

geofísica
internacional

Geofísica Internacional

ISSN: 0016-7169

silvia@geofisica.unam.mx

Universidad Nacional Autónoma de México
México

Malischewsky Auning, Peter G.

The ellipticity of Rayleigh waves at infinite depth

Geofísica Internacional, vol. 47, núm. 1, enero-marzo, 2008, pp. 77-79

Universidad Nacional Autónoma de México

Distrito Federal, México

Disponible en: <http://www.redalyc.org/articulo.oa?id=56847106>

- Cómo citar el artículo
- Número completo
- Más información del artículo
- Página de la revista en redalyc.org

redalyc.org

Sistema de Información Científica

Red de Revistas Científicas de América Latina, el Caribe, España y Portugal

Proyecto académico sin fines de lucro, desarrollado bajo la iniciativa de acceso abierto

Short Note

The ellipticity of Rayleigh waves at infinite depth

P. Malischewsky Auning

Institut für Geowissenschaften, Friedrich-Schiller-Universität, Jena, Germany

Received: October 18, 2007; accepted: November 9, 2007

Resumen

Se presenta una fórmula analítica y una aproximación para calcular la elipticidad de las ondas de Rayleigh en un semi-espacio homogéneo a profundidad infinita, en función del módulo de Poisson. Se compara el resultado con las fórmulas correspondientes para elipticidad en la superficie.

Palabras clave: Ondas de Rayleigh, elipticidad.

Abstract

I present an analytical formula and an approximation for the ellipticity of Rayleigh waves in a homogeneous half-space at infinite depth in terms of Poisson's ratio. The result is compared with the corresponding formulas for surface ellipticity.

Key words: Rayleigh waves, ellipticity.

Introduction

The ellipticity or H/V ratio χ_0 of seismic Rayleigh waves propagating on the surface of the Earth has attracted the attention of experimental seismologists (see, e. g., Lermo and Chávez-García, 1994; Bard, 1998); Flores-Estrella, 2004) as well as theoreticians (see, e. g., Malischewsky and Scherbaum, 2004). The ellipticity of Rayleigh waves at infinite depth χ_∞ is a significant parameter that belongs to a complete theoretical description of the wave field. Weichert (2007) pointed out that the ellipticity adopts a constant value at infinite depth. This is indeed the case: its value may be simply determined as a function of Poisson's ratio ν . It should be noted, however, that the meaning of "infinite depth" is frequency-dependent. It can extend over an interval of many kilometers for long-period Rayleigh waves, or of a few meters for very high-frequency waves. This feature can be significant in some geophysical situations, e. g. for borehole measurements.

An analytical formula for χ_∞

Representations of the Rayleigh eigenfunctions for a homogeneous half-space lead to the simple formula

$$\chi_\infty(\nu) = \sqrt{1 - x(\nu)}, \quad x = c^2 / \beta^2, \quad (1)$$

where c is the phase velocity and β is the shear-wave velocity. By using Malischewsky's formula for the Ray-

leigh-wave velocity in a half-space (see Malischewsky, 2004), the ellipticity at infinite depth may be expressed analytically as a function of Poisson's ratio as

$$\chi_\infty = \sqrt{1 - 2g_4(\nu)} \quad (2)$$

where the following abbreviations are used:

$$g_3(\nu) = 17 + 3\sqrt{33 - 24\bar{\nu}^3 + \frac{321}{4}\bar{\nu}^2 - 93\bar{\nu} - \frac{45}{2}\bar{\nu}},$$

$$g_4(\nu) = \frac{1}{3} \left[4 + \frac{2(1-3\bar{\nu})}{\sqrt[3]{g_3(\nu)}} - \sqrt[3]{g_3(\nu)} \right], \text{ and}$$

$$\bar{\nu} = 1 - \frac{\nu}{1-\nu}, \quad (3)$$

and the main values of the cubic roots are used throughout. For the reader's convenience we recall the formula for χ_0 (see Malischewsky *et al.*, 2007):

$$\chi_0 = \frac{\sqrt{1 - 2g_4(\nu)}}{1 - g_4(\nu)} = \frac{\chi_\infty}{1 - g_4(\nu)}. \quad (4)$$

The ellipticities χ_0 and χ_∞ in terms of Poisson's ratio are shown in Fig. 1 together with the difference between

both values. Negative Poisson's ratios do not arise in seismology, except in some particular crystallization phases of ice (Bormann, 2002). However, they do have some importance in material science, and they are included here for completeness. At infinite depth, the ellipse described by particle motion is always flatter than it is on the surface. In Fig. 1, the difference between the ellipticities has a maximum at $\nu = 0$ ($\chi_0 - \chi_\infty = 0.3$), and the relative deviation $(\chi_0 - \chi_\infty)/\chi_0$ is maximum at $\nu = 0.5$. In the valley of Mexico, Poisson's ratio is as high as 0.499 and the ellipticity at infinite depth reaches 54.4% of the surface ellipticity.

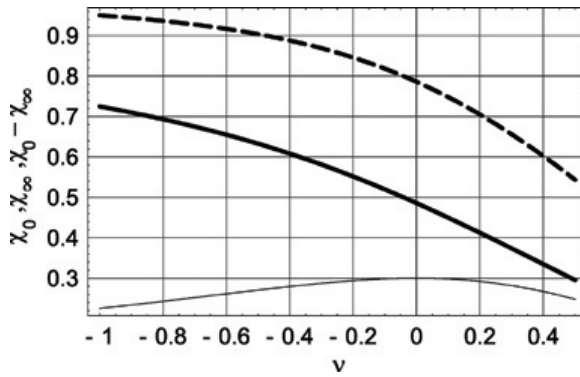


Fig. 1. The ellipticities χ_∞ (heavy line), χ_0 (dashed line) and $\chi_0 - \chi_\infty$ (thin line), as a function of Poisson's ratio ν .

Finally, we may make use of a method proposed by Pham Chi Vinh and Malischewsky (2006) which involves carrying out a Taylor expansion of Equations (2) and (4) in the interval $\nu \in (0, 0.5)$. The approximation is very accurate, with a relative error of less than 0.1 % in the whole interval:

$$\begin{aligned}\chi_\infty &= 0.486 - 0.355\nu - 0.080\nu^2 + 0.057\nu^3, \\ \chi_0 &= 0.786 - 0.349\nu - 0.292\nu^2 + 0.041\nu^3.\end{aligned}\quad (5)$$

These formulas may be easily inverted to obtain Poisson's ratio. Let

$$m_1(u) = \left[-0.027 + 0.0877 \left(u + \sqrt{(u + 0.0881)(u - 0.7046)} \right) \right]^{1/2} \quad (6)$$

$$m_2(u) = \left[0.0517 + 0.0454 \left(u + \sqrt{(u + 3.1621)(u - 0.8829)} \right) \right]^{1/2}; \quad (7)$$

then the equations for the corresponding ellipticities are

$$\nu = \operatorname{Re} \left[0.4678 - \frac{0.2472 - 0.4282i}{m_1(\chi_\infty)} - (2.3208 + 4.0197i)m_1(\chi_\infty) \right]. \quad (8)$$

$$\nu = \operatorname{Re} \left[2.374 - \frac{0.6565 - 1.1372i}{m_2(\chi_0)} - (3.2264 + 5.5883i)m_2(\chi_0) \right], \quad (9)$$

where i denotes the imaginary unit and the main values of the cubic roots should be used. These formulas may also be useful in possible new applications of the H/V -method for non-destructive testing (see Malischewsky *et al.*, 2006, and Weichert, 2007).

Acknowledgements

The support of the Bundesministerium für Bildung und Forschung (BMBF) in the framework of the joint project "WTZ Germany-Israel: System Earth" under Grant No. 03F0448A is gratefully acknowledged.

Bibliography

- Bard, P.-Y., 1998. Microtremor measurements: A tool for site effect estimation. Proc. Second International Symposium on the Effect of Surface Geology on Seismic Motion, Yokohama, Japan, 1-3 December 1998, pp. 25-33.
- Bormann, P. (Ed.), 2002. IASPEI New Manual of Seismological Observatory Practice, GeoForschungsZentrum, Potsdam.
- Flores-Estrella, H., 2004. The SPAC method: An alternative for the estimation of model velocities in the valley of Mexico (in Spanish), MSc thesis, UNAM, Mexico.
- Lermo, J. and F. J. Chávez-García, 1994. Are microtremors useful in site response evaluation? *Bull. Seismol. Soc. Am.*, 84, 1350-1364.
- Malischewsky, P. G. and F. Scherbaum, 2004. Love's formula and H/V -ratio (ellipticity) of Rayleigh waves. *Wave Motion*, 40, 57-67.
- Malischewsky Auning, P. G., 2004. A note on Rayleigh-wave velocities as a function of the material parameters. *Geofisica Internacional*, 43, 507-509.
- Malischewsky, P. G., J. D. Schnapp and A. Ziegert, 2006. The ellipticity of Rayleigh waves and non-destructive testing. Abstracts, 9th European Conference on NDT (ECNDT), Berlin, Germany, September 25-29, 2006, We. 2. 4. 3.

Malischewsky, P. G., F. Scherbaum, C. Lomnitz, Tran Thanh Tuan, F. Wuttke and G. Shamir, 2007. The domain of existence of prograde Rayleigh-wave motion for simple models, submitted to *Wave Motion*.

Pham Chi Vinh and P. G. Malischewsky, 2006. Explanation for Malischewsky's approximate expression for the Rayleigh wave velocity. *Ultrasonics*, 45, 77-81.

Weichert, CH, 2007. Investigation of material properties by using the ellipticity of Rayleigh waves, Ph. D. thesis, Friedrich-Schiller University, Jena, Germany, 226 pp.

Peter G. Malischewsky Auning
*Friedrich-Schiller-Universität Jena,
Institut für Geowissenschaften, Burgweg 11,
07749 Jena, Germany
E-mail: p.mali@uni-jena.de*