Golikov, Víctor; Lebedeva, Olga; May Alarcón, Manuel; Méndez Martínez, Francisco; Rodríguez
Blanco, Marco; Islas Chuc, Mayolo Salvador

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Víctor Golikov, Olga Lebedeva, Manuel May Alarcón, Francisco Méndez Martínez,
Marco Rodríguez Blanco, and Mayolo Salvador Islas Chuc

Facultad de Ingeniería, Universidad Autónoma del Carmen, Ciudad del Carmen, Campeche,
Mexico

{vgolikov, olebedeva, mmay, fmendez}@pampano.unacar.mx

Abstract. In this paper, matched detector (MD) and matched subspace detector (MSD) are studied when the structured background power is different under the null and the alternative hypotheses. The distributions of two test statistics are derived under these conditions. It has been analytically shown that these detectors can suffer a drastic degradation in performance for background power deviations under alternative hypothesis. We discuss the differences between the performances of these detectors in the case of the structured and unstructured backgrounds with uncorrelated Gaussian noise. The theoretical results are compared with simulated data and good agreement is reported. We present experimental results of small floating object detection on an agitated sea surface using spectral digital video experiments which validate the theoretical results.

Keywords. Hypothesis dependent power, subpixel targets, performance loss.

Sobre la eficiencia de detección de objetos subpixeleados con potencia de fondo estructurado que depende de hipótesis

Resumen. El detector acoplado (MD) y el detector de subespacio acoplado (MSD) son estudiados cuando la potencia de fondo estructurado es diferente bajo las hipótesis alternativa y nula. Las distribuciones de las dos pruebas estadísticas son realizadas bajo las mismas condiciones. Ha sido analíticamente demostrado que esos dos detectores pueden sufrir una degradación drástica de su eficiencia para las desviaciones de la potencia de fondo bajo hipótesis alternativas. Se discuten las diferencias entre los rendimientos de esos detectores en el caso de fondos estructurados y no estructurados con ruido Gaussiano no correlacionado. Los resultados teóricos son comparados con los datos simulados y una buena concordancia es reportada. Se presentan resultados experimentales de la detección de objetos pequeños flotando en la superficie agitada del mar, usando el experimento del video digital espectral, que demuestra la validación de los resultados teóricos.

Palabras clave. Potencia que depende de hipótesis, objetos subpixeleados, pérdida de eficiencia.

1 Introduction

Detection problems arise in a wide variety of applications such as sonar, radar, data communication, medical and optical remote systems [1-9]. In optical remote systems, it is typical that the background has the same covariance structure but different variances under hypotheses H_0 and H_1 [6], which is directly related to the fill factor of a target, that is, the percentage of the pixel area unoccupied by an object.

There are two background models frequently used in detectors: unstructured background model, which represents the background as a time stochastic correlated process, and the structured background model, which represents the background as a spectral deterministic function of frequency. The detector performance analysis has been considered [8] for the case of unstructured background models where the purpose is to perform analytical and numerical analysis of two well-known detectors (MD and MSD) which would at least partially explain the behavior of these detectors when the structured background power is different under the null and the alternative hypotheses. It is well known [1, 6, 9], that these detectors are designed for a specific signal waveform and a given noise probability

density function. A common drawback of the works referred to previously is the assumption that the noise power relation under hypothesis H_0 and under hypothesis H_1 remains fixed [2-4,8].

Unfortunately, it turns out that in many cases the optimum detectors can suffer a drastic degradation in the performance for small deviations from the nominal assumptions [3-6]. It has been demonstrated [8] that the MSD performance loss for high-dimensional target subspace $p>10$ achieves a great value in presence of the Gaussian unstructured background with different variance for hypotheses H_0 and H_1, but MD with the target subspace $p=1$ is robust in these conditions.

In the literature [4, 6, 7], the study of these classical detectors is not complete when the structured background power is different under the null and alternative hypotheses. In optical systems (including the hyperspectral subpixel target detection), for example, it is typical that the background has the same spectrum structure but different variances under hypotheses H_0 and H_1 [6], which is directly related to the pixel fill factor b, that is, the percentage of the pixel area unoccupied by an object. The choice of the mathematical model used to describe the variability of target and background spectra (subspace versus statistical) leads to different families of detection algorithms.

The variability of the background can be described using either a subspace model (structured background) or a statistical distribution (unstructured background). In this paper, we investigate theoretically the detection performance losses in the case of background power variations between two hypotheses in the structured background environment for the canonical MD and MSD.

Then we compare numerically the performance losses for MD and MSD in the case of the structured and the unstructured background models. We present simulation and experimental results of detection of two floating objects on an agitated sea surface using spectral digital video experiments.

2 Problem Statement and System Analysis

The target detection problem is often defined in the literature [1, 4, 6] as the hypothesis-testing problem for testing the null hypothesis H_0: the target response scaling $\mu = 0$ versus $\mu > 0$ for the alternative hypothesis H_1. We consider a subpixel model [1, 8], where each pixel is a sum of two vectors, one from the target spectrum subspace and the other from the background spectrum subspace. For targets occupying multiple pixels, detection can exploit both spatial and spectral properties. In contrast, the detection of subpixel targets can be achieved only by exploiting spectral properties. When the background variability is modeled using a subspace model, the target detection problem involves choosing between the following competing hypotheses:

$$H_0: \boldsymbol{x} = \alpha \boldsymbol{B}\boldsymbol{a}_b + \boldsymbol{n}, \quad H_1: \boldsymbol{x} = \mu \boldsymbol{S}\boldsymbol{a}_t + \alpha b \boldsymbol{B}\boldsymbol{a}_b + \boldsymbol{n} \qquad (1)$$

where $\boldsymbol{x}$ is the pixel spectrum under test, $\boldsymbol{B}$ is a $N \times Q$ matrix representing the background spectrum subspace, $\boldsymbol{a}_b$ and $\boldsymbol{a}_t$ are the normalized ($\|\boldsymbol{S}\boldsymbol{a}_t\| = 1$ and $\|\boldsymbol{B}\boldsymbol{a}_b\| = 1$) unknown abundance vectors of the target spectrum and background spectrum, respectively, α is the common background spectrum scale factor, b is the pixel fill factor (the percentage of the pixel area occupied by a background), $\boldsymbol{S}$ is a $N \times p$ matrix representing the target subspace, and $\boldsymbol{n}$ is a N-dimensional error vector accounting for lack-of-fit and noise effects typically assumed to be a zero-mean N-dimensional normal distribution $N(\boldsymbol{0}, \sigma^2 \boldsymbol{I}_N)$, $N > Q > p$.

The orthogonal subspace projection (OSP) detector seems to be a reasonable detector for this linear mixing model containing the target and background spectrum signatures [1, 4]. On the other hand, in practice, MD or MSD may sometimes outperform the OSP, as demonstrated in [7], and, additionally, practitioners often use these detectors for their simplicity. The well-known MD and MSD test statistics [7] are

$$T_{\text{MD}} = \boldsymbol{s}^T \boldsymbol{x} / \sqrt{\sigma^2 \boldsymbol{s}^T \boldsymbol{s}} \text{ and } T_{\text{MSD}} = \boldsymbol{x}^T \boldsymbol{P}_S \boldsymbol{x} / \sigma^2 \qquad (2)$$

where the target response is $s=Sa_t$ and $P_S=S(S^TS)^{-1}S^T$ is the orthogonal p-rank projection matrix on the signal spectrum subspace. To make a decision, we need to compare test statistic $T(x)$ to a given threshold η and accept H_1 when $T>\eta$ and H_0 otherwise. Since the threshold determines both detection probability P_D and false alarm probability P_F, we need to determine the probability distribution of $T(x)$. It can be shown [1, 7], that the MD test statistics are

$$T_{MD}\sim^{H_0} N(0,1) \text{ and } T_{MD}\sim^{H_1} N(m,1) \quad (3)$$

with noncentrality parameter

$$m = \frac{\mu d}{\sigma}+(b-1)\frac{\alpha s^TBa_b}{\sigma d} = \frac{\mu}{\sigma}+(b-1)Kr$$
$$d=(s^Ts)^{1/2}=1, \quad (4)$$

where $K=s^TBa_b$ is the target-background cross-correlation factor, and $r=a/\sigma$ is the background-to-noise ratio. In the case of the MSD test statistics it is known [1, 7] that the random variable P_Sx is distributed as

$$P_Sx\sim^{H_0}N(P_s\alpha Ba_b,P_s), \quad (5)$$

$$P_Sx\sim^{H_1} N(\mu P_s Sa_t+P_s\alpha bBa_b,P_s). \quad (6)$$

Therefore, the MSD test statistics are chi-squared distributed:

$$T_{MSD}\sim^{H_0}\chi_p^2(\lambda_0^2)\text{ and } T_{MSD}\sim^{H_1}\chi_p^2(\lambda_1^2) \quad (7)$$

with noncentrality parameters

$$\lambda_0^2=\frac{\alpha^2a_b^TB^TP_sBa_b}{\sigma^2} = r^2K_1^2, \quad (8)$$

$$\lambda_1^2=\frac{(\mu Sa_t+abBa_b)^TP_s(\mu Sa_t+abBa_b)}{\sigma^2}$$
$$=\frac{\mu^2}{\sigma^2} + b^2r^2K_1^2 + \frac{2\mu}{\sigma}brK \quad (9)$$

where K_1 is the value of the background projection onto signal spectrum subspace and $K_1=K$ for $a_t^T=[1,1,…,1]$. Formulas 7, 8, 9 show that the MSD efficiency depends on the signal amplitude μ and the factor $\gamma=brK$. When the

signal spectrum subspace does not coincide with the background spectrum subspace, we obtain $K_1=K=0$ and $\gamma=0$. In this case $\lambda_0^2 = 0$ and the MSD efficiency achieves the maximum and even for a small signal amplitude μ one can see that $\lambda_1^2 > \lambda_0^2$. In the case of $r^2K_1^2 > 1$, the pixel factor b reduction diminishes the noncentrality parameter λ_1^2 (9) and then the detection performance reduces. In this case it can be seen that for the small signal amplitude μ, the value of λ_1^2 can be smaller than λ_0^2. It is necessary to increase the signal amplitude μ to achieve a fixed value of the detection probability (or to achieve the relationship $\lambda_1^2 = \lambda_0^2$). This additional magnification of the signal amplitude, which is necessary in the case of $b<1$, causes the detection performance loss. It is interesting that in the case of subpixel targets, MSD efficiency depends not only on a relationship between target S and background B spectrum subspaces (see K), signal-to-noise μ^2/σ^2 and background-to-noise r ratios, but also on pixel fill factor b, i.e., the relationship between pixel and target areas and their relative positions.

Unstructured background models assume that the additive noise has been included in the background, which in turn is modeled by a multivariate normal distribution. It has been demonstrated in [8] that in the case of the unstructured background, MSD statistics are given by

$$T_{MSDU}(x)=(1/\sigma^2)x^HR^{-1}S(S^HR^{-1}S)^{-1}S^HR^{-1}x$$
$$\sim \begin{cases} \chi_p^2(0) & \text{under } H_0 \\ b^2\chi_p^2(\lambda_s^2) & \text{under } H_1 \end{cases} \quad (10)$$

where R is the background covariance matrix and $\lambda_S^2=\frac{(\mu Sa_t)^HR^{-1}(\mu Sa_t)}{b^2\sigma^2}$. Based on Expression 10, the detection probability depends on three factors: the pixel fill factor b, the degree of freedom p, and noncentrality parameter λ_S^2. When the pixel fill factor b decreases, this leads to reduction of the detection probability due to the coefficient b^2 in (10). In contrast, when b decreases, the noncentrality parameter λ_S^2 increases, and that leads to improvement of detectability. In general, as a result of the action of two opposite factors,

the detection performance increases not strongly [8].

In the next section we carry out a numerical evaluation of the MD and MSD performance losses in the case of the structured background model with different parameters K and b.

3 Detection Performance Analysis and Numerical Illustrations

The aim of this section is threefold. First, we assess the validity of the theoretical formulas 7, 8, 9 by comparing them with the actual receiver operating characteristics (ROC) obtained through Monte Carlo simulations. Second, we assess the performance loss of the classical MD and MSD in the case of the background power variation under hypothesis H_1. Third, we compare the MSD performance in the presence of the structured background model with respect to the unstructured background model under the pixel fill factor variation.

We set the desired value of the false alarm probability $P_F=10^{-3}$ and determine the corresponding threshold value by simulation.

We generate standard white Gaussian noise data, namely, independent, identically-distributed, Gaussian vectors (of size N) with zero mean and identity variance. In the estimation of probability by counting techniques, the number of independent trials that guarantees a relative r.m.s. error less than 10% is 100/P_F; thus, we generate a vector of 10^5 value of the statistic for each detector from which the threshold is estimated as the 100th value in a descending order, i.e., as the 1-P_F sample quantile. Finally, following the same considerations, the detection probability P_D is estimated using the evaluated threshold on 10^5 independent trials.

The ROC of MD and MSD, namely, curves of their probability of detection versus the signal-to-noise ratio, are plotted in Fig. 1 for MD and in Fig. 2 for MSD. Both analytical and simulation results (simulations marked by symbols) are shown for the same set of parameters of the structured background model.

It can be seen in Fig. 1, 2 that the analytical expressions 7, 8, 9 give very precise approximations of the real test performances. The MD performance is considerably better than the MSD performance, but MD requires an actual target signature which is rarely available in practice. We assess the MD and MSD performance losses in the presence of the structured background to unknown pixel fill factor $b<1$ (background power deviation under H_1) and compare them with the losses of these detectors in the case of the unstructured background [8]. The performance loss is defined as the incremental signal-to-background ratio (SBR) necessary for achieving the same detection performance (P_D=0.5). The MD performance loss (3) is a function of the noncentrality parameter m only and depends on the value of $(b-1)rK$ (4). It is evident that this term represents the degradation in m for $b<1$. It should not seem surprising that this loss depends on the voltage background-to-noise ratio r. It is interesting to notice that the loss depends on the cross-correlation factor K and in the case of $K\rightarrow0$ (the target spectrum does not coincide with the background spectrum) the loss decreases to zero. Next, we analyze the MSD performance loss. Fig. 3 shows that MSD is less sensitive to the factor b in the case of the unstructured background (see the MSDU line).
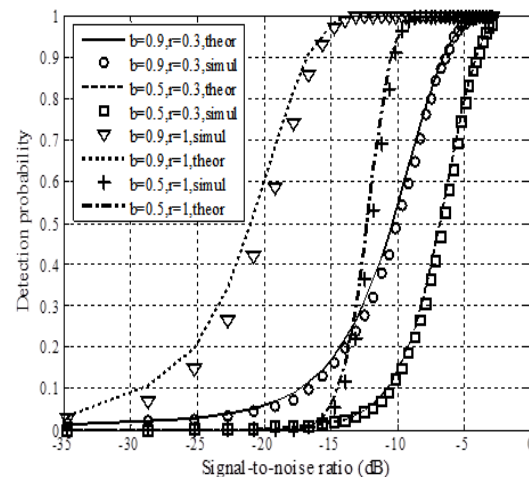

Fig. 1. Detection probability vs. signal-to-background ratio for MD and structured background model, $N = 60$, P_F=10-3
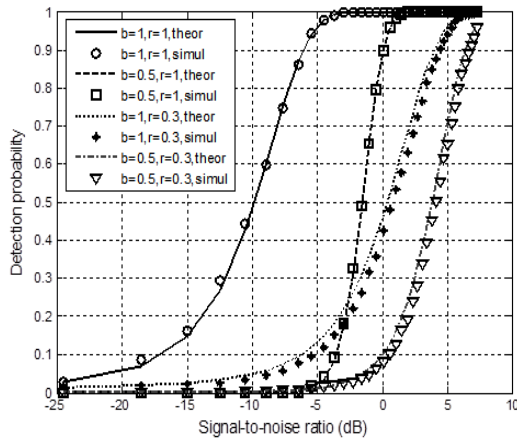
Fig. 2. Detection probability vs. signal-to-background ratio for MSD and structured background model, $N = 60$, $P_F = 0\text{-}3$
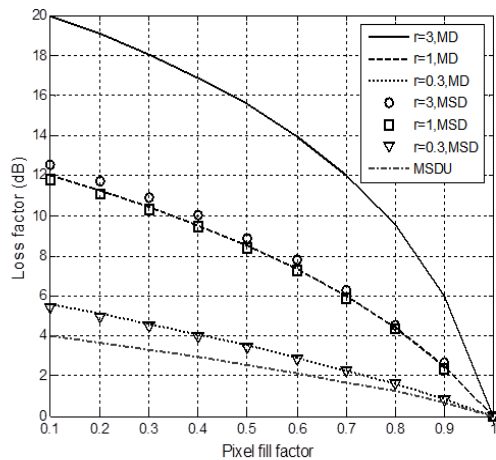

Fig. 3. Detection loss vs. pixel fill factor for MD and MSD in the case of structured and unstructured background models, $N = 60$, $P_F = 10^{-3}$

A typical MSD performance loss for signals in a correlated background, where the unstructured background is specified in terms of its covariance matrix **R**, was shown in [8], (Fig. 1, 2, 4), and the maximum loss is about 6 dB for $b=0.1$.

It can be seen in Fig. 3 that the MSD performance loss is essentially bigger in the case of the structured background model and can achieve 20 dB for small white noise power (or for great r). There are two main causes of the performance loss difference between two

models:1) the unstructured background model is stochastic and therefore MSD carries out the incoherent weakening of the background power, 2) the structured background model is deterministic and MSD can implement the background coherent reduction.

4 Experimental Example

In practice, there are situations where we do not have sufficient information to estimate the target abundance vectors a_t and therefore we cannot use MD. In this section, we illustrate the MSD losses for different pixel fill factor values b and target-background cross-correlation factor K using the experimental video sequences of floating subpixel targets on agitated sea surface. These sequences are about 200000 frames long, taken in coastal sea waters using stationary digital video camera.

We have selected two pixels at the distances of about 200 m and 1000 m. The digital video camera is placed at about 2m above the sea surface leading to a very small observation angle of 1°. Therefore, the pixel area on the sea surface is approximately an ellipse with different great and small axes. Concerning the targets, we have chosen a red boat of a size of 30×7 m (a distance of 1000 m) and a swimmer (a distance of about 100 m).

In Fig. 4 one can see the first image of the experimental image sequence where the swimmer, the boat and the sea surface with sea wave altitudes of less than 0.2 m are presented. The pixel sizes depend on the distance and for the experimental digital camera they were 0.01m²/pixel (an ellipse with axes 0.01×0.8 m) for the 100 m distance and approximately 1m²/pixel (0.1×8 m) for the 1000 m distance. Furthermore, the performance evaluation of detection algorithms in practice is challenging due to the limitations imposed by the limited amount of target data. As a result, the establishment of accurate detection probability curves is quite difficult.

Therefore we divided our research into two stages. At the first stage, we experimentally

evaluated the target signatures at a small distance (30-40 m) and the sea signatures at distances of 100 and 1000 m.
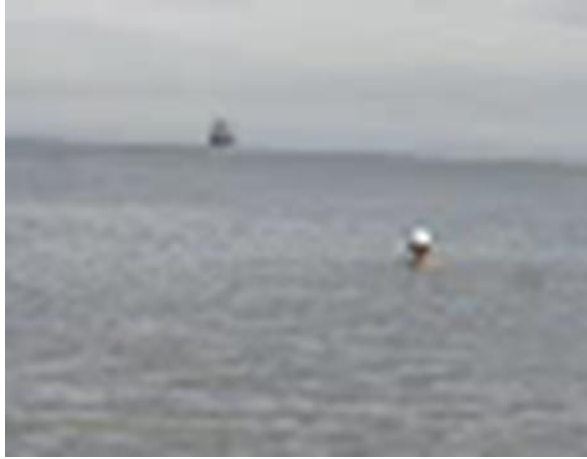

Fig. 4. The first image of the image sequence used in the experiment

For the experiment to be pure, all measurements were conducted in an equal weather situation and altitude of waves. Using the relation between the spectrum bands of the target signature and the sea signatures at distances of 100 m and 1000 m, we estimated the factor K. We assumed that the target reflections from large distance are equal to the obtained target signature plus a white normal noise. We have mixed additively these target reflections with the experimental data of the sea reflections at the distances of 100 m and 1000 m. Such artificial method allowed us to vary the pixel fill factor b and to calculate the MSD detection performances (receiver operating characteristics (ROC) depending on the pixel fill factor for both targets with different factor K (Fig. 5). One can observe in Fig. 5 the MSD performance losses in the case of different factors K and b. The MSD performance losses increase when the factor K gets bigger and the factor b gets smaller.

At the second stage, we experimentally evaluated the MSD performance of the swimmer on the sea surface (a distance of about 100 m) and the boat (a distance of about 1000 m). The pixel fill factor b is approximately equal to 0.4 (the target covers about 60% of pixel area). When the pixel area covering the target is enlarged, the
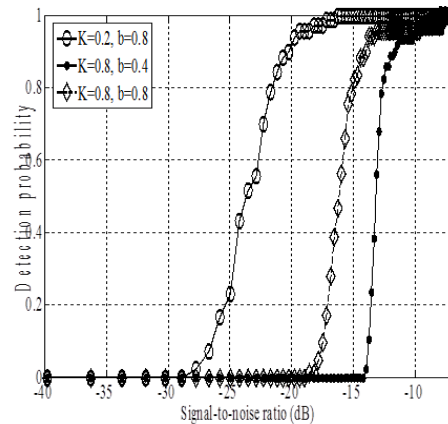
Fig. 5. Comparative experimental results: detection probability vs. signal-to-noise ratio for MSD, $p = 10$, $N = 200$, $P_F = 10^{-2}$. The boat has $K = 0.2$ and the swimmer has $K = 0.8$
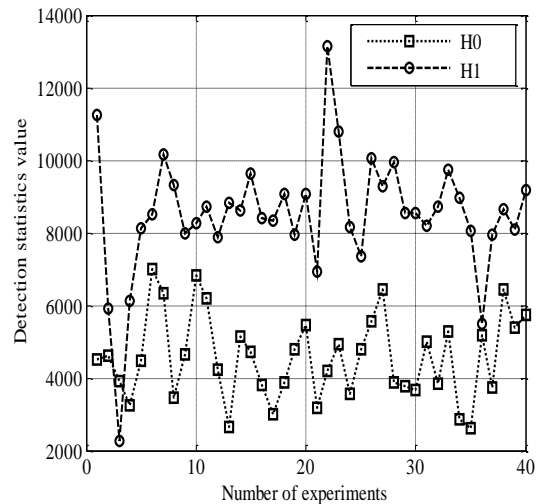

Fig. 6. Experimental results: MSD statistics in the case of the swimmer (N=200, pixel fill factor b=0.2)

target amplitude increases. To eliminate this effect, we normalized the maximum spectral line of the signal reflected from the targets.

Fig. 6 shows the statistical test (MSD) values for 40 experiments with N=200. We chose various pixels so that the pixel fill factor b had values of 0.2 (Fig. 6) and 0.8 (Fig. 7).The comparison of Figures 6 and 7 shows that a pixel fill factor
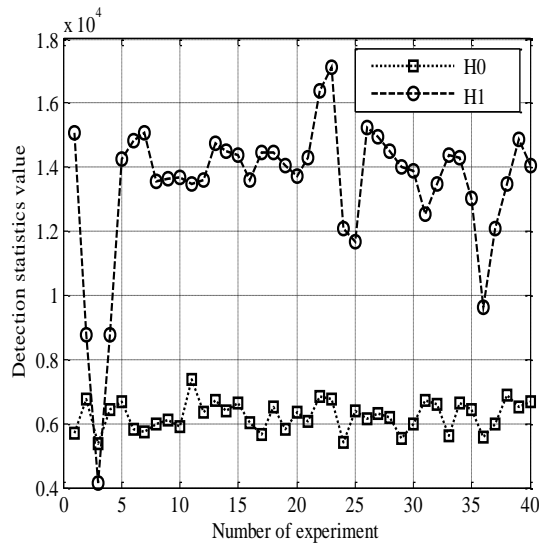
Fig. 7. Experimental results: MSD statistics in the case of the swimmer (N=200, pixel fill factor b=0.8)

decrease at the constant target amplitude is the cause of MSD performance loss. These experimental results demonstrate that both the target-background cross-correlation factor increasing and the pixel fill factor decreasing can result in MSD performance losses.

5 Conclusions

In the case of structured background power deviation under H_1, the distributions and the ROC for MD and MSD were derived analytically and validated via simulations.

The theory and simulation demonstrated that MD and MSD suffer a drastic degradation in performance for the structured background power deviations under the alternative hypothesis.

The simulation and experimental results showed that target-background cross-correlation factor and pixel fill factor play a significant role in MSD performance. Future research will focus on a more effective technique to estimate the pixel fill factor to be used in the detection algorithm.

Acknowledgements

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Victor Golikov (Ph.D.'1976, Dr. Sc.'1992, Full Professor'1998) works as Professor at University Autonomous of Carmen (Ciudad del Carmen, Camp., Mexico). His main research interests are statistical signal/image/ videoprocessing and remote sensing. He is the author of more than 100 international scientific and conference papers, and also 17 patents of the former USSR.

Olga Lebedeva (M.S.'1974) works as Professor at University Autonomous of Carmen (Ciudad del Carmen, Camp., Mexico). Her main research interests are statistical signal/image/video processing and remote sensing. She is the author of more than 45 international scientific and conference papers.

Manuel May Alarcon (Dr. Sc' 2003) works as Professor at University Autonomous of Carmen (Ciudad del Carmen, Camp., Mexico). His main research interests are optoelectronic and fiber optic sensors. He is the author of five international scientific and conference papers.

Francisco Méndez Martínez (Dr. Sc' 2006) works as Professor at University Autonomous of Carmen (Ciudad del Carmen, Camp., Mexico). His main research interests are optoelectronic, fiber application of *Sagnac* interferometer and noise reduction. He is the author of five international scientific and conference papers.

Marco Antonio Rodríguez Blanco (Dr. 2009) works as Professor at University Autonomous of Carmen (Ciudad del Carmen, Camp., Mexico). His main research interest is electronic semi-conductor devices. He is the author of more than 15 international scientific and conference papers.

Mayolo Salvador Islas Chuc (M.C. 2011) works as Professor at University Autonomous of Carmen (Ciudad del Carmen, Camp., Mexico). His main research interests are statistical signal/image/video processing and remote sensing. He is the author of two international scientific and conference papers.