processing.

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Introduction

GPR (Ground Penetrating Radar) constitutes an important methodology to reveal subsurface features. Shallow buried features such as fractures, mines, caves and anthropogenic structures (pipes, connecting lines, etc.) can be successfully detected. Traditionally the processing of GPR data may be done in the Time Domain (TD) or the Frequency Domain (FD) by employing the Fourier transform. However, it is not always possible to obtain a reasonable resolution from GPR data, due to variations in the relative dielectric permittivity (RDP). Therefore, GPR interpretation requires the application of TD or FD filters to separate important information from noise. Unfortunately, such filtering may be contaminated with spurious information. Also, the estimated events or anomalies may be well-defined in FD, but not in TD. A time-frequency analysis (TF) can reveal the time variation of the frequencies contained in a 1D GPR signal.

In the TF linear distributions as well as in the Wavelet transform, the signal is decomposed in the time-frequency domain based on amplitude, over a family of functions derived from a function base to determine the scale on which depends the resolution (Vera, 2003; Auger *et al*., 1995).

In the case of discrete Wavelet transform, it corresponds to a dyadic sampling of the time-frequency plane in which the function family constitutes an orthonormal base with a wavelet well localized in time and frequency (Auger *et al*., 1995; Rosado, 2000; Rivera-Recillas *et al*., 2005).

The Wigner-Ville distribution is a time-frequency bilinear transform, which computes the distribution of signal energy in two variables: time and frequency, as energy density. Its spectrum depends on time, preserving mathematical properties such as the marginal and instantaneous frequency. It employs the analytic signal from a real signal and a parametric function of constant value (kernel = 1).Thus, the interference terms appearing in the Wigner distribution due to negative frequency components of the real signal can be suppressed, (Ramos, 1997).

The idea is to extract the time series profile that make up the radargram and define a function in TF domain, which shows features related to anomalous bodies or sub-surface structures. In this case, the Wigner-Ville Distribution (WVD) is applied to define these structural objectives (Lopera *et al*., 2008). It is proposed to obtain the frequency information contained in time, for a 1D GPR signal processed by the WVD, in order to identify an event such as a stratigraphic change,

a cavity or fracture. First, a 1D theoretical model is presented to show the ability of the WVD, and then a real example is shown from a test site where the characteristics are known. The test site is located in the facilities of the National Autonomous University of México, Magnetic Observatory at Teoloyucan town, in the State of Mexico, approximately 60 km NE of Mexico City.

Methodology

The WVD of a discrete signal $s(t)$ is given by (Cohen, 1989):

$$WVD_s(t_i, f_j) = \frac{T}{\pi} \sum s(t_i + kT) s^*(t_i - kT) e^{-i4\pi f_j kT}$$

(1)

If a signal is sampled at an interval T, the maximum cutoff frequency where the signal is folded in the Fourier spectrum is fc = 1/2T. However, the WVD time-frequency spectrum is folded at fc/2. It means that the original signal must be sampled at an interval T*=T/2 to avoid aleasing effects. Therefore, the new cutoff frequency can be expressed as fc*=1/2T*=1/T=2fc.

For the theoretical analysis a scan synthetic radargram was obtained for a 1D stratigraphy land, where the electric field at the Earth's surface can be obtained. Following the method of Weng (1971), based on the fact that a plane wave enters the Earth's surface and the total field obtained on the surface is the incident field plus the reflected, considering a dissipative dielectric N-layered earth model and a generalized reflexion coefficient given by Stokes (Bellman and Wing, 1975) and solved in a recursive form as:

$$\tilde{R}_{i,i+1} = \frac{R_{i,i+1} + \tilde{R}_{i+1,i+2} e^{2i\gamma_{i+1,z}(d_{i+1} - d_i)}}{1 + R_{i,i+1}\tilde{R}_{i+1,i+2} e^{2i\gamma_{i+1,z}(d_{i+1} - d_i)}}$$

. (2)

Later in the frequency domain, the Ricker's wavelet, given by (Ricker, 1977), is used:

$$S(f) = \frac{f^2}{f_c^2} e^{\frac{f^2}{f_c^2}}$$

(3)

Where f = frequency

fc = central frequency of oscillation

Which is characterized by a slow construction and a decline of energy at the extremes of the function, in which the standard of the wavelet is smooth and continuous in its 1st and 2nd

derivatives and can be optimized to meet the center frequency of a GPR antenna, which is designed to match the input impedance at the central feed point and the origin of the wavelet is used to force the electric field vector in the central feed point (Daniels, 2004; Cassidy and Murdie, 2000) Figure 1.

This is done through the Convolution Theorem radagram obtaining the synthetic trace in the frequency domain (Lázaro-Mancilla and Gómez-Treviño, 1996).

$$R\,(f) = E\,(f)\,S\,(f) \qquad (4)$$

Where

E (f) = the electric field in the frequency domain

S (f) = the Ricker pulse in the frequency domain

Finally, the inverse Fourier transform is computed to go into the time domain and produce a physical meaning to the estimated radagram (Annan and Chua, 1992; Díaz, 2003).

$$R(t) = F^{-1}\{E(f)S(f)\} \qquad (5)$$

It is expected that reflections produced by each layer represent the double travel time done by the GPR signal, given by:

$$t_r = \frac{2}{c}\sum\nolimits_{j=1}^{n-1} h_j \sqrt{\varepsilon_{rj}} \qquad (6)$$

The pulse travel time along the layered medium are calculated independently, the speed of propagation of electromagnetic waves is obtained through the formula.

$$v = \frac{c}{\sqrt{\varepsilon_r}} \qquad (7)$$

Where   c = speed of light

ε_r = *relative permitivity*
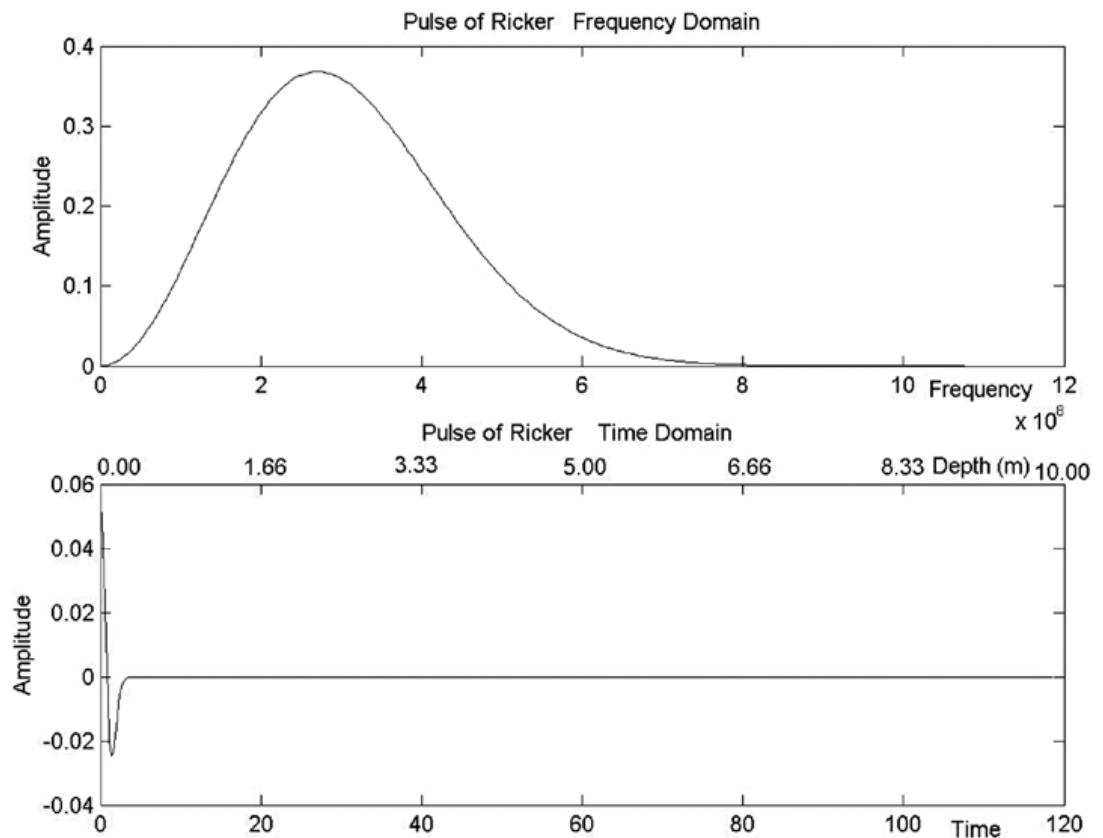
h = layer thickness

Figure 1. Features Ricker pulse used

The simple travel time of each layer is

$$t = \frac{h\sqrt{\varepsilon_r}}{c} \tag{8}$$

And the depth of the objects known can be estimated following, Peniche (2008).

$$depth = \frac{sample\ time\ travel}{speed} \tag{9}$$

Finally, we calculated the WVD to obtain the time-frequency plane, which is represented in graphical form by his absolute value.

Analysis and results of the GPR traces

Table (1) presents the model used to generate the synthetic trace (Figure 2). The synthetic trace model (Figure 2a) is represented in a sampling window of 60 ns, that would be obtained by an antenna of 270 MHz with 512 samples per record.

First we calculated the electric field, and the synthetic trace. Then, the time-frequency representation for this trace is obtained. However, small features such as a layer change can be diminished by the direct wave and its reflection surface, so the direct wave and its reflection can be removed by applying a filter (a simple average moving autoregressive to eliminate the unwanted echoes). The final result displays only important reflection components (Lopera, 2007).

The position of high amplitudes indicate events as at the times $t_1 = 18$ and $t_2 = 39$ ns. It is interesting to note that the amplitudes are compacted into the centre of the plane in a frequency range from 100 to 300 MHz. These represent the energy spectrum of the signal, which has a maximum value of 270 MHz (Figure 2).

Finally, a practical example is presented. A GPR profile at the test site, obtainid using a mono-static antenna, whose offset is zero, in which no lateral effects have a significant influence in giving a 1D representation; in the experimental site a cylindrical drum is buried at
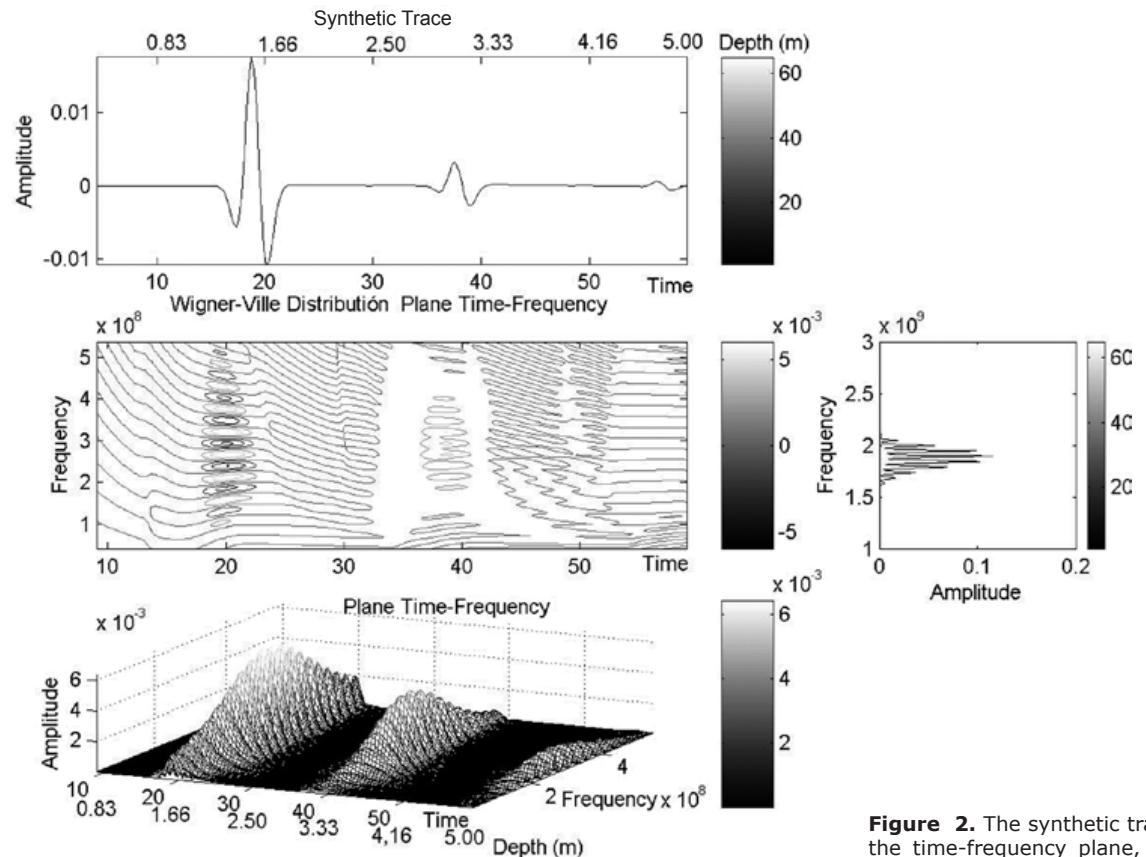

Figure 2. The synthetic trace, the time-frequency plane, the spectrum energy of the signal.

Table 1. Data used to generate the synthetic trace.

Layer	Material	Depth (m)	Magnetic Permeability (h/m)	Electric Conductivity(s/m)	Electric Permitivity (f/m)
1	Sandy soil	4	$\mu 0$	0.001	$8\varepsilon_0$
2	Air	3	$\mu 0$	0	ε_0
3	Sandy soil	–	$\mu 0$	0.001	$8\varepsilon_0$

depth of 1.0 m, with the following dimensions: diameter of 0.60 m. An area of 1.20 x 8.00 m was scanned. GPR data were analyzed following the methodology already described, employing the WVD. Each trace presents a similar behavior (Figure 3), providing the possibility to locate with better resolution the position at depth of the drum.

We have taken a trace of the GPR profile. Then, the time-frequency representation for this trace is obtained; the position of high amplitudes can indicate the location where the cylindrical drum was buried; this event is indicated at the time $t_1 = 18$ ns. These plots show us different events and are represented in a time–frequency plot as high energy values. It is interesting to note that the amplitudes are compacted into the centre of the plane in a frequency range from 100 to 300 MHz. These represent the energy spectrum of the signal, which has a maximum value of 270 MHz (Figure 4).

Conclusions

WVD time-frequency representation may provide another view of the GPR signal that complements what other methods may achieve. We provide a more accurate representation of the evolution of a non-stationary signal, which enables a more complete analysis of the signal providing classification, modeling and pattern identification parameters.

Non-stationary properties of the GPR signal allowed the application of WVD based analysis. A time–frequency representation was obtained, which allowed visualization of the frequency range contributing to the signal in the time scale studied. It allows a better visualization of the position of high amplitude that we can indicate the location of events of interest.

The spectral components represented in the time-frequency plane and depth, can be correlated with the characteristic events of the temporal signals or indirectly allow the characterization of the information contained in the GPR signal in terms of depths.

The study objectives from wich the were to allow characterization of the information contained in the GPR signal, applying the Wigner-Ville transform to non-stationary signals,
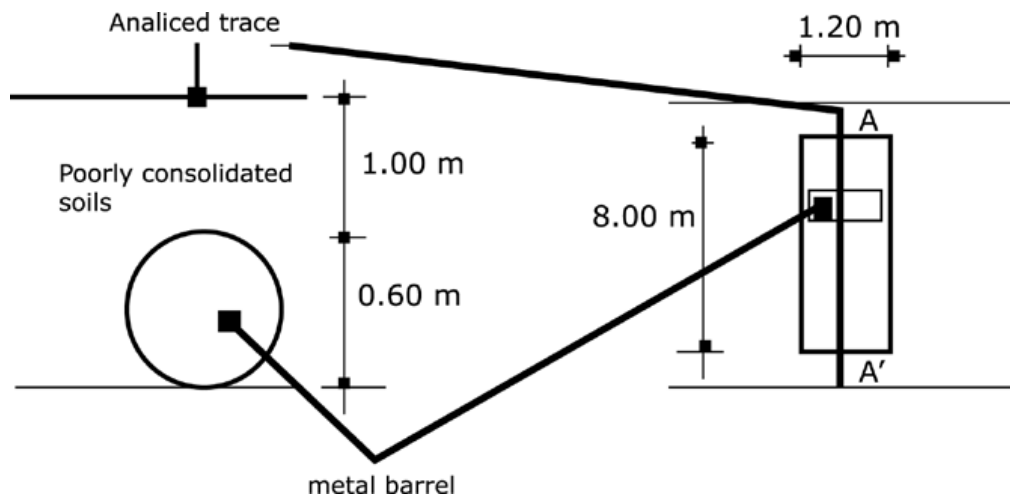
Figure 3. Diagram showing the location of the buried drum, analyzed GPR transect is also shown.
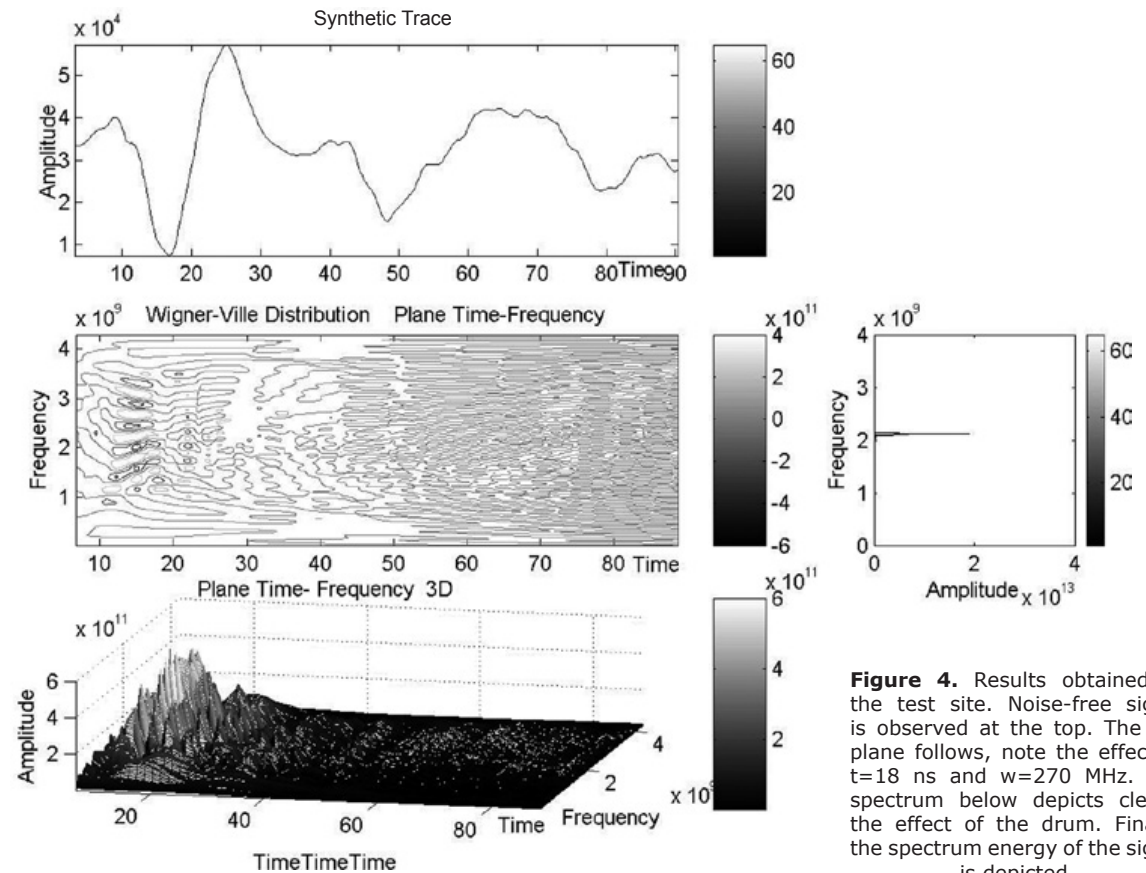
Figure 4. Results obtained at the test site. Noise-free signal is observed at the top. The t-w plane follows, note the effect at t=18 ns and w=270 MHz. The spectrum below depicts clearly the effect of the drum. Finally, the spectrum energy of the signal is depicted.

to visualize the evolution over time of the signal in a frequency range, to help in the classification, modelling and identification of patterns with their local geological correlation.

One of the goals of developed GPR methodologies was to recognize with adequate resolution structures lying at depths of 5.00 m at the most. New numerical approximations should be applied in the future to deal with noise and to enhance useful GPR signals to detect anomalies in the subsoil.

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