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The Kodama state for topological quantum field theory beyond instantons in topological Yang-Mills theory

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We constructed a symplectic structure that preserves the symmetries including the topological invariance for topological Yang-Mills theory, and it is shown that the Kodama (Chern-Simons) state traditionally associated with a topological phase of unbroken diffeomorphism invariance for instantons, exists actually for the complete topological sector of the Yang-Mills theory.

Keywords: Teorías topológicas; estado de Kodama; instantones.

Se construye una estructura simpléctica que preserva las simetrías, incluida la invariancia topológica, de la teoría de Yang-Mills topológica, se muestra que el estado de Kodama (Chern-Simons) generalmente asociado a instantones existe per se para el sector topológico de la teoría, sin la necesidad de imponer condiciones de auto-dualidad.

Descriptores: Topological theories; Kodama state; instantons.

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1. Introduction

Yang-Mills theory in four dimensions admits the so called Chern-Simons wavefunction as an exact zero energy eigenfunction of the Schrödinger equation [1]. This solution presents deep problems, since such a state is neither normalizable, nor invariant under CPT and negative helicity states not only have negative energy but also negative norm. Therefore the Chern-Simons state is not admissible as the ground state of the quantum Yang-Mills theory [2]. Despite all these properties, it is important to understand what this intriguing state describe.

The self-duality condition on the fields associated with instantons play a important role producing the deformation of the original action into a topological action, picking up thus a topological phase among various ground states of that theory. That state turn out to be the only quantum state for the topological quantum field theory obtained [5, 6]. Therefore, it is natural to associate the Chern-Simons (or Kodama) state with the sector corresponding to instantons.

We shall show that the Chern-Simons state is associated actually with the whole of the topological sector of the Yang-Mills theory (TYM), without using the self-duality conditions for instantons, provided that we start from the appropriate topological action, given in Yang-Mills case by the second Chern class.

2. TYM theory

We can construct a YM theory from a topological invariant, the second Chern class,

$$S_{TYM}(A) = \beta \int_M \text{Tr} (F \wedge F),$$  \hspace{1cm} (1)

where $\beta$ is a parameter, $A$ is the gauge connection and $F = dA + A \wedge A$ its curvature; $d$ and $\wedge$ correspond to the exterior derivative and the wedge product on $M$, which we assume as the four-dimensional Minkowski spacetime.

From Eq.(1) we obtain the Bianchi identity

$$dF + [A, F] = 0. \hspace{1cm} (2)$$

Note that when the connection is self-dual or anti-self-dual $F = \pm \ast F$, the Bianchi identity (2) automatically implies the Yang-Mills equations, and additionally the YM action turns out to be a topological action, independent on the metric of $M$.

The topological Yang-Mills action (1) is the subject of the present study forgotten at all the self-duality condition.

3. The symplectic structure for TYM theory

We can construct from the action (1) a symplectic structure that preserves all relevant symmetries of the theory given by [8, 10]

$$\omega = \int_{\Sigma} 4\beta \text{Tr} \delta \tilde{F}^{\mu\nu} \delta A_\mu d\Sigma_\nu, \hspace{1cm} (3)$$

where $\Sigma$ is a Cauchy hypersurface, and $\delta$ corresponds to the exterior derivative on the phase space $Z$ of the theory [9].

It is important remark that $\omega$ retains all the symmetries of the topological action. $\omega$ is an gauge invariant symplectic structure, is a topological invariant and independent on the choice of $\Sigma$ because it is covariantly conserved [10].
4. Classical and Quantum Hamiltonian for TYM theory and the Chern-Simons state

Our simplectic structure give us the canonical variables and the (symmetric and gauge-invariant) energy-momentum tensor for TYM theory [9, 10].

If we considering that $d\Sigma_{\mu}$ is a time-like vector field in Eq. (3) we can obtain in particular the following (non-covariant) description of the phase space,

$$\omega = \int_\Sigma 4\beta \text{Tr} (\delta \tilde{F}_{0i} \land \delta A_i),$$

(4)

in order to make contact with [2, 5]. Equation (4) shows explicitly that the canonical variables for TYM theory are given by $(2\beta) \tilde{F}_{0i}$ and $A_i$.

In other hand the energy-momentum tensor for TYM theory is given by

$$T^{\mu\nu} = -\text{Tr} 4\beta (F^{(\mu}_\alpha \tilde{F}^{\nu)\alpha} - \frac{1}{4} g^{\mu\nu} \tilde{F}^{\alpha\beta} F_{\alpha\beta}),$$

(5)

which is classically zero, as expected for a topological action.

Using the component $T_{00}$ and the classical-quantum correspondence

$$(2\beta) \tilde{F}_{0i} \rightarrow i \frac{\delta}{\delta A_i},$$

(6)

we find the quantum Hamiltonian

$$H_Q = \int_\Sigma d\Sigma \text{Tr} F_{0i} \left( i \frac{\delta}{\delta A_i} - \beta \epsilon_{ijk} F_{jk} \right).$$

(7)

In the temporal gauge $A_0 = 0$, it is easy to show that $[F_{0i}, \delta/\delta A_i]=0$, and thus we have no ordering ambiguity in the quantum Hamiltonian (7). Thus, any wave function $\psi$ satisfying

$$\left( i \frac{\delta}{\delta A_i} - \beta \epsilon_{ijk} F_{jk} \right) \psi = 0,$$

(8)

will correspond to a state of zero energy for the Hamiltonian (7). Therefore, the solution for Eq. (8) is given by

$$\psi(A) = e^{-4\pi i\beta I(A)},$$

(9)

where $I$ is the Chern-Simons functional

$$I = \frac{1}{4\pi} \int d^3x \text{Tr} (\epsilon^{ijk} (A_i \partial_j A_k + \frac{2}{3} A_i A_j A_k)).$$

(10)

Equation (9) is essentially the Chern-Simons wave function [3, 4].

We conclude then that the Chern-Simons state corresponds strictly to a (topological) state of the TYM theory, without invoking the self-duality condition on the fields. Thus, such a state is associated with the topological phase of unbroken diffeomorphism invariance of the complete topological sector.

The Hamiltonian (7) is purely a combination of constraints, with $F_{0i}$, the dual of the canonical momentum, playing the role of a Lagrange multiplier field [10].

5. Concluding remarks

The Chern-Simons state exists for the complete topological sector of the theory, and in order to establish its existence, neither the self-dual condition nor the Yang-Mills equations are required. The Bianchi identity becomes the generator of gauge symmetries at quantum level. As a particular case, the topological phases of Yang-Mills theory can be obtained invoking self-duality.

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