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Inter-well tracer tests in oil reservoirs using different optimization methods: A field case

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RESUMEN

En la interpretación de pruebas de trazadores entre pozos en yacimientos petroleros, geotérmicos y en acuíferos se emplean diversos métodos de regresión no lineal para determinar algunas de las propiedades físicas promedio del sistema roca fluido. Con este propósito se ajustan modelos analíticos a los datos de campo de surgencia del trazador y se determinan los parámetros libres del modelo. La no linealidad inherente al problema puede en ocasiones dar lugar a soluciones múltiples, las cuales corresponden a distintos mínimos locales. En la metodología de interpretación tradicional se hace uso de un solo método de optimización, y se considera diversos valores iniciales de los parámetros para analizar la existencia de varias soluciones. En general, este procedimiento resulta complicado y requiere de largos tiempos de cómputo. Además, para obtener resultados confiables es necesario proponer valores iniciales cercanos al óptimo global, los cuales en muchos de los casos de campo se desconocen. El empleo de distintos métodos de búsqueda para obtener el óptimo global resulta entonces una herramienta de gran utilidad. En este trabajo presentamos una nueva metodología que consiste en el uso simultáneo de varios métodos de optimización y de tan sólo pocos valores iniciales. De esta manera se pueden encontrar soluciones al problema inverso de forma relativamente simple y confiable.

PALABRAS CLAVES: Pruebas de trazadores, problema inverso, optimización no lineal, transporte de fluidos, medios porosos.

ABSTRACT

In the interpretation of inter-well tracer tests to determine reservoir properties in oil and geothermal reservoirs as well as in aquifers, different non-linear regression methods are used. Analytical flow models are employed to fit tracer breakthrough data in order to determine the free parameters in the models. Non-linearity can yield multiple solutions for the fitting parameters. Traditionally, a single optimization method and several initial parameter values are employed. This procedure is often cumbersome and computer time-consuming. Moreover, an initial point close to the global optimum must be provided, what in many field cases is not available. We propose an approach which employs several optimization methods simultaneously, using a few initial points. Thus, different solutions can be found in a relatively simple way, and the reliability of the solutions is improved.

KEY WORD: Tracer tests, inverse problem, non linear optimization methods, fluid transport, porous media.

1. INTRODUCTION

Since the beginning of the last century tracer tests have been employed in underground porous media to extract information of flow directions, flow barriers and communication channels. A tracer test may provide quantitative information of reservoir properties. This can be achieved by matching mathematical models for tracer flow to field data. With this procedure the free parameters of the model are determined, and some reservoir properties, such as porosity, dispersion coefficient, fracture width, block size, etc., can be calculated. In general, optimization methods yield multiple solutions because of the inherent non-linearity of the problem, or when it is ill-posed. Finding these solutions in the region of interest commonly requires a large amount of computing time. The solution is traditionally obtained from a single optimization method. Until now, the problem of finding different solution has been only partially discussed, for example in the determination of groundwater flow direction (Steinich *et al.* 1996), or of permeability and oil saturation in oil reservoirs (Lliassov and Datta-Gupta, 2001), in fractured reservoirs (Ramírez, *et al.* 1994), in trapped gas saturation (Radke and Gillis, 1999), or in residual oil saturation (Tang and Zhang, 2000). In this paper we examine simultaneously distinct optimization methods that make use of different search procedures to reach the optimal value. Therefore, from a single initial point, several solutions can be obtained. We find that the procedure reliability is also increased by this mean.

We analyze two typical mathematical models from the tracer test literature, each model having two free parameters. We use ten different optimization methods to match tracer data. In Section 2, we describe the models, and in Section 3 we present a methodology to perform the optimization. Section 4 is devoted to its application on synthetic data, and Section 5 to its application on real field data. The conclusions are drawn in Section 6.

2. DESCRIPTION OF THE MODELS

The mathematical models employed in this work describe the behavior of tracers in one dimensional linear reservoirs subject to convection and dispersion. The first model corresponds to continuous tracer injection, and the second to pulse injection. The tracer concentration in continuous injection is given by Coats and Smith (1964) and Bear (1972), as follows:

$$C_c(x_D,t_D)=\frac{C_0}{2}\left[\operatorname{erfc}\left(\frac{x_D-t_D}{\sqrt{4t_D/Pe}}\right)+\exp(x_D Pe)\operatorname{erfc}\left(\frac{x_D+t_D}{\sqrt{4t_D/Pe}}\right)\right], \quad (1)$$

where the dimensionless space and time variables, $x_D = x/L$ and $t_D = tu/L$ are introduced. Here u is a convective constant speed and L a characteristic length, which can be set equal to the distance between the injection and the observation well. Pe is a Peclet number defined by $Pe = uL/D$, where D is a constant dispersion coefficient. In porous media it holds $D = \alpha u + D^*$, where α is hydrodynamic dispersivity and D^* a molecular dispersion coefficient (Bear, 1972). For typical reservoir conditions it can be assumed that $D^* << \alpha u$. C_0 is the tracer concentration at the injection site. The tracer concentration at any position $x_D > 0$ is initially zero and increases monotonically reaching C_0 asymptotically at large times. The tracer breakthrough curve (concentration as a function of time) obtained from Eq. (1) contains two free parameters, x_D and Pe. From x_D the effective transit length could be obtained using $x = x_D L$, and from Pe the dispersion coefficient can be calculated as $D=uL/Pe$.

The model to be employed for a short pulse injection, known as 'spike injection', is (Kreft and Zuber, 1978; Ramírez, 1988):

$$C_s(x_D,t_D)=\frac{E\,x_D}{\sqrt{4\pi\,t_D^3/Pe}}\exp\left[-\frac{Pe\,(x_D-t_D)^2}{4t_D}\right]. \quad (2)$$

For a given position, $x_D > 0$, the breakthrough curve is zero during an initial time period. Later, it increases abruptly and reaches a maximum. Finally, it smoothly reduces to zero at large times. The expression in Eq. (2) contains the nonlinear fitting parameters x_D and Pe, and the linear parameter E, which is a scale factor associated to the production rate (Kreft and Zuber, 1978).

3. METHODOLOGY

Solving the inverse problem means finding the values of the fitting parameters in the models of Eq. (1) and Eq. (2) that minimize the square of the differences for a given data set. To this purpose an objective function is defined as

$$OF(\beta_1,...,\beta_k)=\sum_{i=1}^{N}\left[C(\beta_1,...,\beta_k;t_i)-c_i(t_i)\right]^2, \quad (3)$$

and non-linear regression methods are employed to obtain the parameter set $\{\beta_1 \ldots \beta_k\}$ that minimizes $OF(\beta_1,...,\beta_k)$ in Eq. (3). Here $C(\beta_1 \ldots \beta_k; t_i)$ is the function given by Eq.(1) or Eq.(2), $\{c_i(t_i)\}$ is a data set, and N is the total data number in the set. As mentioned before, ten different non-linear optimization methods are to be employed. They can be grouped in (i) methods that require the evaluations of the objective function, such as the Complex method (Nelder and Mead, 1965), (ii) methods that demand the evaluation of its first derivatives, such as the Gauss-Newton, Levenberg-Marquardt, (Levenberg, 1944 and Marquardt, 1963), VARPRO (Golub and Pereyra, 1973) and Rosenbrock (1960) methods, (iii) methods that employ Hessian updating such as the Fletcher (1963), Broyden (1970), Shanno (1970), Gill (1984), Goldfarb (1970) methods, and (iv) Sequential Quadratic Programming methods, such as the method by Biggs (1975), which invoke the solution of an auxiliary quadratic problem.

The Nelder and Mead method is suited for problems that are highly nonlinear or involve discontinuous functions. In the Simplex method, the simplex is characterized in a k-dimensional space by k+1 distinct vertices, and the objective function is evaluated in each of them. At each step of the search, a new point in or near the current simplex is generated. The function value at the new point is compared with the value at the other vertices and, usually, one of the vertices is replaced by the new point, giving a new simplex. This step is repeated until the diameter of the simplex is less that the specific tolerance. The gradient methods use information on the function slope to define a search direction, where the minimum seems to lie. The steepest descent method is the simplest of this group of methods, here the search is performed in direction $-\nabla[OF(\beta_1,...,\beta_k)]$. An important issue in the implementation of the Levenberg-Marquardt method is the setting of an effective strategy for controlling the regularization parameter at each iteration, in such a way that the method becomes efficient for a broad spectrum of problems. The strategy employed here consists in estimating the relative no linearity of the objective function OF in Eq.(2), by using a linear predicted sum of squares of the objective function and a cubicly interpolated estimation of its minimum (Optimization Toolbox User's Guide, 2000). The Gauss method is obtained when the regularization parameter is set to zero. In the Rosenbrock algorithm the first variable β_1 is set at a distance S_1 along the gradient of OF, and the function is evaluated there. If the resulting value of OF decreases, the move is considered successful and S_1 is increased by a factor larger than the unity. If the value of OF increases, the move is termed a failure and S_1 decreased by a positive factor lower or equal than the unity, and the direction of move-

ment is reversed. The same procedure is applied to the whole set of parameters $\{\beta_i\}$. The methods employing Hessian updating consider the behavior of *OF* and its gradient in order to acquire appropriate curvature information. An updating technique is employed to approximate the Hessian without calculating it numerically. These methods use a quadratic model for *F0* given by $\min_{\beta} \frac{1}{2}\beta^T H\beta + c^T\beta + b$, where H is the Hessian matrix, which is symmetric and positive defined, c a constant vector, and b a constant scalar. A large number of Hessian update algorithms have been developed, some of them have been cited above.

To study the characteristics of these methods, two different types of data are analyzed in relation to the models under consideration. One type corresponds to synthetic data, and the other type to field results obtained from a tracer test performed in the Loma Alta Sur field in Argentina (Somaruga, 2003). The first data type results from adding random noise to deterministic values obtained from the models in Eq. (1) and Eq. (2). For continuous injection synthetic data are generated from Eq. (1), with $x_D = 1$, $Pe = 1$ and $C_0 = 1$ was used, and noise following a Gaussian distribution with zero mean and standard deviation $\sigma = C_0/10$ was added. The effect of different noise amplitudes will be briefly analyzed below. For spike injection Eq. (2) was employed with $x_D = 2$, $Pe = 4$ and E=1, and a standard deviation $\sigma = E/10$. Ten different realizations of synthetic data were generated for each model, and ten different optimization methods were applied in each case. Several data realizations are used here to analyze variability and to perform realization averages. For illustration purposes one of these synthetic data sets is displayed for continuous injection in Figure 1, and equivalently for spike injection in Figure 2.

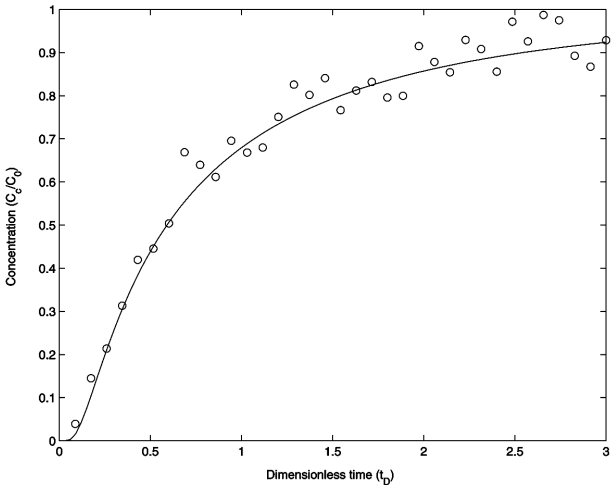

Fig. 1. Synthetic tracer breakthrough data obtained from the model for continuous injection (solid line) plus a Gaussian random noise (circles).

4. RESULTS USING SYNTHETIC DATA

In Table 1 the results for the continuous injection model in Eq.(1) obtained from each of the ten synthetic data sets after application of the optimization methods mentioned before are shown. The original parameters used were x_D = 1 and Pe = 1. The parameters resulting from the application of the different optimization methods to a data set are in all cases the same. The starting values used were x_D = 0.6 and Pe = 1.5. The obtained average value (over the ten data sets) is Pe = 1.023 and x_D= 0.976. For the noise level employed here, 10%, the minimum and maximum variations from the original value are 1% and 12% for Pe, and 3% and 26% for x_D, respectively. Thus, the original values were recovered within a relative small error.

An interesting issue is the sensitivity of the optimization methods to different starting value x_D and Pe. For this purpose, initial parameter values were chosen between 0.1 and 10.0. The results are shown in Table 2. The resulting differences from the original value x_D = 1 and Pe = 1 can become up to 500%. This table shows that the most robust optimization methods are Nelder-Mead (Complex), Gauss-Newton and Rosenbrock, since they converge and reach the correct value even when the initial point is far from the original value. On the contrary, the worst method is the Steepest-Descent method, since it converges only for small deviations from the original value. Finally, when the initial point is 900% away from the original value all methods fail.

Table 3 shows the results for spike injection using ten different synthetic data realizations. Synthetic data were generated employing x_D = 2.0 and Pe = 4.0, and noise is added in the same way as in the previous case. Iterations for the optimization methods started with x_D = 1.0 and Pe = 10.0,

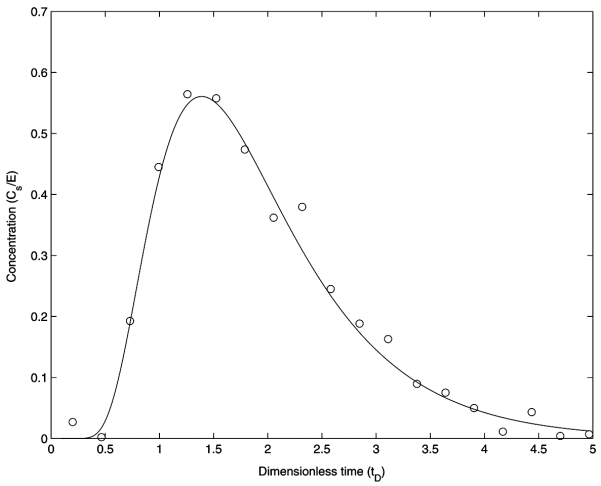

Fig. 2. Synthetic tracer breakthrough data obtained from the model for spike injection (solid line) plus a Gaussian random noise (circles).

Table 1

Optimized parameter values for ten different realization sets of synthetic data representing continuous injection.

Continuous Injection Data set	Optimized x_D Value	Optimized P_e Value	Objective Function Value
data01	0.89	1.01	0.0484
data02	0.97	1.02	0.0499
data03	1.24	0.93	0.0657
data04	0.97	1.02	0.0499
data05	1.09	1.02	0.0563
data06	0.87	1.06	0.0568
data07	0.75	1.12	0.0519
data08	1.13	0.98	0.0880
data09	1.06	0.97	0.0713
data10	0.78	1.10	0.0805

i.e., taking a value 100% and 150% away from the original values. The average value of x_D and Pe, considering the ten synthetic data sets, are 1.99 and 4.01, respectively. When the methods converge, they all reach the same result, with a difference around 0.1%. Within the ten data sets the minimum and maximum deviations from the real value of Pe are 1% and 16%. For x_D, the minimum and maximum deviations are 0.5% and 3.5%, respectively. Here the same noise amplitude as in the previous case of Table 1 was used.

In Table 4 spike injection results using different initial points for parameter values between 0.5 and 10.0 are shown. The algorithms Complex, Gauss-Newton, Levenberg-Marquardt and SQP converge even when the initial point for x_D is 500% larger than the original value. The Steepest – Descent method was again the worst optimization method.

Different noise levels for the continuous and spike injection cases were also treated: {0.10, 0.13, 0.17, 0.20}. It was found that the values of the fitting parameters are practically the same when the noise level is lower than 0.17. However, when it becomes 0.20, the parameters obtained show an important deviation from the real values. It is to be mentioned that the typical noise level in tracer tests is lower than 0.17.

5. RESULTS USING FIELD DATA

Oil field data from a spike injection tracer test performed in Loma Alta Sur in Argentina (Somaruga 2003) were used to study the optimization methods performance. There were analyzed two cases: well A and well B. The average transit time was calculated from the first moment of the tracer breakthrough curve in each case. The ground surface distance between the injection and the production wells is 142.0 *m* and 131.0 *m* for case A and B respectively, and the corresponding average transit time is 19.7 and 6.1 days. Consequently, the mean velocity is $u = 7.2$ *m/d* and *21.6 m/d*, respectively.

Since the tracer was injected as a short pulse, then Eq. (2) is used to model the tracer test in the Loma Alta Sur field

Table 2

Optimization methods convergence for various starting points in the case of continuum injection. The real values are $x_D = 1$ and $P_e = 1$. Here, √ means convergence and no symbol means no convergence

Starting Point $[x_D, Pe]$	Nelder Mead Complex	Gauss Newton	Rosen-brock	Broyden Fletcher Goldfarb Shanno	Davidon Fletcher Powell	Levenberg-Marquardt	Gill-Murray	Sequential Quadratic Programming	Steepest Descent
[0.1 0.1]	√	√	√	√	√	√	√		√
[1.5 1.5]	√	√	√	√	√	√	√	√	
[2.0 2.0]	√	√	√					√	√
[0.1 2.0]	√	√	√	√	√	√	√		
[2.0 0.1]	√	√	√	√			√	√	
[2.0 0.6]	√	√	√					√	
[0.6 2.0]	√	√	√			√			
[2.5 2.5]	√	√	√	√	√				
[3.0 3.0]	√	√	√						
[5.0 5.0]	√	√	√						
[10.0 10.0]									

Table 3

Optimized parameter values for ten sets of synthetic data representing spike injection.

Spike Injection data set	Optimized x_D Value	Optimized P_e Value	Objetive Function Value
datap01	2.05	3.67	0.1393
datap02	1.98	3.90	0.1096
datap03	2.02	4.04	0.1310
datap04	2.00	4.22	0.1076
datap05	1.93	4.63	0.1038
datap06	2.03	3.71	0.1417
datap07	2.00	3.55	0.1884
datap08	1.97	4.09	0.2101
datap09	2.03	3.86	0.1918
datap10	1.94	4.48	0.2103

for both cases. This equation involves 3 fitting parameters, Pe, x_D and E. The linear parameter E, which is related to the total amount of tracer per unit of cross section that arrives at the production well, is found by the application of the linear least squares method. A similar data fitting treatment was made by Jensen (1983) using the VARPRO optimization method. For comparison purposes this method is also included in our analysis.

When fitting Eq. (2) to field data, the time should be changed to the dimensionless time $t_D=ut/L$, this is done by employing the values of u and L mentioned before. There is no reservoir information that suggests specifically what initial value for x_D and Pe to take. However, by using various optimization methods we can obtain several solutions from a single initial parameter value.

The iteration procedure for well A was started with x_D = 1.0 and Pe = 4.0. No initial value to calculate the parameter E is required, since C_s (x_D, t_D) has a linear dependence on it. Optimization results are shown in Table 5, where two sets of parameters can be identified. They are approximately (x_D, Pe) = (0.95, 5.63) and (x_D, Pe) = (1.06, 4.01). The parameter Pe is used to calculate D through $D = uL/Pe$, and the parameter x_D is employed to estimate the effective travel length by $x = x_D/L$. One of the parameter sets obtained gives a dispersion coefficient and a travel length of $2.10\text{x}10^{-3} m^2/s$ and 134.9 m, respectively. The other parameter set yields D = $2.95\text{x}10^{-3} m^2/s$ and x=150.7m (the corresponding initial values used were x = 142.0 m and $D = 2.90\text{x}10^{-3} m^2/s$). The breakthrough curves resulting from the two parameter sets are shown in Figure 3.

For the case of well B the initial parameters chosen are x_D=0.8 and Pe=4.0. Optimization results are shown in Table 6, where three sets of parameters can be identified. They are: (x_D, Pe) = (0.67, 3.99), (x_D, Pe) = (0.69, 3.66), and (x_D, Pe) = (0.71, 3.47) and the corresponding physical properties are (x_D)=(87.8m, $8.20\text{x}10^{-3} m^2/s$), (90.9m, $8.93\text{x}10^{-3} m^2/s$) and (92.9m, $9.42\text{x}10^{-3} m^2/s$). The breakthrough curves resulting from the three parameter sets are shown in Figure 4. In the

Table 4

Optimization methods convergence for various initial points in the case of spike injection. The real values are x_D = 2 and Pe = 4. Here, √ means convergence and no symbol means no convergence

Starting Point [x_D, Pe]	Nelder Mead Complex	Gauss Newton	Rosen-brock	Broyden Fletcher Goldfarb Shanno	Davidon Fletcher Powell	Levenberg-Marquardt	Gill-Murray	Sequential Quadratic Programming	Steepest Descent
[0.5 0.5]									
[0.6 0.6]									
[0.8 0.8]			√						
[0.9 0.9]	√	√	√	√	√	√	√	√	
[1.0 1.0]	√	√	√	√	√	√	√	√	
[1.0 4.0]	√	√	√	√	√	√	√	√	√
[1.0 6.0]	√	√	√	√	√	√	√	√	
[6.0 1.0]	√	√	√	√	√	√	√	√	
[6.0 6.0]	√	√			√	√	√	√	
[10.0 10.0]	√	√				√		√	

Table 5

Results for different optimization algorithms applied to the case of well A using Eq. (2)

Objective Function Value	Number of Iterations	Optimized x_D Value	Optimized P_e Value	Optimized E[fraction]	Resulting Parameter D[m²/seg]	Resulting Parameter x[m]	Method's name
9.76E-04	39	0.95	5.63	3.32E-03	2.10E-03	134.9	Nelder-Mead
1.07E-03	3	1.06	4.01	3.60E-03	2.95E-03	150.7	Broyden-Fletcher Goldfarb-Shanno
1.07E-03	3	1.06	4.01	3.60E-03	2.95E-03	150.7	Davidon-Fletcher Powell
1.07E-03	20	1.06	4.04	3.59E-03	2.93E-03	150.3	Steepest-Descent
1.07E-03	3	1.06	4.01	3.60E-03	2.95E-03	150.7	Gill-Murray
9.76E-04	17	0.95	5.63	3.32E-03	2.10E-03	134.9	Sequential Quadratic Programming
9.76E-04	4	0.95	5.63	3.32E-03	2.10E-03	134.9	Levenberg-Marquardt
9.76E-04	4	0.95	5.63	3.32E-03	2.10E-03	134.9	Gauss-Newton
1.08E-03	3	1.07	3.95	3.62E-03	2.99E-03	152.1	Rosenbrock
9.70E-04	8	0.95	5.63	3.32E-03	2.10E-03	134.9	VARPRO

Table 6

Results for different optimization algorithms applied to the case of well B using Eq. (2)

Objective Function Value	Numbers of Iterations	Optimized x_D Value	Optimized P_e Value	Optimized E[fraction]	Resulting Parameter D[m²/seg]	Resulting Parameter x[m]	Method's name
8.47E-03	53	0.69	3.66	5.43E-03	8.93E-03	90.9	Nelder-Mead
8.53E-03	6	0.67	3.99	5.35E-03	8.20E-03	87.8	Broyden-Fletcher Goldfarb-Shanno
8.53E-03	6	0.67	3.99	5.35E-03	8.20E-03	87.8	Davidon-Fletcher Powell
8.53E-03	5	0.67	3.99	5.35E-03	8.20E-03	87.8	Steepest-Descent
8.53E-03	6	0.67	3.99	5.35E-03	8.20E-03	87.8	Gill-Murray
8.53E-03	5	0.67	3.99	5.35E-03	8.20E-03	87.8	Sequential Quadratic Programming
8.49E-03	3	0.71	3.47	5.47E-03	9.42E-03	92.9	Levenberg-Marquardt
8.49E-03	3	0.71	3.47	5.47E-03	9.42E-03	92.9	Gauss-Newton
8.47E-03	3	0.69	3.66	5.43E-03	8.93E-03	90.9	Rosenbrock
8.47E-03	8	0.69	3.66	5.43E-03	8.93E-03	90.9	VARPRO

case of well B it is found that the solutions yield dispersion coefficients with a difference of around 13% between them. This percentage difference can be compared against the case of well A, which is 33%.

Additional information would be needed to prefer one of these curves over the other. In well A, for example, if we assign more physical relevance to the first 12 data points than to the last 5 data points (at the curve tail), the solid line in Figure (3) should be chosen. A procedure using weights in the field data according to their reliability would be helpful in this case.

6. CONCLUDING REMARKS

The benefits of using simultaneously several optimization methods to analyze inter-well tracer tests in reservoirs have been described. By employing this procedure to solve the inverse problem, different local minima in the region of interest can be obtained in a relatively simple way. This is especially useful when real field data are analyzed, particularly when no information for the initial parameters is available. The procedure presented here should be compared against the standard procedure consisting in finding a solution for each set of fitting parameters in a multidimensional

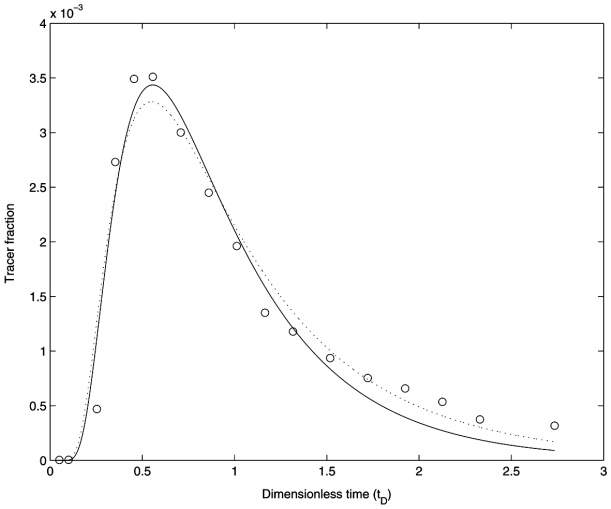

Fig. 3. Tracer breakthrough curve of well A for the field test performed in Loma Alta Sur. Circles represent measured tracer concentration. Solid (x_D = 0.95, P_e = 5.63) and broken lines (x_D = 1.06, P_e=4.01) provide the two fitting curves obtained. The tracer fraction arriving per day is given in the vertical axes.

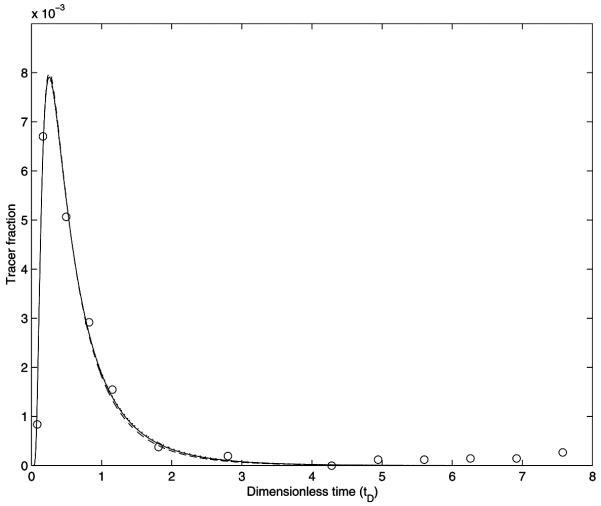

Fig. 4. Tracer breakthrough curve of well B for the field test performed in Loma Alta Sur in B well. Circles represent measured tracer concentration. Solid (x_D = 0.69, P_e = 3.66), broken (x_D =0.67, P_e=3.99) and doted (x_D=0.71, P_e=3.47) lines show the three fitting curves obtained.

parameter grid. In the case when several parameters are involved, the standard procedure becomes cumbersome and computer time consuming. Further, the procedure discussed here can provide additional reliability on the solution, when the same solution appears using different optimization methods.

The analysis of synthetic data done here has shown that the best optimization methods are Nelder-Mead (Complex), Gauss-Newton and Rosenbrock for the case of continuous injection, and Nelder-Mead, Gauss-Newton, Levenberg-Marquardt and SQP in the spike injection case. However, none of them can be favored over the others. The worst method in all the cases was the Steepest-Descent method. It is to be mentioned that by increasing the noise level above certain range, the convergence of the different optimization methods is importantly affected. In the cases analyzed here, the noise level limit is about 0.17.

The treatment of the data from Loma Alta Sur Field has illustrated the situation when two or more solutions appear (several local minima). The same solution is obtained by several optimization methods, what is a solid evidence for the existence of multiple solutions. To prefer one solution over the others, additional information would be required. Different techniques to treat data, such as smoothing abrupt changes or assigning weights accordingly to data relevance or uncertainty would be useful in analyzing multiple solutions.

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