geofísica internacional

Fractal geometry and seismicity in the Mexican subduction zone

F. Angulo-Brown[1], A. H. Ramírez-Guzmán[2], E. Yépez[1], A. Rudolf-Navarro[1] and C. G. Pavía-Miller[3,1]

1 Departamento de Física, Escuela Superior de Física y Matemáticas, IPN, Zacatenco, México D.F.

2 Sección de Graduados, Escuela Superior de Ingeniería y Arquitectura, IPN, Zacatenco, México D.F.

3 Area de Física, Departamento de Ciencias Básicas, DCBI, UAM-A, México D.F.

Received: April 28, 1997; accepted: January 27, 1998.

RESUMEN

Mediante el método de conteo por cajas se calcula la dimensión fractal ***D*** de una distribución de fallas, fracturas y lineamien-tos en una región de la costa del sur del Pacífico mexicano. Los resultados sugieren que el valor ***b*** de la ley de Gutenberg-Richter y la dimensión fractal están positivamente correlacionadas. Proponemos también que ***D*** depende de la profundidad de la zona sismogénica.

PALABRAS CLAVE: Geometría fractal, sismos, Guerrero, México.

ABSTRACT

By means of the box counting method we calculate the fractal dimension ***D*** of the distribution of faults, fractures and lineaments in a region of the southern Mexican Pacific coast. The results suggest that in the spatial sense the ***b***-value of the Gutenberg-Richter law and the fractal dimension are positively correlated. We also suggest that ***D*** depends on the depth of the seismogenic zone.

KEY WORDS: Fractal geometry, earthquakes, Guerrero, Mexico.

INTRODUCTION

The earth's crust can be conceived as a hierarchical set of objects of many shapes and sizes suitable for a fractal description (Turcotte, 1989). Scale invariance is a well known property of many geological structures and phenomena. Self-similar behavior is reflected in several empirical power-laws in geology and geophysics (Turcotte, 1992; Ito and Matzuzaki, 1990). Seismicity, for example, has fractal structure with respect to time, space and magnitude (Hirata 1989a). Recently, some authors (Bak and Tang, 1989; Sornette and Sornette, 1989) have suggested that the structure of the crust can be interpreted as arising from a self-organized critical process. The concept of self-organized criticality (SOC) was proposed by Bak *et al*. (1987) as a general organizing principle governing the behavior of spatially extended dynamical systems with both temporal and spatial degrees of freedom. According to this principle, composite open systems having many interacting elements organize themselves into a stationary critical state with no length or time scales other than those imposed by the finite size of the system. The critical state is characterized by spatial and temporal power laws. Sornette and Sornette (1989) suggested that SOC is relevant for understanding earthquakes as a relaxation mechanism which organizes the crust both at spatial and temporal levels. This idea allows to rationalize observations on occurrences and magnitudes of earthquakes. Also in a SOC context Sornette *et al.* (1991) proposed a general mechanism which accounts for fluctuations in the ***b/c***-value as being related to the variations of the fractal dimension of the set of faults on which the Gutenberg-Richter law is measured (***b*** and ***c*** are the slopes of the magnitude-frequency and the moment-magnitude rela-tions respectively). Those authors suggest that the "bare" ***b*** value is close to 0.75 (taking ***c*** = 1.5) and it fluctuates around this value depending on regional fractal dimensions of fault sets. Lomnitz-Adler (1992) proposed an approach where the ***b***-value depends on two effects, one associated with the dynamics of fracture propagation within a single planar fault, and the other linked to fractal properties of the complete fault system. Okubo and Aki (1987) calculated the fractal dimension of the San Andreas fault system and suggest that differences between observed seismic activity inside and outside the 1857 rupture area might be attributed to differences in fault complexity and fractal dimensions. The SOC nature of the earth's upper crust is supported by the seismicity in plate boundaries and in the plate interiors, and by earthquakes induced by dams (Barriere and Turcotte, 1994). Spatial fault distributions are linked with seismicity levels. Aki (1981) suggested that the Gutenberg-Richter relation

where ***N*** is the number of earthquakes with a magnitude greater than ***M***, and ***a***, **b** are constants, is equivalent to the fractal distribution

where ***A*** is the area of the rupture and (imagen) is the fractal dimension of the fault plane. If c = 1.5, then

Thus, the fractal dimension in a region is twice the ***b***-value (Turcotte, 1989; 1992). This result was also obtained by King (1983) by means of a fractal faulting model in three dimensions. Recently, Barriere and Turcotte (1994) found an excellent agreement between Eq. (3) and ***b***-values by means of a SOC cellular-automata model based on the working hypothesis that each fault is associated with a characteristic earthquake. All this was discussed early by Lomnitz-Adler (1992). However, as Guo and Ogata (1995) point out Aki's relation ***D*** = 2***b***, has not been fully confirmed by observational data. Hirata (1989a) analyzed variations of ***b*** and ***D***2 (correlation dimension) in two regions (Tohoku and southwest Japan) and found that the temporal variations of ***b*** and ***D***2 in each region exhibit a negative correlation, but a positive correlation in the spatial variation. Using a box-counting algorithm over fault maps, Hirata (1989b) found that the surface fractal dimension in Japan varied from 1.05 to 1.60 which are not so far from ***D*** = 2***b***. Recently, Guo and Ogata (1995) found a positive correlation between ***b*** and ***D***2 for aftershocks sequences in Japan, with a general behavior ***D***2 > 2***b***.

In the present work, we calculate the surface fractal dimension of a distribution of faults, fractures and lineaments over a region at the western coast of Guerrero State in southern Mexico, an active seismic zone linked to the Middle American Trench. Our results suggest that the fractal dimension of the fault distribution embedded both in a surface and in a volume is positively correlated with the ***b***-value. In the case of the two-dimensional approach (shallow earthquakes) ***D*** and ***b*** approximate Aki's relation.

Fractal dimension and seismicity in southern Mexico

The box counting method consists in dividing a box containing the fractal object into small boxes of linear dimension ***r***, and count how many smaller boxes ***N***(***r***) contain points from the object:

where ***D*** is the fractal dimension (Mandelbrot, 1982; Sander *et al*., 1994).

The western coast of Guerrero has been identified as a seismic gap, in which no large earthquakes have occurred at least since 1908 (Suárez *et al*., 1990). The data used in our study were obtained from geologic maps at 1:250,000 (INEGI, 1985) for two regions: The Zihuatanejo sheet between 16.875°-18° N and 100°-102° W and the Acapulco sheet between 16°-17° N and 98°-100° W (Figure 1). The surfaces of these areas are 26,563 km2 and 23,780 km2 respectively. A third geological sheet (González, 1992) corresponds to the Chilpancingo region between 17°-18° N and 99°-100° W with a surface of 11,890 km2 (Figure 1). We consider all linear features, without distinguishing between faults, fractures or lineaments. Our maps correspond to case one of Hirata (1989b), where the fractal dimension is a measure of the spatial distribution of the faults. We use boxes with lengths between 2.5 km and 12.5 km. Figure 2 shows an example for the Zihuatanejo sheet.

Fig. 1. A) Zihuatanejo region; B) Acapulco region; C) Chilpancingo region. The isodepth contours were taken from Pardo and Suárez (1993).

Fig. 2. Zihuatanejo geologic map including faults, fractures and lineaments. Some box sizes are shown.

For the total region we obtain $\boldsymbol{D}_t = 1.6437 \pm 0.0028$ by means of a least square fit in a double-log plane (Figure 3). Singh *et al*. (1983) found a ***b***-value between 0.82 and 0.84, for this region. Thus $2\boldsymbol{b} = 1.64$ (=) $\boldsymbol{D}_t$ in good agreement with Eq. (3). For the Acapulco-Zihuatanejo subarea, which contains most of the seismic activity, we obtain $\boldsymbol{D}_{AZ}$(=)1.7, close to 1.68 which is twice the value of $\boldsymbol{b} = 0.84$. For the Chilpancingo region alone we obtain $\boldsymbol{D}_C=1.4$. Thus our results approximately support Aki's formula (Eq. 3). In the absence of a hypocenter catalog, we may estimate the fractal dimension of a 3-d distribution by means of $\boldsymbol{D}_{3d} = \boldsymbol{D}_{2d} + 1$ (Okubo and Aki, 1987). Thus $\boldsymbol{D}_{t3}$(=)2.64, $\boldsymbol{D}_{AZ3}$(=)2.7 and $\boldsymbol{D}_{C3}=2.4$ respectively. These results are in the order of those found by Guo and Ogata (1995) for hypocenter distributions in Japan.

In conclusion, in both cases $\boldsymbol{D} >= 2\boldsymbol{b}$, suggesting a positive correlation between ***b*** and ***D*** in the spatial variation. The Chilpancingo ***b***-value is lower than for Acapulco-Zihuatanejo. From the Harvard and PDE (USGS) catalogs in the period from 1969 up to 1995 we calculate $\boldsymbol{b}_A$(=)$\boldsymbol{b}_Z=0.67$ and $\boldsymbol{b}_C=0.52$ respectively.

Pardo and Suárez (1993) report focal-isodepth contours for the Mexican subduction region from 20 km along the Pacific coast, up to 80 km inland. The Acapulco-Zihuata-nejo sub-area is mainly in the 20 km-isodepth contour and the Chilpancingo sheet in the 40 km contour (Figure 1). Since the Chilpancingo value $\boldsymbol{D}_C$(=)1.4 is lower than $\boldsymbol{D}_{AZ}$(=)1.7, the fractal dimension may be correlated with the depth of the seismogenic zone: The larger the seismogenic depth the smaller the fractal dimension. This behavior is consistent with the finding by Pacheco *et al*. (1992) that for a system containing many faults, fewer large events will occur than expected from extrapolating the distribution of small earthquakes, since large events are bounded by the down-dip width of the rupture.

Fig. 3. Double-log graph of ***N*** versus ***r*** a slope ***D(=)***-1.64. The box lengths used were 2.5, 5.0, 7.5, 10.0 and 12.5 km.

Conclusion

From available geologic maps of the distribution of faults, fractures and lineaments in southern Mexico, we find that ***b*** and ***D*** are positively correlated in the spatial sense. For both 2d and 3d cases we find that ***D***>=2.1. Our results also suggest that the earth's surface dimension may be linked to the depth of the seismogenic zone. Thus, an estimation of ***D*** could give an idea of the depth of the seismogenic zone in some earth's area.

Acknowledgments

This work was partially supported by Secretaría General de Obras DDF and COFFA-IPN-México. We also thank Prof. K. Ito for his valuable comments.

BIBLIOGRAPHY

Aki, K. 1981. A probabilistic synthesis of precursors phenomena. Earthquake Prediction. Maurice Ewing series, 4, (ed. Simson and Richards) (AGU, Washington D.C.) 566-575.

Bak, P., C. Tang and K. Wiesenfeld, 1987. Self-organized criticality: an explanation of 1/f noise. *Phys. Rev. Lett. 59*, 381-384.

Bak, P. and C. Tang, 1989. Earthquake as a self-organized critical phenomenon. *J. Geophys. Res. 94*. 15635-15637.

Barriere, B. and D. L. Turcotte, 1994. Seismicity and self-organized criticality. *Phys. Rev. E. 49*, 1151-1160.

GonzAlez, M. E. 1992. Estudio geomorfológico comparativo con modelos magnetométricos del área del Cerro de Toro, Chilpancingo, Guerrero. M. Sc. Thesis ESIA-IPN México.

Guo, Z. and Y. Ogata, 1995. Correlation between characteristic parameters of aftershock distributions in time space and magnitude. *Geophys. Res. Lett. 22*, 993-996.

Hirata, T. 1989a. A correlation between de ***b*** value and the fractal dimension of earthquakes. *J. Geophys. Res. 94*, 7507-7514.

Hirata, T. 1989b. Fractal dimension of fault systems in Japan: Fractal structure in rock fracture geometry at various scales. *Pageoh. 131*, 157-170.

INEGI. 1985. E-14-11 Hoja Acapulco, México.

INEGI. 1985. E-14-7 Hoja Zihuatanejo, México.

Ito, K. and M. Matsuzaki, 1990. Earthquake as a self-organized critical phenomenon. *J. Geophys. Res. 95*, 6853-6860.

King, G. 1983. The accommodation of large strains in the upper lithosphere of the earth and other solids by self-similar fault systems: the geometrical origin of ***b***-value. *Pageoh. 121*, 761-815.

Lomnitz-Adler, J. 1992. Interplay of fault dynamics and fractal dimension in determining Gutenberg-Richter's ***b***-value. *Geophys. J. Int. 108*, 941-944.

Mandelbrot, B. B. 1982. The fractal geometry of nature. (Freeman, San Francisco).

Okubo, P. G. and K. Aki, 1987. Fractal Geometry in the San Andreas Fault System. *J. Geophys. Res. 92*, 345-355.

Pacheco, J. F., H. Christopher Scholz and R. Lynn Sykes, 1992. Change in frequency-size relationship from small to large earthquakes. *Nature. 335*, 71-73.

Pardo, M. and G. SuArez, 1993. Steep subduction geometry of the Rivera Plate beneath the Jalisco block in western Mexico. *Geophys. Res. Lett. 20*, 2391-2394.

Sander, E., L.M. Sander and R.F. Ziff, 1994. Fractals and fractal correlations. *Comput. Phys. 8*, 420-425.

Singh, S.K., M. Rodriguez and L. Esteva, 1983. Statistics of small earthquakes and frequency of occurrence of large earthquakes along the Mexican subduction zone. *Bull. Seism. Soc. Am., 73,* 1779-1796.

Sornette, A. and D. Sornette, 1989. Self-organized criticality and earthquakes. *Europhys. Lett. 9*, 197-202.

Sornette, D., C. Vanneste and A. Sornette. 1991. Dispersion of ***b***-values in Gutenberg-Richter law as a consequence of a proposed fractal nature of continental faulting. *Geophys. Res. Lett. 18*, 997-900.

SuArez, G., T. Monfret, G. Wittlinger and C. David, 1990. Geometry of subduction and depth of the seismogenic zone in the Guerrero gap, Mexico. *Nature, 345,* 336-338.

Turcotte, D.L. 1989. A fractal approach to probabilistic seismic hazard assessment. *Tectonophysics 167*, 171-177.

Turcotte, D. L., 1992. Fractals and chaos in geology and geophysics. (Cambridge University. Press. Cambridge).

F. Angulo-Brown1, A. H. Ramirez-Guzmán2, E. Yépez1, A. Rudolf-Navarro1 and C. G. Pavía-Miller3,1

1 Departamento de Física, Escuela Superior de Física y Matemáticas, IPN, Edif 9, U.P. Zacatenco 07738 México D.F.

2 Sección de Gaduados, Escuela Superior de Ingeniería y Arquitectura, IPN, Edif 12, U.P. Zacatenco 07738 México D.F.

3 Area de Física, Departamento de Ciencias Básicas, DCBI, UAM-A, Col Reynosa 02200 México D.F.