Fast Adaptive Trajectory Tracking Control for a Completely Uncertain DC Motor via Output Feedback

Control Adaptable Rápido por Realimentación de Salida para Seguimiento de Trayectorias en un Motor de C.C. Completamente Desconocido

H. Sira Ramírez, E. Barrios Cruz and R. Márquez Contreras
Departamento de Ingeniería Eléctrica, Cinvestav,
Av. Instituto Politécnico Nacional, San Pedro Zacatenco, C.P. 07360,
A.P. 14-740, México, D.F. 07300, México.
{hsira,ebarrios,rmarquez}@cinvestav.mx

Article received on March 10, 2008; accepted on September 04, 2008

Resumen
Utilizando el método algebraico para la identificación de los parámetros desconocidos en sistemas lineales se sintetiza un controlador adaptable rápido, implementado en línea, para el control de seguimiento de referencia de velocidad angular en un motor de corriente continua sujeto a una perturbación de carga constante. Para resolver el problema de seguimiento de salida en presencia de perturbaciones, se propone un controlador del tipo Proporcional Integral Generalizado (GPI) por retroalimentación de salida escrito en forma de red de compensación clásica. La adaptación rápida de los parámetros del sistema se lleva a cabo tanto en los parámetros que definen la red de compensación clásica como en la expresión de la señal de pre-compensación de entrada. El método propuesto se valida mediante resultados experimentales.
Palabras clave: Identificación algebráica, Motores de c.c., Control adaptable

Abstract
An algebraic parameter identification method, developed for fast, on-line, computation of unknown linear system parameters, is here used for the fast adaptive output feedback control of a completely unknown dc motor, subject to constant perturbation load torques while solving a reference trajectory tracking task. An output feedback controller of the Generalized Proportional Integral (GPI) type, written in classical compensation network form, is proposed for the perturbed output trajectory tracking problem. The fast adaptation of system parameters is carried out, both, on the classical compensating network parameters and on the conformation of the feed-forward control input signal. Experimental results validate the effectiveness of the proposed approach.
Keywords: Algebraic identification, DC motors, Adaptive control

1 Introducción

The identification of dynamic systems, in general, and of DC motor models, in particular, has been the subject of sustained interest over the years (6). Most of the literature concerning DC motor identification is dominated by, off-line, least squares based, linear, identification techniques (See, for instance the article by Kara and Elker (8)). Closed loop identification, also called ``on-line" identification,-for feedback control purposes- is of utmost interest in scientific and industrial environments (see the work of Lakshminarayanan *et al*. (9)). Due to the nonlinear perturbations present in DC motor dynamics (such as Coulomb friction terms, saturations, dead zones, etc.), the techniques proposed range from applications of extended Kalman filtering to Neural Networks (see, for instance, Jang and Leon (7)). An interesting work is that of Elker (2), which combines traditional root means square errors with discrete time identification algorithms for the on-line control of a mechanical system actuated by a DC motor.

In recent years, algebraic techniques have been developed for the fast, on line, reliable estimation, or identification, of system parameters, states, failures and input perturbations. The fundamentals of the approach, for the linear systems case, may be found in the article by Fliess and Sira-Ramírez (4) and in the book chapter (5). The idea, which may be suitably extended for a large class of nonlinear systems, is to obtain a linear set of time-varying equations for the unknown parameters which are independent of the initial conditions and of the structured perturbations. The initial conditions and the structured perturbations are conveniently eliminated by algebraic manipulations including: multiplications of the differential equations by suitable powers of the time variable, combined with a finite number of time differentiations of the equations and followed by a sufficient number of integrations. In the linear case, the non-commutative procedures are greatly simplified thanks to ``Operational Calculus" and use of the ``Algebraic Derivative" in the complex domain (see section IV and (4), for the mathematical basis, examples, and further details). The net result is a linear system of equations, independent of unknown initial conditions and of classical perturbations, from which the parameters can be efficiently computed on the basis solely of measured inputs and outputs, while being robust with respect to zero mean measurement and plant noises whose statistics become unnecessary.

Generalized Proportional Integral (GPI) control of linear systems was developed as an output linear feedback control technique which may be shown to be robust with respect to the, so called, classical perturbation inputs (steps, ramps, polynomial perturbation inputs). GPI controllers were developed by Fliess *et al*. in (3) as an alternative means of circumventing the use of traditional observers. Connections with sliding mode control, without state measurements, can be found in Sira-Ramírez (11). Applications of GPI control in the regulation of bilinear power electronics models is the subject of the article (12) and the recent book by Sira-Ramírez and Silva-Ortigoza (13).

In this article, we suitably combine 1) the algebraic identification method for fast on-line computation of unknown linear system parameters with 2) the GPI output feedback controller scheme. The combination of techniques is proposed for the solution of a reference trajectory tracking task in a completely unknown, load perturbed, DC motor system. The dynamic GPI controller, in classical compensation network form, is regarded as a certainty equivalence controller and adaptation is carried out on the nonlinear algebraic expressions capturing the dependence of the controller's transfer function parameters on the unknown system parameters. Adaptation is also carried out, on line, on the feed-forward expression of the nominal control input, written in terms of the given desired angular velocity reference trajectory.

Section II deals with the main assumptions about the uncertain, load perturbed, DC motor. The output feedback regulation scheme, of the GPI type, is developed in Section III. The GPI controller is shown to adopt a classical compensation network form which may be classed as a generalization of the traditional lead-lag compensator. In the context of a certainty equivalence feedback controller, the proposed GPI controller is shown to be robust with respect to constant load perturbations including unidirectional Coulomb friction terms. In section III, we also develop in detail the fast algebraic identification scheme for the unknown parameters of the motor system. The identification is shown to be completely independent of the initial conditions and of the constant load perturbations. Section IV is devoted to an experimental validation depicting the performance of the proposed fast adaptive output feedback control scheme for the load perturbed DC motor in a rest-to-rest angular velocity reference trajectory tracking task. The appendix contains a proof of the proposition made in section III.

2 The D.C. motor model

We consider the classical direct current (DC) motor model (See Levine (10))

$$\begin{aligned} L\tfrac{d}{dt}i(t) &= Eu(t) - Ri(t) - k_e\omega(t) \\ J\tfrac{d}{dt}\omega(t) &= -B\omega(t) + k_m i(t) - \tau(t) \\ y &= \omega(t) \end{aligned} \tag{1}$$

$y =$where *i(t)* denotes the armature circuit current and y=ω(t) represents the angular velocity of the motor shaft. *L* is the armature circuit inductance while *J* is the motor inertia. The voltage *E* is assumed to be fixed. *E* actually represents the maximum available voltage, in absolute value, which excites the machine. The signal *u* is an input voltage modulation signal, acting as the ultimate control input, with values restricted to the closed set [-1,1] of the real line. The parameter *ke* is the back electromotive force parameter and km is addressed as the motor gain. *R* is the armature circuit resistance and *B* is the rotational viscous friction parameter. The load torque perturbation, denoted by τ, is assumed to be constant but, otherwise, unknown. This load term will include Coulomb friction terms for unidirectional motions. None of the motor parameters: *L, J, E, R, ke, B* and *km* are assumed to be known. The only available measurement on the system variables is that of the angular velocity here denoted by *y*. The input *u* is naturally considered to be available.

In the absence a of load torque perturbation input *τ(t)*, the transfer function relating the control input *u* to the angular velocity *y* is readily found to be given by,

$$y(s) = \left[\frac{\gamma}{s^2 + \gamma_1 s + \gamma_0}\right] u(s) \tag{2}$$

where,

$$\gamma = \frac{k_m E}{JL}, \quad \gamma_1 = \frac{B}{J} + \frac{R}{L}, \gamma_0 = \frac{k_m k_e + RB}{JL} \tag{3}$$

Note that knowledge of the parameter set, Θ={γ,γ1,γ0}, does not allow to uniquely determine the unknown motor parameters *L, J, E, R,* k_e , *B* and k_m. Hence, the set of motor parameters is *not linearly identifiable*. Nevertheless, the relevant set of parameters, Θ, is indeed *linearly identifiable* (See (4)). It is not difficult to realize that the set of parameters, Θ, is all that is needed to actually control the system and solve a given angular velocity reference trajectory tracking task.

3 Problem formulation and main result

Definition

A feasible angular velocity reference trajectory, y*(t), for the motor model (2) is one for which a corresponding control input u*(t) exists, such that u*(t) ε [-1,1] forall *t*, and for which the open loop controlled, unperturbed, response of the system *y(t)* perfectly coincides with y*(t), forall *t*, provided the initial value, y(0), of *y* and a finite number of its time derivatives, $y^{(i)}(0)$, i=1,2,..., at time t=0, coincide, respectively, with the corresponding values of y*(0) and with the same finite number of time derivatives, $[y^*(0)]^{(i)}$, i=1,2,... of the given reference trajectory y*(t).

Problem Formulation

Given a feasible angular velocity reference trajectory, y*(t) for the completely uncertain motor model (2), devise an output feedback controller which processes the angular velocity measurements y(t) (possibly subject to additive, zero mean, noise), the desired reference trajectory, y*(t), and the nominal control input u*(t), which guarantees the global asymptotic convergence of y(t) to y*(t) in spite of the constant, unknown, value taken by the perturbation load torque input τ (t).

3.1 A certainty equivalence GPI controller in classical form

Assume momentarily, in the spirit of classical adaptive feedback control (see Aström and Wittenmark (1)), that *all* system parameters are perfectly known and no perturbation inputs are present. Under these circumstances, consider the following (certainty equivalence) GPI output feedback controller, written in classical compensation network form:

$$u = u^*(t) - \frac{1}{\gamma}\left[\frac{k_2 s^2 + k_1 s + k_0}{s(s+k_3)}\right](y - y^*(t)) \tag{4}$$

where we have abusively, as it is also customarily done in the adaptive control literature, combined frequency domain and time domain quantities in the same equation.

We denote by eu the input tracking error e_u=u-u*(t) and by e_y the output tracking error e_y=y-y*(t). The output tracking error is evidently related to the input tracking error by means of the transfer function

$$e_y(s) = \left[\frac{\gamma}{s^2 + \gamma_1 s + \gamma_0}\right] e_u(s) \tag{5}$$

[1]The basic problem to be solved here entitles a controlled equilibrium to equilibrium angular velocity transfer. Hence, the typical Coulomb friction term: k_C sign ω, may be assumed to be an unknown constant, of absolute value given by k_C.

The output error feedback controller is then expressed as:

$$e_u(s) = -\frac{1}{\gamma}\left[\frac{k_2 s^2 + k_1 s + k_0}{s(s+k_3)}\right] e_y(s) \tag{6}$$

Substituting the feedback controller expression (6) into the tracking error dynamics (5) leads to

$$\left\{1 + \left[\frac{1}{s^2 + \gamma_1 s + \gamma_0}\right]\left[\frac{k_2 s^2 + k_1 s + k_0}{s(s+k_3)}\right]\right\} e_y(s) = 0 \tag{7}$$

The closed loop characteristic polynomial p(s) of the tracking error dynamics is readily found to be:

$$p(s) = s^4 + (k_3 + \gamma_1)s^3 + (k_2 + k_3\gamma_1 + \gamma_0)s^2 + (k_3\gamma_0 + k_1)s + k_0 \tag{8}$$

It follows that under precise knowledge of the set of system parameters, Θ, the gains, k_3, k_2, k_1, k_0, of the controller can be suitably adjusted so that the poles of the closed loop system coincide with the roots of a desired closed loop characteristic polynomial $p_d(s)$

$$p_d(s) = s^4 + \alpha_3 s^3 + \alpha_2 s^2 + \alpha_1 s + \alpha_0 \tag{9}$$

which may be, in particular, proposed to be given by

$$p_d(s) = (s^2 + 2\zeta\omega_n s + \omega_n^2)^2 = s^4 + 4\zeta\omega_n s^3 + (2\omega_n^2 + 4\zeta^2\omega_n^2)s^2 + \ 4\zeta\omega_n^3 s + \omega_n^4 \tag{10}$$

where ζ and ω_n are strictly positive constants appropriately chosen to obtain a desirable closed loop behavior. For this, one simply sets:

$$\begin{gathered} k_3 = 4\zeta\omega_n - \gamma_1 \\ k_2 = 2\omega_n^2 + 4\zeta^2\omega_n^2 - (4\zeta\omega_n - \gamma_1)\gamma_1 - \gamma_0 \\ k_1 = 4\omega_n^3\zeta - (4\zeta\omega_n - \gamma_1)\gamma_0 \\ k_0 = \omega_n^4 \end{gathered} \tag{11}$$

Regarding the feed-forward signal u*(t), appearing in the controller (4) and implicitly present in (6), this signal can be readily computed on the basis of the knowledge of the given reference trajectory, y*(t), and the (known) system transfer function parameters:

$$u^*(t) = \frac{1}{\gamma}\left[\ddot{y}^*(t) + \gamma_1\dot{y}^*(t) + \gamma_0 y^*(t)\right] \tag{12}$$

We have thus shown, for the unperturbed case: $\tau(t)=0$, the following result:

Proposition
Given the perturbed DC motor model (1), with all the system parameters perfectly known, and given a desired angular velocity reference trajectory y*(t), the output feedback controller (4), with parameter gains chosen according to (11) and u*(t) computed according to (12), induces the motor angular velocity response y(t) to asymptotically exponentially track the given angular velocity reference trajectory, y*(t), regardless of the constant value taken by the load torque perturbation input τ (t).

Proof (See the Appendix)

Clearly, the prescription of the appropriate controller parameters (11), as well as the synthesis of the feed-forward compensation signal u*(t) in (12), demands the precise knowledge of the parameter set Θ. A separate on-line, fast and precise, parameter identification process will be proposed to obtain, quite robustly with respect to possible zero mean noise measurement process, a parameter estimate set, denoted by $\Theta=\{\gamma_{1e},\gamma_{0e},\gamma_e\}$ which will replace the unknown parameters in a controller identical to the certainty equivalence controller (6), (11), (12), except that it is written in terms of estimated parameter values, rather than the actual parameters, as follows

$$u = u_e^*(t) - \frac{1}{\gamma_e}\left[\frac{k_2 s^2 + k_1 s + k_0}{s(s+k_3)}\right](y - y^*(t)) \tag{13}$$

$$u_e^*(t) = \frac{1}{\gamma_e}\left[\ddot{y}^*(t) + \gamma_{1e}\dot{y}^*(t) + \gamma_{0e}y^*(t)\right] \tag{14}$$

$$\begin{gathered} k_3 = 4\zeta\omega_n - \gamma_{1e} \\ k_2 = 2\omega_n^2 + 4\zeta^2\omega_n^2 - (4\zeta\omega_n - \gamma_{1e})\gamma_{1e} - \gamma_{0e} \\ k_1 = 4\omega_n^3\zeta - (4\zeta\omega_n - \gamma_{1e})\gamma_{0e} \\ k_0 = \omega_n^4 \end{gathered} \tag{15}$$

We thus propose to use the output feedback tracking controller (13)-(15) as a *certainty equivalence* output feedback controller. The elements of the unknown parameter set, Θ, needed in the controller gain specification and in the feed-forward compensation signal, will be estimated, on line, using the fast algebraic estimation method to be presented next.

4 On-line parameter estimation: The algebraic method

Consider the perturbed system dynamics with τ(t)=T for all t.

$$\ddot{y} + \left(\frac{B}{J} + \frac{R}{L}\right)\dot{y} + \left(\frac{BR}{LJ} + \frac{k_e k_m}{LJ}\right)y = \frac{k_m E}{JL}u - \frac{R}{LJ}T \tag{16}$$

which, according to the previous parameters definition, may be rewriten as follows:

$$\frac{d^2}{dt^2}y + \gamma_1\frac{d}{dt}y + \gamma_0 y = \gamma u - \frac{R}{LJ}T \tag{17}$$

Taking Laplace transforms in (17) results in:

$$s^2 y(s) - sy(0) - \dot{y}(0) + \gamma_1[sy(s) - y(0)] + \gamma_0 y(s) = \gamma u(s) - \left(\frac{R}{LJ}\right)\frac{T}{s} \tag{18}$$

Multiplying out by the factor s and taking three derivatives of the resulting expression with respect to the complex variable s, one obtains an expression which is free of the influences of the initial conditions: y(0), d/dt y(0), and of the value of the constant perturbation input T. One finds:

$$6y(s) + 18s\frac{dy(s)}{ds} + 9s^2\frac{d^2y(s)}{ds^2} + s^3\frac{d^3y(s)}{ds^3} + \gamma_1\left[6\frac{dy(s)}{ds} + 6s\frac{d^2y(s)}{ds^2} + s^2\frac{d^3y(s)}{ds^3}\right]$$
$$+\gamma_0\left[3\frac{d^2y(s)}{ds^2} + s\frac{d^3y(s)}{ds^3}\right] - \gamma\left[3\frac{d^2u(s)}{ds^2} + s\frac{d^3u(s)}{ds^3}\right] = 0 \tag{19}$$

Multiplying (19) by the factor s^{-3} gets rid of all the intrinsic derivations represented by the positive powers of the complex variable s. To further filter out, and attenuate, the effects of the zero mean noise present in y (and, hence, in u), while improving the signal-to-noise ratio in the computations to be performed, we propose to multiply (19) by s^{-5}, instead of s^{-3}. Reverting the resulting expression to the time domain, one obtains:

$$-q_1(t) + p_{11}(t)\gamma_1 + p_{12}(t)\gamma_0 + p_{13}(t)\gamma = 0$$

where

$$\begin{aligned}
q_1(t) &= -6(\int^{(5)} y) + 18(\int^{(4)} ty) - 9(\int^{(3)} t^2y) \\
p_{11}(t) &= -6(\int^{(5)} ty) + 6(\int^{(4)} t^2y) - (\int^{(3)} t^3y) \\
p_{12}(t) &= 3(\int^{(5)} t^2y) - (\int^{(4)} t^3y) \\
p_{13}(t) &= -3(\int^{(5)} t^2u) + (\int^{(4)} t^3u)
\end{aligned}$$

Using differential equations rather than iterated integrations one obtains the following linear equation for the unknown parameters in the set Θ:

$$p_{11}(t)\gamma_1 + p_{12}(t)\gamma_0 + p_{13}(t)\gamma = q_1(t) \tag{20}$$

With

$$\begin{array}{llll}
\dot{p}_{11} = p_{11a} & \dot{p}_{12} = p_{12a} \\
\dot{p}_{11a} = p_{11b} & \dot{p}_{12a} = p_{12b} \\
\dot{p}_{11b} = p_{11c} - t^3y & \dot{p}_{12b} = p_{12c} \\
\dot{p}_{11c} = p_{11d} + 6t^2y & \dot{p}_{12c} = p_{12d} - t^3y \\
\dot{p}_{11d} = -6ty & \dot{p}_{12d} = 3t^2y \\
\\
\dot{p}_{13} = p_{13a} & \dot{q}_1 = q_{1a} \\
\dot{p}_{13a} = p_{13b} & \dot{q}_{1a} = q_{1b} + t^3y \\
\dot{p}_{13b} = p_{13c} & \dot{q}_{1b} = q_{1c} - 9t^2y \\
\dot{p}_{13c} = p_{13d} + t^3u & \dot{q}_{1c} = q_{1d} + 18ty \\
\dot{p}_{13d} = -3t^2u & \dot{q}_{1d} = -6y
\end{array} \tag{20A}$$

One now generates a set of three linear independent equations in the three unknowns γ_1, γ_0, γ by repeated integration of the linear expression (20). It is not difficult to see that the thus obtained set of equations are indeed linearly independent. If they were not, this would mean that y and u satisfy a higher order time-varying differential equation which is completely independent of the hypothesized system parameters. This is a contradiction due to the assumed validity of the system model.

$$\begin{aligned} p_{11}(t)\gamma_1 + p_{12}(t)\gamma_0 + p_{13}(t)\gamma = q_1(t) \\ p_{21}(t)\gamma_1 + p_{22}(t)\gamma_0 + p_{23}(t)\gamma = q_2(t) \\ p_{31}(t)\gamma_1 + p_{32}(t)\gamma_0 + p_{33}(t)\gamma = q_3(t) \end{aligned} \tag{21}$$

i.e.

$$\begin{array}{llllllll} \dot{p}_{21} & = p_{11} & \dot{p}_{22} & = p_{12} & \dot{p}_{23} & = p_{13} & \dot{q}_2 & = q_1 \\ \dot{p}_{31} & = p_{12} & \dot{p}_{32} & = p_{22} & \dot{p}_{33} & = p_{23} & \dot{q}_3 & = q_2 \end{array} \tag{22}$$

Denote the matrix with elements $p_{ij}(t)$ in (21) by either $\{p_{ij}(t)\}$ or P(t). Hence, $P(t)\Gamma = Q(t)$ where $\Gamma = (\gamma_1,\gamma_0,\gamma)^T$ and $Q(t)=(q_1(t),q_2(t),q_3(t))^T$. Clearly, from the expressions in the time domain defining the coefficients $p_{1j}(t)$,j=1,2,3 in (20), it follows that $p_{11}(0)=p_{12}(0)=p_{13}(0)=q_1(0)=0$ and, hence, at time t=0, the system (21) is singular. A solution for the unknown parameters in Γ may then be uniquely obtained after a small interval $[0,\varepsilon]$, $\varepsilon > 0$, has elapsed. Note that by the same argument given in the previous footnote: det $\{p_{ij}(t)\}$ =0 over any open interval, however small, of the real line, constitutes a contradiction.

The differential equations (20A) represent linear time-varying unstable filters excited only by the system input and the system output. Being linear, they cannot exhibit finite escape times and, more importantly, since they accomplish their parameter identification purpose in a small time interval, $[0,\varepsilon]$, once the parameters are computed they can be safely switched off.

We adopt the following estimates for the unknown parameters

$$\Gamma_e = \begin{cases} \text{arbitrary} & \text{for } t \in [0, \epsilon] \\ \\ P^{-1}(t)Q(t) & \text{for } t > \epsilon \end{cases} \tag{23}$$

The components of the, on line, fast estimated parameter vector Γ_e, are used in the feed-forward compensated output feedback controller expressions: (13), (14) and (15).

5 Experimental Results

Experiments were carried out in order to assess the effectiveness of the proposed fast adaptive GPI controller, for angular velocity reference trajectory tracking, in a typical DC motor with parameter values obtained from the manufacturer's nominal characteristics:

R $= 5.6\Omega$, $L = 8.9mH$, $J = 15.93\mu Kg - m^2$, $E = 24V$, $k_m = k_e = 0.0603N - m/A$, $B = 15.61\mu N - m - seg/rad$.

It was desired to smoothly bring the motor angular velocity from a value of 100 [rad/s] towards a final value of 300 [rad/s] in 2 [s]. A 16th order Bézier polynomial, smoothly interpolating between the initial and final values, was prescribed as the desired angular velocity reference trajectory y*(t). The output feedback controller gains (15) were obtained using the following design values: ζ=0.8 an ω_n=400

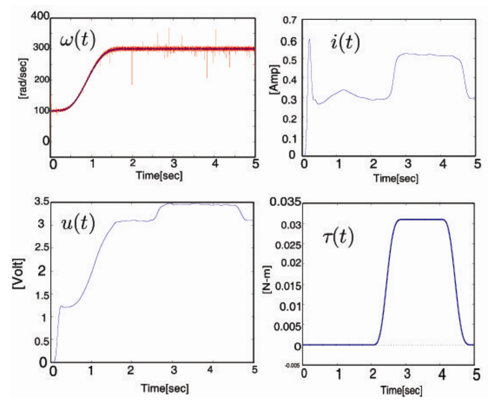

Fig. 1. Experimental response of uncertain load perturbed DC motor

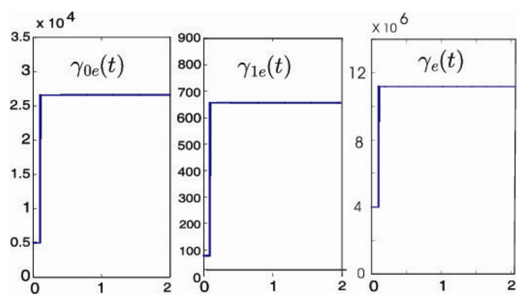

Fig. 2. On line experimental parameter estimation for uncertain motor

Figures 1 and 2 depict the performance of the fast adaptive GPI controller in achieving the desired angular velocity profile. The system was subject to a nearly constant, smoothly rising, load torque perturbation input of 0.03 [N-m] occurring at time t=2 [s] which also smoothly decreased to zero at time t=4 [s]. This load profile was obtained using a magnetic brake acting as a nearly constant load for the motor. The control system reacts to this perturbation input while maintaining the desired angular velocity at the corresponding desired constant steady state value. The figure depicts the measured angular velocity, including a significant measurement noise process. The armature circuit current, and the feedback generated control input are also shown, exhibiting the effect of the rising and decreasing load torque perturbation as well as the efforts caused by the unknown Coulomb friction term. Figure 2 depicts the fast parameter estimation process taking place in an interval of duration: ε=0.15 [s]. The estimators were switched off at time t=0.4 [s], and their last computed value was saved, as the parameter estimates, for the remaining of the computation time.

The actual values of the system transfer function parameters, obtained from the motor data, are found to be:

$$\gamma_1 = 630.5, \quad \gamma_0 = 2.63 \times 10^4, \quad \gamma = 1.02 \times 10^7$$

while the obtained values, for these parameters, were

$$\gamma_{1e} = 645, \quad \gamma_{0e} = 2.7 \times 10^4, \quad \gamma_e = 1.12 \times 10^7$$

The experimental results exhibit a reasonable proximity with the nominally computed values.

6 Conclusions

In this article, we have examined, and tested in a laboratory setup, the relevance and effectiveness of the algebraic parameter identification method in the fast adaptive output feedback control of a completely unknown DC motor which is also subject to unknown load torques. The algebraic identification method was developed in recent years for the fast on-line computation of unknown parameters in linear systems. The method enjoys immediate extensions to linearly parameterizable nonlinear systems in combination with algebraic on-line state estimation. We have shown that an angular velocity reference trajectory tracking task for a DC motor can be efficiently handled via a combination of GPI output feedback control (which evades the need for state estimation while being robust with respect to classical perturbation inputs) and the algebraic identification method for on-line obtaining the parameters of the transfer function of the DC motor. The fast adaptation of the uncertain parameters is carried out both on the classical compensating network parameters and on the conformation of the feed-forward control input signal. Since the DC motor may have up to 6 unknown parameters, it is impossible to compute them all in an on-line simultaneous fashion based only on input output measurements due to a lack of linear identifiability of all the parameters. The system is neither weakly linearly identifiable. Thus, instead of trying to identify each one of the motor parameters, an on-line identification of the parameters actually needed for the output (GPI) feedback controller is carried out. These parameters are, precisely, the unknown parameters appearing in the transfer function of the motor relating the angular velocity to the input voltage and they are nonlinear functions of the original parameters.

7 Future Works

Future works will contemplate the simultaneous identification of the uncertain transfer function parameters and time-varying (i.e., not necessarily nearly constant) load torques but widely varying loads. This problem may be approached by proposing a polynomial approximation to the unknown signal and then proceed to compute the unknown parameters. This idea must be used in combination with periodic *resettings* of the estimation algorithm so that the polynomial approximation remains valid. An interesting possibility for further work, in this totally uncertain context, is the problem of sensor-less control; in which the angular velocity of the motor shaft is not available for measurement and only the armature circuit current is measured.

Appendix

Proof of Proposition 1. Let the perturbation input, $\tau(t)=T$, be an unknown nonzero constant, possibly including an unidirectional Coulomb friction term of unknown constant amplitude. The dynamics of the tracking error, $e=y-y^*(t)$, satisfies the following perturbed controlled differential equation:

$$\ddot{e}_y + \left(\frac{B}{J} + \frac{R}{L}\right)\dot{e}_y + \left(\frac{BR}{LJ} + \frac{k_e k_m}{LJ}\right)e_y = \frac{k_m E}{JL}e_u - \frac{R}{LJ}T \tag{A1}$$

which, using the definitions in equation (3), is written as

$$\ddot{e}_y + \gamma_1\dot{e}_y + \gamma_0 e_y = \gamma e_u - \frac{R}{LJ}T \tag{A2}$$

Denote by $\hat{\dot{e}}_y$ a structural estimate of the time derivative of the tracking error, e_y, obtained by direct integration of equation (2) while *neglecting initial conditions and the effects of the unknown constant load torque term*:

$$\hat{\dot{e}}_y = -\gamma_1 e_y - \gamma_0\left(\int e_y\right) + \gamma\left(\int e_u\right) \tag{A3}$$

where $(\int \phi)$ denotes: $\int_0^t \phi(\sigma)d\sigma$. Henceforth, the term $(\int^{(j)} \phi)$ denotes an iterated integral of the form:

$$\int_0^t \int_0^{\sigma_1} \cdots \int_0^{\sigma_{j-1}} \phi(\sigma_j)d\sigma_j \cdots d\sigma_1$$

Clearly, the relation between $\dot{\hat{e}}_y$ and the actual time derivative $\dot{e}_y$ is, in this case, given by

$$\dot{e}_y = \hat{\dot{e}}_y + \dot{e}_y(0) - \gamma_1 e_y(0) - \frac{R}{LJ}Tt \tag{A4}$$

i.e., the structural estimate $\dot{\hat{e}}_y$ differs, at most, by a first order polynomial in t, with respect to the actual value $\dot{e}_y$.

Consider the feedback controller:

$$e_u = \frac{1}{\gamma}\left[(\gamma_1 - \alpha_3)\, \hat{\dot{e}}_y + (\gamma_0 - \alpha_2)\, e_y - \alpha_1(\int e_y) - \alpha_0(\int^{(2)} e_y)\right] \tag{A5}$$

where the set of parameters $\{\alpha_3,\alpha_2,\alpha_1,\alpha_0\}$are design constants chosen from the coefficients of a desired, hurwitzian, closed loop characteristic polynomial: $p_d(s)=s^4+ \alpha_3 s^3+,\alpha_2 s^2+ \alpha_1 s+ \alpha_0$. The closed loop tracking error system is found to evolve, after substituting (A.5) in (A.2}) and using relation (A.4), according to

$$\ddot{e}_y + \alpha_3\dot{e}_y + \alpha_2 e_y + \alpha_1(\int e_y) + \alpha_0(\int^{(2)} e_y) = -\frac{R}{LJ}T - (\gamma_1 - k_3)\left(\dot{e}_y(0) - \gamma_1 e_y(0) - \frac{R}{LJ}Tt\right) \tag{A6}$$

Note that the integral term of the output error compensates the constant term of the time polynomial while the double integral compensates for the linearly growing term of the first degree polynomial excitation.

The characteristic polynomial of the integro-differential equation system (A.6), excited in the right hand side by a first order time polynomial, is clearly given by:

$$p(s) = s^4 + \alpha_3 s^3 + \alpha_2 s^2 + \alpha_1 s + \alpha_0 \tag{A7}$$

In other words, the closed loop tracking error dynamics can be made globally exponentially asymptotically stable while its stability features are found to be completely independent of the value of the constant perturbation input, T.

In order to find a more compact expression for the controller (A.5) and evade the intrinsically unstable nature of the iterated integrations terms, we proceed as follows: Substitute (A.3) into (A.5) and separate the expressions in e_u from those in e_y. We obtain the following dynamic description for e_u:

$$e_u + (\alpha_3 - \gamma_1)(\int e_u) = -\frac{1}{\gamma}\left[(\alpha_2 - \gamma_1(\alpha_1 - \gamma_1) - \gamma_0)e_y + (\alpha_1 - \gamma_0(\alpha_3 - \gamma_1))(\int e_y) + \alpha_0(\int^{(2)} e_y)\right] \tag{A8}$$

Taking Laplace transforms, and solving for $e_u(s)$, we obtain

$$e_u(s) = -\frac{1}{\gamma}\left[\frac{k_2 s^2 + k_1 s + k_0}{s(s+k_3)}\right] e_y(s) \tag{A9}$$

where

$$\begin{aligned} k_3 &= \alpha_3 - \gamma_1 \\ k_2 &= (\alpha_2 - \gamma_1(\alpha_1 - \gamma_1) - \gamma_0) \\ k_1 &= (\alpha_1 - \gamma_0(\alpha_3 - \gamma_1)) \\ k_0 &= \alpha_0 \end{aligned} \tag{A10}$$

which is precisely the proposed controller (4), (6). The α coefficients, for the desired characteristic polynomial, may be borrowed from the expression (10).

References

1. **Aström, K. J., and B. Wittenmark**, *Adaptive Control*, 2nd Edition. Addison-Wesley Publishing Company, 1995.
2. **Elker, I**., ``Open loop and closed loop experimental on line identification of a three mass electromechanical system" *Mechatronics*, 14: 549-665, 2004.
3. **Fliess, M., R. Márquez, E. Delaleau and H. Sira-Ramírez**, `` Correcteurs Proportionnels-Intègraux Généralisés", *ESAIM: Control, Optimization and Calculus of Variations*, 7(2):23-41, 2002.
4. **Fliess, M., and H. Sira-Ramírez** ``An algebraic framework for linear identification", *ESAIM, Control, Optimization and Calculus of Variations*, 9:151-168, 2003.
5. **Fliess, M., and H. Sira-Ramírez** ``Closed-loop parametric identification for continuous time linear systems" in *Continuous Time Model Identification from Sampled Data*, H. Granier & L. Wang (Eds.), Springer, Berlin, 2007.
6. **Forssell, U.. and L. Ljung**, ``Closed-loop identification revisited". *Automatica*, 35:1215–1241, 1999.
7. **Jang, J.O., and G.J. Jeon**, ``A parallel neuro-controller for DC motors containing nonlinear friction". *Neurocomputing*, 30:233–248, 2000.
8. **Kara, T., and Y. Elker,** ``Nonlinear modeling and identification of a DC motor for bidirectional operation with real time experiments", *Energy Conversion and Management*, 45(7-8): 1087-1106, 2004.
9. **Lakshminarayanan, S., G. Emoto, S. Ebara, K. Tomida and S.L. Shah**, ``Closed-loop identification and control loop reconfiguration: an industrial case study". *J. Process Control*, 11:587–599, 2001.
10. **Levine, W.S**., *Control System Applications*, CRC Press, Boca Raton, 2000.
11. **Sira-Ramírez, H**., ``Sliding Modes, Delta-modulators, and Generalized Proportional Integral Control of Linear Systems" *Asian Journal of Control*, 5(4):467-475, 2003.
12. **Sira-Ramírez, H., R. Márquez and M. Fliess**, ``Sliding Mode Control of DC-to-DC Power Converters using Integral Reconstructors" *International Journal of Robust and Nonlinear Control*, 12(13):1173-1186, 2002
13. **Sira-Ramírez, H., and R. Silva-Ortigoza**, *Control Design Techniques in Power Electronics* Devices, Springer-Verlag, Power Systems Series, London, 2006.

ISSN 1405-5546

H. Sira Ramírez *es Investigador Titular adscrito a la Sección de Mecatrónica del Departamento de Ingeniería Eléctrica del Centro de Investigación y de Estudios Avanzados del Instituto Politécnico Nacional, México D.F., México.*

E. Barrios Cruz *es Maestro en Ciencias, egresado del Departamento de Ingeniería Eléctrica del Centro de Investigación y de Estudios Avanzados del Instituto Politécnico Nacional, México D.F., México.*

R. Márquez *es Profesor del Departamento de Sistemas de Control de la Escuela de Ingeniería de Sistemas de la Facultad de Ingeniería de la Universidad de Los Andes, Mérida, Venezuela.*