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The threshold anomaly from the simultaneous calculation of elastic scattering and fusion cross sections for the systems $^9$Be+$^{144}$Sm and $^9$Be+$^{64}$Zn for energies around the barrier

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The energy dependence of the optical potential is used to study the threshold anomaly for reactions with the weakly bound projectile $^9$Be on $^{144}$Sm and $^{64}$Zn for energies around the Coulomb barrier. The energy dependent potential parameters are obtained from a simultaneous $\chi^2$-analysis of elastic scattering and fusion data. There are signatures that in fact, the so-called breakup threshold anomaly shows up for these systems. This finding is in agreement with other calculations involving weakly bound projectiles.

Keywords: Nuclear reactions; weakly bound nuclei; threshold anomaly.

1. Introduction

One of the main interests in the field of low energy nuclear reactions has been the role that the breakup process plays on fusion and other reaction mechanisms in reactions involving weakly bound nuclei [1]. This subject is usually studied by two different approaches. The first is the direct investigation of the behavior of fusion excitation functions and coupled channel calculations for fusion reactions. The second approach is more indirect, and it is concerned with the behavior of the energy dependence of optical potentials derived from elastic scattering data, particularly on the presence of the threshold anomaly in the elastic scattering of those nuclei. Most of such studies, in both approaches, use the following weakly bound nuclei as projectiles: $^9$Be, $^6$Li, $^7$Li and $^6$He. For all of them, the breakup separation energies is much smaller than the usual 8 MeV/A required to separate nucleons from tightly bound nuclei. For the nuclei mentioned, the breakup threshold energies range from $S_{\alpha}=2.47$ MeV for the $^7$Li $\rightarrow \alpha + t$ breakup to $S_{2n}=0.98$ MeV for the $^6$He $\rightarrow 4^\text{He}+2\text{n}$ breakup. A very interesting nucleus to be investigated is $^8$Be, which breaks up as $^9$Be $\rightarrow ^8$Be+$n$ $\rightarrow \alpha + \alpha + n$ with $S_n = 1.67$ MeV. Reactions with these weakly bound projectiles show that the breakup yield has considerable values even for energies well below the Coulomb barrier. For the scattering of tightly bound systems, the threshold anomaly (TA) usually is observed. Since this phenomenon is associated to the closing of reaction channels as the bombarding energy decreases towards the Coulomb barrier, it is interesting to investigate whether the TA is present in the scattering of weakly bound projectiles. It has recently been proposed that reactions involving the most weakly bound nuclei show a different type of anomaly, the so-called Breakup Threshold Anomaly (BTA) [2-4]. The origin of the BTA is the fact that the breakup process for weakly bound nuclei has large cross section even at sub-barrier energies, and so the absorptive part of the nuclear potential does not show a sharp decrease as the bombarding energy decreases, but keeps constant values or increasing values at energies slightly below the barrier. So, if BTA occurs, the energy dependence of the real nuclear potential derived from the dispersion relation does not show the usual bell-shape associated to the TA around $V_B$ [5].

At this point, it is important to mention that the energy dependence of the optical potential is produced by polarization potentials originated from the coupling between the elastic scattering and different reaction mechanisms, such as inelastic excitation, transfer of nucleons or clusters of nucleons, breakup, fusion, fission and quasi-fission. These mechanisms may produce polarizations of different signs, attractive or repulsive[6,7]. The net effect on the energy dependence of the total optical potential depends on the importance and strength of the different specific polarization potentials. For systems containing only tightly bound nuclei, couplings to bound ex-
excited states or transfer channels produce an attractive polarization potential and, consequently, leading to enhancement of the fusion cross section at energies near and below the Coulomb barrier, when compared with no-coupling calculations. For these systems, the energy dependence of the optical potential shows the usual TA [8,9]. When weakly bound nuclei are involved in the interaction, the breakup channel may remain open at energies below the Coulomb barrier, and so the imaginary part of the potential does not vanish near the barrier. Actually, for stable weakly bound nuclei, such as $^6$Li and $^9$Be, it has been observed [2,3,10,11] that the strength of the imaginary part of the optical potential increases as the energy is reduced toward the nominal Coulomb barrier. From this characteristic of the imaginary potential and because of the dispersion relation, the strength of the real part of the potential decreases at this energy region. For halo or neutron skin nuclei, not only the breakup channel but also transfer channels may remain open, with cross sections much larger than that for fusion for energies below the fusion barrier.

In this work, we consider the extensive measurements for the reaction between $^9$Be with the medium mass targets $^{64}$Zn [12–15] and $^{144}$Sm [16,17] and perform a theoretical study of them using the optical model for direct reactions. Specifically, a simultaneous calculation of elastic scattering, complete fusion and breakup cross sections is done within the optical model. Particular emphasis is focused into the effect that the breakup process has on fusion and on the threshold anomaly. The approach used is the one proposed by Udagawa et al. [18,19] to describe fusion within the framework of direct reaction theory. Thus, a Woods-Saxon optical potential $V_a = V_a + iW_a$ for the entrance channel $a$ is considered, where the imaginary potential $V_a$ is split into volume and surface parts, that is $W_a = W_{a,F} + W_{a,DR}$. It will be assumed that the volume part $W_{a,F}$ is solely responsible for the complete fusion absorption process while the surface part $W_{a,DR}$ is so for all other absorption processes. Therefore, we propose that, by means of the decomposition of $W_a$, the effect of the breakup of the projectile on fusion will be more clearly isolated. It is expected that the conjugated energy dependence of the fusion potential $W_{a,F}$ and the direct reaction potential $W_{a,DR}$ will tell us how strong the breakup effect on fusion. For the systems $^9$Be+$^{64}$Zn and $^9$Be+$^{144}$Sm, direct reactions are mostly breakup reactions, particularly near and below the Coulomb barrier, hence these should be treated by $W_{a,DR}$.

The Woods-Saxon parameters of the optical potentials will be extracted by a simultaneous fit of complete fusion and elastic scattering data. The direct reaction cross section $\sigma_{DR} = \sigma_R - \sigma_{CF}$ includes incomplete fusion $\sigma_{ICF}$, inelastic scattering $\sigma_{inel}$ and non-capture breakup $\sigma_{NCBU}$. In fact, $\sigma_{NCBU}$ for the nuclear systems under consideration, accounts for most of the total reaction cross section at energies below the Coulomb barrier. The threshold anomaly is discussed by studying the behavior of the energy dependent potentials $V_{a,F}(r, E), V_{a,DR}(r, E), W_{a,F}(r, E)$ and $W_{a,DR}(r, E)$ at the strong absorption radius $R_{sa}$. Finally the effect of breakup reactions on complete fusion is discussed by studying the effects of $V_{a,DR}$ and $W_{a,DR}$ on the fusion cross section calculation.

2. Basic equations

The Hamiltonian $H$ for the nuclear system is of the form,

$$H_a = T_a + V_a,$$

where the distorted wave $\chi_a^{(+)}$ satisfies the following expression,

$$(T_a + V_a)\chi_a^{(+)} = E_a\chi_a^{(+)}$$

and the potential $V_a$ is defined by,

$$V_a(r, E) = V_{Coul}(r) - V_{a,0}(r) - U_a(r, E).$$

Here, $V_{Coul}(r)$ is the Coulomb potential, $V_{a,0}(r)$ is the energy independent average nucleus-nucleus potential and $U_a(r, E)$ is the nuclear polarization potential which is given by [20-23],

$$U_a(r, E) = V_{a}(r, E) + iW_a(r, E).$$

In order to simplify the notation, we drop the subindex $a$ which refers to the incident elastic channel. Then, the imaginary part $W$ is assumed to have two parts, i.e.,

$$W(r, E) = W_F(r, E) + W_{DR}(r, E),$$

where $W_F$ accounts for complete fusion and $W_{DR}$ for all other absorption processes. Thus, Eq. (4) can be written as $U = U_F + U_{DR}$, where $U_F = V_F + iW_F$, $U_{DR} = V_{DR} + iW_{DR}$ with $V = V_F + V_{DR}$.

The strength of the real polarization potential $V(E)$ can be derived from the imaginary polarization potential strength $W(E)$ by the dispersion relation,

$$V_i(E) = V_i(E_s) + \frac{(E - E_s)}{\pi} \int \frac{W_i(E')}{(E' - E_s)(E' - E)} dE',$$

where $V_i(E_s)$ is a value of the potential at the reference energy $E_s$ as defined in Ref. 5. So, once the energy dependent forms for $V_i, i = F, DR$ are calculated, the corresponding real potential parts $V_i$ can be found.

The energy independent nuclear potential $V_0(r)$ and the fusion absorption potential $W_F(r, E)$ are assumed to have the geometrical forms[24],

$$V_0(r) = V_0 f(r)$$

$$W_F(r, E) = W_F f(r)$$
and,

\[ W_F(r, E) = W_F(E)f(r) \]  

(8)

where

\[ f(x_i) = \frac{1}{1 + \exp(x_i)}, \quad x_i = \frac{r - R_i}{a_i}, \quad i = 0, F. \]  

(9)

with \( R_i = r_i(A_1^{1/3} + A_2^{1/3}) \), \( r_i \) being the reduced radius parameter and \( a_i \) is the diffuseness parameter.

The surface imaginary potential \( W_{DR}(r, E) \) is define by,

\[ W_{DR}(r, E) = 4a_{DR}W_{DR}(E)\frac{df(x_{DR})}{dr}, \]  

(10)

where \( a_{DR} \) stands for the direct reaction diffuseness and \( x_{DR} = (r - R_{DR})/a_{DR} \). The potentials \( V_F(r, E) \) and \( V_{DR}(r, E) \) are assumed to have the same forms as \( W_F(r, E) \) and \( W_{DR}(r, E) \) respectively, with the same diffuseness and reduced radius. The parameters of \( V_0(r) \), \( W_F(r, E) \), \( W_{DR}(r, E) \) as well as the strengths of \( V_F(r, E) \) and \( V_{DR}(r, E) \) will be extracted from a simultaneous \( \chi^2 \)-analysis of the elastic and complete fusion data, as it will be shown in the next section. It should be pointed out that the breakup cross section may include contributions from Coulomb and nuclear interactions, and therefore the direct reaction potential may include both effects. Also, the Hartree-Fock potential \( V_0(r) \) of Eq. (3) may have an energy dependence due to non-locality effects coming from knock-on-exchange contribution. We shall not consider such effects since they are negligible [25].

The angle-integrated total reaction cross section is calculated by using the full absorption potential \( W \), i.e.,

\[ \sigma_R(E) = \frac{2}{hv} \left< \chi_a^{(+)} | W(E) | \chi_a^{(+)} \right>, \]  

(11)

here we have rewritten the sub-index \( a \) to emphasize the elastic channel. The fusion and direct reaction cross sections are similarly obtained by

\[ \sigma_i(E) = \frac{2}{hv} \left< \chi_{a_i}^{(+)} | W_i(E) | \chi_{a_i}^{(+)} \right>, \quad i = F, DR. \]  

(12)

The calculated relative motion distorted waves \( \chi_a^{(+)} \) obtained with the Woods-Saxon potential \( U_a \) of Eq. (4) will be used throughout the calculations. So, in this sense all the calculated cross sections will be consistent with elastic scattering.

3. \( \chi^2 \)-analysis of elastic scattering and complete fusion.

We begin by performing a simultaneous \( \chi^2 \)-analyses of elastic scattering and CF data for the system \(^9\)Be+\(^{144}\)Sm at the energies \( E_{lab} = 33, 34, 35, 37, 39 \) and \( 41 \) MeV [16,17]. The parameters of the real energy independent average potential \( V_0(r) \) of Eq. (7), were found by fitting the elastic scattering data at \( 32 \) MeV, with a volume Woods-Saxon absorption potential with radial parameter \( r_W = 1.4 \) fm, strength \( W = 64.3 \) MeV and diffuseness \( a_W = 0.36 \) fm. The derived parameters for \( V_0(r) \) are \( V_0 = 25 \) MeV, \( r_0 = 1.22 \) fm and \( a_0 = 0.52 \) fm, which correspond to a shallow potential as it is required in the fit of elastic scattering data of projectiles such as \(^6\)Li and \(^9\)Be [26]. Concerning the potential \( W_F(r, E) \), the radius parameter \( r_F \) was fixed at 1.4 fm thus the strength \( W_F(E) \) and the diffuseness \( a_F \) are calculated by the \( \chi^2 \)-analysis. As for the surface potential \( W_{DR}(r, E) \), we set \( a_{DR} = 0.72 \) fm, then \( W_{DR}(E) \) and the reduced radius \( r_{DR} \) are calculated. Since the geometric forms of \( V_F(r, E) \) and \( V_{DR}(r, E) \) are assumed to be the same as \( W_F(r, E) \) and

<table>
<thead>
<tr>
<th>( E_{cm} ) (MeV)</th>
<th>( a_F ) (fm)</th>
<th>( r_{DR} ) (fm)</th>
<th>( \chi^2/N )</th>
</tr>
</thead>
<tbody>
<tr>
<td>31.1</td>
<td>0.57</td>
<td>1.61</td>
<td>0.21</td>
</tr>
<tr>
<td>32.0</td>
<td>0.51</td>
<td>1.63</td>
<td>0.65</td>
</tr>
<tr>
<td>32.9</td>
<td>0.56</td>
<td>1.63</td>
<td>0.45</td>
</tr>
<tr>
<td>34.8</td>
<td>0.53</td>
<td>1.66</td>
<td>0.56</td>
</tr>
<tr>
<td>36.7</td>
<td>0.67</td>
<td>1.67</td>
<td>0.90</td>
</tr>
<tr>
<td>38.6</td>
<td>0.67</td>
<td>1.68</td>
<td>0.90</td>
</tr>
</tbody>
</table>
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Figure 2. Results of the ratios of elastic scattering cross sections to Rutherford cross sections for the system $^9\text{Be}+^{144}\text{Sm}$. The experimental data are taken from Ref. 16.

$W_{DR}(r, E)$ respectively with the same reduced radius and diffuseness parameters, therefore only the energy dependent strengths $V_F(E)$ and $V_{DR}(E)$ are to be determined. In Figs. 1a and 1b, we show the results for the potential strengths $W_F(E)$, $W_{DR}(E)$, $V_F(E)$ and $V_{DR}(E)$ and in Table I, the values for $a_F$ and $r_{DR}$.

The straight lines in Fig. 1a are linear fit to the extracted potentials given by,

\[
W_F(E) = \begin{cases} 
0 & \text{if } E \leq E_{CF} = 28.73 \\
0.49(E - 28.73) & 28.73 < E \leq 33.06 \\
2.122 & E > 33.06 
\end{cases} \tag{13}
\]

and

\[
W_{DR}(E) = \begin{cases} 
0 & \text{if } E \leq E_{DR} = 24.74 \\
0.031(E - 24.74) & 24.74 < E \leq 33.33 \\
0.2663 & E > 33.3 
\end{cases} \tag{14}
\]

Figure 3. Calculated complete fusion and total reaction cross sections. Data taken from Ref. 16.

Figure 4. Results for the direct reaction cross section $\sigma_{DR} = \sigma_R - \sigma_{CF}$. Data taken from Ref. 16

Here, $E_{CF}$ and $E_{DR}$ correspond to the complete fusion and direct reaction threshold energies, which are found by the zero of the linear fit to the values $S_i = \sqrt{\langle E_{cm}\sigma_{i,exp} \rangle}$, $i = CF, R$, as prescribed by P.H. Stelson et al. [27]. It is assumed that the threshold energy of direct reactions $E_{DR}$ coincides with that of the total reaction cross section $E_R$. This is so since, for energies below the sub-barrier regime, breakup reactions account for almost all of the total reaction cross section. The lines in Fig. 1b correspond to the calculation of the real polarization potentials $V_F(E)$ and $V_{DR}(E)$ by means of the dispersion relation [5], where the reference energy has been set at $E_0 = 32.94$ MeV with reference potentials $V_F(E_0) = 1.5$ MeV and $V_{DR}(E_0) = -0.05$ MeV. As seen, from Figs. 1a and 1b our calculations show that the dispersion relation is satisfied. The elastic scattering cross section shown in Fig. 2, is determined with the use of the FRESCO code [28]. Figure 3 shows the total reaction $\sigma_R$ and complete fusion $\sigma_{CF}$ cross section results in the center of mass system and in Fig. 4, the calculations for $\sigma_R - \sigma_{CF}$ are given in comparison with the "data" for $\sigma_{NCBU} + \sigma_{ICF} + \sigma_{inelastic}$. As regards to the system $^9\text{Be}+^{64}\text{Zn}$, we follow the same lines as done for the case of $^{144}\text{Sm}$. We consider the experimental data of [12–15] and perform a simultaneous $\chi^2$ analysis of elastic scattering and complete fusion data at the laboratory energies of 21, 23, 26 and 28 MeV. As before the Coulomb potential radius is set at $r_C = 1.25$ fm. The optical potential parameters of the energy independent potential $V_0(r)$ are fitted at $V_0 = 66.0$ MeV, $a_0 = 0.52$ fm and $r_0 = 1.22$ fm. In this calculation, we have fitted the diffuseness parameters at $a_F = 0.35$ fm and $a_{DR} = 0.25$ fm. In Table II the determined values from the $\chi^2$-fitting for the radii parameters are given. As there are not available complete fusion data at 17 MeV and 19 MeV, we estimated these by a
Table II. Calculated values for the reduced radii (fm) and cross sections (mb). Data taken from Refs. 12 to 15.

<table>
<thead>
<tr>
<th>$E_{lab}$ (MeV)</th>
<th>$\tau_R$</th>
<th>$\tau_{DR}$</th>
<th>$\chi^2/N$</th>
<th>$\sigma_{CF}$</th>
<th>$\sigma_R$</th>
<th>$\sigma_{CF,exp}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
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<td>0.90</td>
<td>5.0</td>
<td>70.0</td>
<td>4.9</td>
</tr>
<tr>
<td>19</td>
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<td>1.89</td>
<td>0.68</td>
<td>524</td>
<td>603</td>
<td>530</td>
</tr>
<tr>
<td>21</td>
<td>1.64</td>
<td>1.93</td>
<td>0.38</td>
<td>329</td>
<td>420</td>
<td>344</td>
</tr>
<tr>
<td>23</td>
<td>1.57</td>
<td>1.7</td>
<td>0.39</td>
<td>524</td>
<td>603</td>
<td>530</td>
</tr>
<tr>
<td>26</td>
<td>1.56</td>
<td>1.7</td>
<td>0.71</td>
<td>774</td>
<td>876</td>
<td>800</td>
</tr>
<tr>
<td>28</td>
<td>1.58</td>
<td>1.7</td>
<td>0.74</td>
<td>976</td>
<td>1049</td>
<td>1000</td>
</tr>
</tbody>
</table>

Wong calculation [29]. That is, the existing data at 21, 23, 26 and 28 MeV are fitted with the parameters $V_B = 17$ MeV, $R_B = 10.0$ fm and $\hbar \omega = 4.0$ MeV.

The determined potentials $V_F$, $V_{DR}$, $V_F'$ and $W_{DR}$ are given in Fig. 5 at the strong absorption radius $R_{sa} = 12.2$ fm. The elastic scattering cross section is presented in Fig. 6. The calculated values of the total reaction cross section shown in Table II are those extracted from elastic scattering data. Also, the values of the complete fusion cross sections are shown with the corresponding experimental measurements.

As seen from Figs. 2, 3, 4, 6 and Table II our results are in close agreement with the data for both nuclear systems.

4. A study of the threshold anomaly and the effect of breakup on fusion

One of the most interesting features of this model, that of separating the incident flu absorption into fusion and direct reaction parts, represented by the potentials $W_F$, $V_F$ and $W_{DR}$, is not present for the systems $^9$Be+$^{64}$Zn and $^9$Be+$^{144}$Sm. This finding is in agreement with some other previously reported works for different nuclear systems involving weakly bound projectiles such as $^6$Li+$^{208}$Po, $^{138}$Ba [30,33], $^{6,7}$Li, $^9$Be+$^{27}$Al [11,34,35] and $^9$Be+$^{209}$Bi [32,36]. However, we believe that the present work shows more clearly what hap-
pens in terms of the two components of the imaginary part of the optical potential, that is $W_{DR}$ and $W_{DR}$. The fact that the imaginary potential is dominated by the direct reaction part as the energy is lowered below the natural barrier threshold, and therefore the "threshold" ceases to be the barrier itself, was pointed out by Hussein et al.,\cite{2} as a characteristic of the Breakup Threshold Anomaly (BTA).

Now, we propose that the effect of breakup reactions on the fusion cross section can be studied by analyzing the effect that the potentials responsible for direct reactions, that is $V_{DR}$ and $W_{DR}$ have on fusion. This is true since for the systems at hand, breakup is by far, the most important contribution to the direct reaction cross section in the low energy region. There are two main physical contributing effects by which $V_{DR}$ and $W_{DR}$ may affect fusion reactions.

(i)- A repulsive $V_{DR}$ tends to lower the Coulomb barrier and therefore suppresses fusion and,

(ii)- The loss of incident flu into direct reactions, represented by $W_{DR}$, suppresses fusion.

From Figs. 5 and 7, it is evident $V_{DR}$ suppresses fusion since becomes repulsive in the energy region under study. On the other hand, $W_{DR}$ which is connected to the loss of flu mainly into the breakup channel, also suppresses fusion. When both potentials $V_{DR}$ and $W_{DR}$ are simultaneously applied, we obtain a net suppression. This result is in agreement with some other works for systems with weakly bound projectiles on heavy targets, where experimental complete fusion cross sections are compared with coupled channel calculations which use potentials deduced from experimental barrier distributions\cite{37-39} or reliable double folding potentials\cite{16,17,40-43} that do not take into account the breakup channel. However, it is important to mention that the fusion suppression due to the breakup effects is not observed when light targets are used\cite{44-46}, and therefore the Coulomb breakup is not so important as for heavier targets.

5. Summary and conclusions

In summary, we have carried out a simultaneous $\chi^2$-analysis of elastic scattering and complete fusion cross sections for the systems $^9$Be+$^{64}$Zn and $^9$Be+$^{144}$Sm, at near fusion barrier energies. In the model, the optical polarization potential has been split into a fusion part and a direct reaction part, each one being responsible for the corresponding fusion and direct reaction absorption processes. The results of the $\chi^2$-fitting show that the extracted potentials satisfy the dispersion relation. It has been shown that the fusion potential $W_F(E)$ exhibits the threshold anomaly as it occurs in reactions with tightly bound projectiles. However, this is not the case for the surface potential $W_{DR}(E)$ that is a smooth function of $E$. Energy dependent forms for $W_F$, $V_{DR}$, $W_F$ and $W_{DR}$ at the strong absorption radius, show that the direct reaction potential is much more important than the fusion potential. As a matter of fact, the forms of the total potentials $V(E)$ and $W(E)$, which are dominated by $V_{DR}(E)$ and $W_{DR}(E)$, are rather smooth functions of the energy around the barrier. Therefore, our results indicate that instead of the usual threshold anomaly, the Breakup Threshold Anomaly shows up for these nuclear systems. We believe that the method used in this work, is more illustrative for this kind of analysis than the ones which do not split the optical potential.

The effect of breakup on fusion cross sections has been studied by analyzing the effect of the potentials responsible for direct reactions, $V_{DR}$ and $W_{DR}$. By separately considering these potentials, it has been determined that, there is a net effect of suppression of complete fusion cross sections for energies above the Coulomb barrier energy.

\begin{thebibliography}{99}
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