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Dynamic Random Fuzzy Cognitive Maps  
*Mapas Cognitivos Difusos Aleatorios Dinámicos*

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**Abstract**

A fuzzy cognitive map is a graphical means of representing arbitrary complex models of interrelations between concepts. The purpose of this paper is to describe a dynamic/adaptive fuzzy cognitive map based on the random neural network model. Previously, we have developed a random fuzzy cognitive map and illustrated its application in the modeling of processes. The adaptive fuzzy cognitive map changes its fuzzy causal web as causal patterns change and as experts update their causal knowledge. Our model carries out inferences via numerical calculation instead of symbolic deduction. We show how the adaptive/dynamic random fuzzy cognitive map can reveal implications of models composed of dynamic processes.  

**Keywords:** Random Neural Network, Fuzzy Logic, Fuzzy Cognitive Maps, Dynamic Systems.

**Resumen**

Un Mapa Cognitivo Difuso es un medio gráfico de representación de modelos complejos de interrelaciones entre conceptos. El propósito de este artículo es describir un Mapa Cognitivo Difuso Dinámico/Adaptivo basado en el Modelo de Redes Neuronales Aleatorias. En trabajos previos, nosotros hemos desarrollado un Mapa Cognitivo Difuso Aleatorio y mostrado su aplicación en el modelado de procesos. Nuestro modelo realiza inferencias a través de cálculos numéricos en vez de deducciones simbólicas. Ahora bien, el Mapa Cognitivo Difuso Adaptivo cambia su red de relaciones causales difusas como un patrón causal cambia y un experto actualiza su conocimiento causal. Nosotros mostramos cómo el Mapa Cognitivo Difuso Dinámico/Adaptivo puede ser usado para describir implicaciones en el modelado de procesos dinámicos.  

**Palabras Clave:** Redes Neuronales Aleatorias, Lógica Difusa, Mapas Cognitivos Difusos, Sistemas Dinámicos.

1. **Introduction**

Modeling a dynamic system can be hard in a computational sense. Many quantitative techniques exist. Well-understood systems may be amenable to any of the mathematical programming techniques of operations research. Insight into less well-defined systems may be found from the statistically based methods of data mining. These approaches offer the advantage of quantified results but suffer from two drawbacks. First, developing the model typically requires a great deal of effort and specialized knowledge outside the domain of interest. Secondly, systems involving significant feedback may be nonlinear, in which case a quantitative model may not be possible. What we seek is a simple method that domain experts can use without assistance in a “first guess” approach to a problem. A qualitative approach is sufficient for this. The gross behavior of a system can be observed quickly and without the services of an operations research expert. If the results of this preliminary model are promising, the time and effort to pursue a quantitative model can be justified. Fuzzy cognitive maps are the qualitative approach we shall take.

Fuzzy Cognitive Maps (FCMs) were proposed by Kosko to represent the causal relationship between concepts and analyze inference patterns [12, 13, 15]. FCMs are hybrid methods that lie in some sense between fuzzy systems and neural networks. So FCMs represent knowledge in a symbolic manner and relate states, processes, policies, events, values and inputs in an analogous manner. Compared with experts system or neural networks, it has several desirable properties such as: it is relative easy to use for representing structured knowledge, and the inference can be computed by numeric matrix operation instead of explicit IF/THEN rules. FCMs are appropriate to explicit the knowledge and experience which has been accumulated for years on the operation of a complex system. FCMs have gained considerable research interest and have been applied to many areas [6, 8, 15, 16, 17, 18, 19, 20]. However, certain problems restrict its applications. A FCM
Dynamic Random Fuzzy Cognitive Maps

does not provide a robust and dynamic inference mechanism, a FCM lacks the temporal concept that is crucial in many applications and a FCM lacks the traditional statistical parameter estimates.

The Random Neural Network (RNN) has been proposed by Gelenbe in 1989 [9, 10, 11]. This model does not use a dynamic equation, but uses a scheme of interaction among neurons. It calculates the probability of activation of the neurons in the network. Signals in this model take the form of impulses that mimic what is presently known as inter-neural signals in biophysical neural networks. The RNN has been used to solve optimization and pattern recognition problems [1, 2, 3, 4]. Recently, we have proposed a fuzzy cognitive map based on the random neural model. The problem addressed in this paper concerns the proposition of a dynamic/adaptive FCM using the RNN. We describe the dynamic/adaptive Random Fuzzy Cognitive Map (DRFCM) and illustrate its application in the modeling of dynamic processes. Our adaptive/dynamic FCM changes its fuzzy causal web as causal patterns change and as experts update their causal knowledge. We shall use each neuron to model a concept. In our model, each concept is defined by a probability of activation, the dynamic causal relationships between the concepts (arcs) are defined by positive or negative interrelation probabilities, and the procedure of how the cause takes effect is modeled by a dynamic system. This work is organized as follows, in section 2 the theoretical bases of the RNN and of the FCM are presented. Section 3 presents the DRFCM. In section 4, we present applications. Remarks concerning future work and conclusions are provided in section 5.

2. Theoretical Aspects

2.1. The Random Neural Network Model

The RNN model has been introduced by Gelenbe [9, 10, 11] in 1989. This model has a remarkable property called "product form" which allows the computation of joint probability distributions of the neurons of the network (the product form is true for the Markovian case [9]). The model consists of a network of n neurons in which positive and negative signals circulate. Each neuron accumulates signals as they arrive, and can fire if its total signal count at a given instant of time is positive. Firing then occurs at random according to an exponential distribution of constant rate, and signals are sent out to other neurons or to the outside of the network. Each neuron i of the network is represented at any time t by its input signal potential k_i(t). Positive and negative signals have different roles in the network. A negative signal reduces by 1 the potential of the neuron to which it arrives (inhibition) or has no effect on the signal potential if it is already zero; while an arriving positive signal adds 1 to the neuron potential. Signals can either arrive to a neuron from the outside of the network or from other neurons. Each time a neuron fires, a signal leaves it depleting the total input potential of the neuron. A signal which leaves neuron i heads for neuron j with probability p+(i,j) as a positive signal (excitation), or as negative signal with probability p-(i,j) (inhibition), or it departs from the network with probability d(i). Clearly we shall have:

\[ \sum_{j=1}^{n} [p^+(i,j)+p^-(i,j)] + d(i) = 1 \quad \text{for } 1 \leq i \leq n. \]

Positive signals arrive to the ith neuron according to a Poisson process of rate \( \Lambda(i) \) (external excitation signals). Negative signals arrive to the ith neuron according to a Poisson process of rate \( \lambda(i) \) (external inhibition signals). The rate at which neuron i fires is r(i). The main property of this model is the excitation probability of a neuron i, q(i), which satisfy the non-linear equation:

\[ q(i) = \frac{\lambda^+(i)}{r(i)+\lambda^-(i)} \]

where,

\[ \lambda^+(i) = \sum_{j=1}^{n} q(j)r(j)p^+(j,i)+\Lambda(i) \]
\[ \lambda^-(i) = \sum_{j=1}^{n} q(j)r(j)p^-(j,i)+\lambda(i) \]

The synaptic weights for positive \((w^+(i,j))\) and negative \((w^-(i,j))\) signals are defined as:

\[ w^+(i,j) = r(i)p^+(i,j) \]
\[ w^-(i,j) = r(i)p^-(i,j) \]

and

\[ r(i) = \sum_{j=1}^{n} [w^+(i,j)+w^-(i,j)] \]
To guarantee the stability of the RNN, the following is a sufficient condition for the existence and uniqueness of the solution in the equation (1)

$$\lambda^+(i) < [ r(i) + \lambda^-(i)]$$

Notice that the model is based on rates, much as natural neural systems operate. Thus, this is a "frequency modulated" model, which translates rates of signal emission into excitation probabilities via equation (1). For instance, $$q(j)r(i)p_{ij}$$ denotes the rate at which neuron $$j$$ excites neuron $$i$$. Equation (1) can also be translated into a special form of sigmoid that treats excitation (in the numerator) asymmetrically with respect to inhibition (in the denominator).

### 2.2. Fuzzy Cognitive Maps

FCMs combine the robust properties of fuzzy logic and neural networks. At first, R. Axelrod used cognitive maps as a formal way of representing social scientific knowledge and modeling decision making in social and political systems [5]. Then, B. Kosko enhanced cognitive maps considering fuzzy values for them [12, 13, 15]. A FCM describes the behavior of a system in terms of concepts, each concept represents a state or a characteristic of the system. A FCM can avoid many of the knowledge-extraction problems which are usually posed by rule based systems.

A FCM illustrates the whole system by a graph showing the cause and effect along concepts. Particularly, a FCM is a fuzzy signed oriented graph with feedback that model the worlds as a collection of concepts and causal relations between concepts. Variable concepts are represented by nodes in a directed graph. The graph's edges are the casual influences between the concepts. The value of a node reflects the degree to which the concept is active in the system at a particular time. This value is a function of the sum of all incoming edges and the value of the originating concept at the immediately preceding state. The threshold function applied to the weighted sums can be fuzzy in nature. This destroys the possibility of quantitative results, but it gives us a basis for comparing nodes – on or off, active or inactive. This is a variation of the "fuzzification" process in fuzzy logic. Fuzzification gives us a qualitative model and frees us from strict quantification of edge weights. Thus, concept values are expressed on a normalized range denoting a degree of activation rather than an exact quantitative value. The causal relationships are expressed by either positive or negative signs and different weights. Once constructed, a FCM of a specific system allows to perform qualitative simulations of the system.

In general, a FCM functions like associative neural networks. A FCM describes a system in a one-layer network which is used in unsupervised mode, whose neurons are assigned concept meanings and the interconnection weights represent relationships between these concepts. The fuzzy indicates that FCMs are often comprised of concepts that can be represented as fuzzy sets and the causal relations between the concepts can be fuzzy implications, conditional probabilities, etc. A directed edge $$E_{ij}$$ from concept $$C_i$$ to concept $$C_j$$ measures how much $$C_i$$ causes $$C_j$$. In simple FCMs, directional influences take on trivalent values {$$-1, 0, +1$$}, where $$-1$$ indicates a negative relationship, $$0$$ no causality relationship, and $$+1$$ a positive relationship. In general, the edges $$E_{ij}$$ can take values in the fuzzy causal interval $$[-1, 1]$$ allowing degrees of causality to be represented:

- $$E_{jk} > 0$$ indicates direct (positive) causality between concepts $$C_j$$ and $$C_k$$. That is, the increase (decrease) in the value of $$C_j$$ leads to the increase (decrease) on the value of $$C_k$$.
- $$E_{jk} < 0$$ indicates inverse (negative) causality between concepts $$C_j$$ and $$C_k$$. That is, the increase (decrease) in the value of $$C_j$$ leads to the decrease (increase) on the value of $$C_k$$.
- $$E_{jk} = 0$$ indicates no relationship between $$C_j$$ and $$C_k$$.

Because the directional influences are presented as all-or-none relationships, FCMs provide qualitative as opposed to quantitative information about relationships. In FCM nomenclature, model implications are revealed by clamping variables and using an iterative vector-matrix multiplication procedure to assess the effects of these perturbations on the state of a model. A model implication converges to a global stability, an equilibrium in the state of the system. During the inference process, the sequence of patterns reveals the inference model. The simplicity of the FCM model consists in its mathematical representation and operation. So a FCM which consists of $$n$$ concepts, is represented mathematically by a $$n$$ state vector $$A$$, which gathers the values of the $$n$$ concepts, and by a $$n \times n$$ weighted matrix $$E$$. Each element $$E_{ij}$$ of the matrix indicates the value of the weight between concepts $$C_i$$ and $$C_j$$. The activation level $$A_i$$ for each concept $$C_i$$ is calculated by the following rule:
Dynamic Random Fuzzy Cognitive Maps

\[ A_{i}^{new} = f\left(\sum_{j=1}^{n} A_{j}^{new} E_{ji}\right) + A_{i}^{old} \]  (2)

\( A_{i}^{new} \) is the activation level of concept \( C_i \) at time \( t+1 \), \( A_{i}^{old} \) is the activation level of concept \( C_i \) at time \( t \) for \( 1 \leq i \leq n \), and \( f \) is a threshold function. So the new state vector \( A \), which is computed by multiplying the previous state vector \( A \) by the edge matrix \( E \), shows the effect of the change in the activation level of one concept on the other concepts. A FCM can be used to answer a “what-if” question based on an initial scenario that is represented by a vector \( S_0 = \{s_i\} \), for \( i = 1 \ldots n \), where \( s_i \) indicates that concept \( C_i \) holds completely in the initial state, and \( s_i = 0 \) indicates that \( C_i \) does not hold in the initial state. Then, beginning with \( k=1 \) and \( A = S_0 \) we repeatedly compute \( A_t \). This process continues until the system convergence (for example, when \( A_{i}^{new} = A_{i}^{old} \)). This is the resulting equilibrium vector, which provides the answer to the “what if” question.

FCMs have been used for decision analysis, for modeling and processing political knowledge, etc. [6, 7, 16, 18, 19, 20] investigate the implementation of the FCM in distributed and control problems. Particularly, FCMs have been used to model and support a plant control system, to construct a system for failure modes and effect analysis, and to model the supervisor of a control system. In [20] is introduced a formal technique based on FCM to represent different types of knowledge in a group of agents. FCMs model the possible worlds as collection of classes and causal relations between classes. In [13] is proposed an extension of the FCM where each concept can have its own value set, depending on how precisely it needs to be described in the network. The value set can be a binary set, a fuzzy set, or a continuous interval. In addition, the procedure of how the causes take effect is modeled by a dynamic system. A novel approach is the use of FCMs as a computationally inexpensive way to "program" the actors in a virtual world [7, 8, 14]. Simulations involving human actors might combine FCMs with expert systems in order to model the soft, emotional aspect of human decision making as well as the formal, logical side. Finally, to overcome the lack of a concept of time and that they cannot deal with occurrence of multiple causes such as expressed by “and” conditions, in [17] is proposed the extended FCM. The features of this model are: weights having non-linear membership functions and conditional time-delay weights.

3. The Dynamic Random Fuzzy Cognitive Maps (DRFCM)

Our RFCM improves the conventional FCM by quantifying the probability of activation of the concepts and introducing a nonlinear dynamic function to the inference process [3]. Similar to a FCM, concepts in RFCM can be causes or effects that collectively represent the system’s state. The value of \( W_{ij} \) indicates how strongly concept \( C_i \) influences concept \( C_j \). \( W_{ij} > 0 \) and \( W_{ij} = 0 \) if the relationship between the concepts \( C_i \) and \( C_j \) is direct, \( W_{ij} > 0 \) and \( W_{ij} = 0 \) if the relationship is inverse, or \( W_{ij} = W_{ji} = 0 \) if it doesn’t exist a relationship among them. The quantitative concepts enable the inference of RFCM to be carried out via numeric calculations instead of a deductive procedure.

The new aspect introduce by the DRFCM is the dynamic causal relationships. That is, the values of the arcs are modified during the runtime of the FCM to adapt them to the new environment conditions. The quantitative concepts allow us develop a feedback mechanism that is included in the causal model to update the arcs. In this way, with the DRFCM we can consider on-line adaptive procedures of the model like real situations. For example, our DRFCM can structure virtual worlds that change with time. The DRFCM does not write down differential equations to change the virtual world. They map input states to limit-cycle equilibrium. A limit cycle repeats a sequence of events or a chain of actions and responses. Our DRFCM change their fuzzy causal web during the runtime using neural learning laws in order to change the causal rules and the limit cycles. In this way, our model can learn new patterns and reinforce old ones. To calculate the state of a neuron on the DRFCM (the probability of activation of a given concept \( C_j \)), the following expression is used [3]:

\[ q(j) = \min_{i=1,n} \left\{ \lambda^+ (j), \max_{i=1,n} \left\{ f(i,j), \lambda^- (j) \right\} \right\} \]  (3)

where \[ \lambda^+ (j) = \max_{i=1,n} \left\{ \min_{i=1,n} \left\{ q(i), W^+ (i,j) \right\} \right\} \]
\[ \lambda^- (j) = \max_{i=1,n} \left\{ \min_{i=1,n} \left\{ q(i), W^- (i,j) \right\} \right\} \]

Such as, \( \lambda(j) = \lambda(i) = 0 \). That means, the external signal inputs are equal to 0.

In addition, the fire rate is
José Aguilar

\[ r(j) = \max_{i=1,n} \{W^+(i,j), W^-(i,j)\} \quad (4) \]

The general procedure of the DRFCM is the following:

1. Define the number of neurons (the number of neurons is equal to the number of concepts).
2. Call the Initialization phase.
3. Call the Execution phase.

3.1 The Initialization Procedure

In this phase we must define the initial weights. The weights are defined and/or update according to the next procedures:

- **Based on expert’s opinion:** each expert defines its FCM and we determine a global FCM. In this case, the knowledge and experience human is exploited. We use two formulas to calculate the global causal opinion.

\[ E_{ji}^G = \max_e \frac{1}{NE} \sum_{e=1}^{NE} E_{ji}^e, \quad \forall e=1, \text{NE (number of experts)} \]

or

\[ E_{ji}^G = \sum_{e=1}^{NE} b_e E_{ji}^e / \text{NE} \]

Where \( E_{ji}^e \) is the opinion of the expert \( e \) about the causal relationship among \( C_j \) and \( C_i \), and \( b_e \) is the expert’s opinion credibility weight.

Then, with the next algorithm we determine the initial weights of the DRFCM:

1. If \( i \neq j \) and if \( E_{ji}^G > 0 \) \( W^+_{ji} = E_{ji}^G \) and \( W^-_{ji} = 0 \)
2. If \( i \neq j \) and if \( E_{ji}^G < 0 \) \( W^+_{ji} = 0 \) and \( W^-_{ji} = E_{ji}^G \)
3. If \( i = j \) or if \( E_{ji}^G = 0 \) \( W^+_{ji} = W^-_{ji} = 0 \)

The causal relationship \( (E_{ji}^e) \) is caught from each expert using the next table:

<table>
<thead>
<tr>
<th>Symbolic value</th>
<th>Real Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>no relationship</td>
<td>0</td>
</tr>
<tr>
<td>Slight</td>
<td>0.2</td>
</tr>
<tr>
<td>Low</td>
<td>0.4</td>
</tr>
<tr>
<td>Somehow</td>
<td>0.6</td>
</tr>
<tr>
<td>Much</td>
<td>0.8</td>
</tr>
<tr>
<td>Direct</td>
<td>1</td>
</tr>
</tbody>
</table>

**Table 1.** Relationship between the concepts

- **Based on measured data:** In this case we have a set of measures about the system. This information is the input pattern:

\[ M = \{D_1, \ldots, D_m\} = \{[d_1^1, d_1^2, \ldots, d_1^n], \ldots, [d_m^1, d_m^2, \ldots, d_m^n]\} \]

Where \( d_i^t \) is the value of the concept \( C_j \) measured at time \( t \). In this case, our learning algorithm follows the next mechanism:

\[ W^t_{ji} = W^{t-1}_{ji} + r \left\{ \frac{\Delta d^t_i \Delta d^t_j}{\Delta^* d^t_i \Delta^* d^t_j} \right\} \]
where \( \Delta d^t_j = d^t_j - d^{t-1}_j \)
\( \Delta d^t_i = d^t_i - d^{t-1}_i \)
\( \Delta^+ d^t_j = d^t_j + d^{t-1}_j \)
\( \Delta^+ d^t_i = d^t_i + d^{t-1}_i \)

and \( \eta \) is the learning rate (\( \eta < 1 \) to guarantee the convergence to a local optimum). On this way we guarantee the values of \( W_{ij} \) in the interval \([0, 1]\), where \( W_{ij} \) can be \( W^+_{ij} \) or \( W^-_{ij} \).

### 3.2 The Execution Phase

The DRFCM can be used like an associative memory. In this way, when we present a pattern to the network, the network will iterate until generate an output close to the information keeps. This phase consists on the iteration of the system until the system convergence. The input is an initial state \( S_0 = \{s_1, \ldots, s_n\} \), such as \( q^0(1) = s_1, \ldots, q^0(n) = s_n \) and \( s_i \in [0, 1] \) (set of initial values of the concepts \( S_0 = Q^0 \)). The output \( Q^m = \{q^m(1), \ldots, q^m(n)\} \) is the prediction of the DRFCM such as \( m \) is the number of the iteration when the system converge. \( Q^m \) must be analyzed for an expert. During this phase, the DRFCM is trained with a reinforced learning procedure. The weights of edges leaving a concept are modified when the concept has a nonzero state change (the weight of edge among two concepts is increased if they both increase or both decrease, and the weight is decreased if concepts move in opposite directions):

\[
W^t_{ij} = W^{t-1}_{ij} + \eta \left( \Delta q^t_i \Delta q^t_j \right)
\]

where \( \Delta q^t_i \) is the change in the \( i^{th} \) concept’s activation value among iterations \( t \) and \( t-1 \).

It is an unsupervised method whose computational load is light. In this way, we take into account the dynamic characteristics of the process. The algorithm of this phase is:

1. Read input state \( Q^0 \)
2. Until system convergence
   2.1 Calculate \( q(i) \) according to the equation (3)
   2.2 Update \( W^t \) according to the equation (5)

### 4. Experiments

In this section we illustrate the DRFCM application. A discrete time simulation is performed by iteratively applying the equation (3) to the state vector of the graph. At the beginning, we must define an initial vector of concept states, and the simulation halts if an equilibrium state is reached. To test the quality of our approach, we compare it with the Kosko FCM [7, 8, 14, 15] and with the RFCM [3]. Obviously, the success of a particular model depends greatly on the selection of concept nodes and the interpretation of state vectors.

#### 4.1. First Experiment: a simple model of a country

In this first experiment we discuss a simple model to determine the risk of a crisis in a country. Our model of assumptions is depicted in Fig. 1. The operative concepts are:

- Foreign inversion (C1): the presence of a strong foreign inversion.
- Employment rate (C2): The level of Employment on the country.
- Laws (C3): the presence or absence of laws.
- Social problems (C4): the presence or absence of social conflict in/within the country.
- Government stability (C5): a good relationship between the congress, the president, etc.
The edge connection matrix (E) for this map is given in table 2. This is an example where the system is non-dynamic, for this reason we can applied all methods.

<table>
<thead>
<tr>
<th>Foreign inversion</th>
<th>Employment Rate</th>
<th>Laws</th>
<th>Social problems</th>
<th>Government stability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Foreign inversion</td>
<td>0</td>
<td>0.8</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Employment Rate</td>
<td>0</td>
<td>0</td>
<td>-0.6</td>
<td>0.8</td>
</tr>
<tr>
<td>Laws</td>
<td>0.4</td>
<td>0</td>
<td>-0.8</td>
<td>0</td>
</tr>
<tr>
<td>Social problems</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-0.8</td>
</tr>
<tr>
<td>Government stability</td>
<td>0.6</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2. The edge connection matrix for the first experiment

The table 3 presents the results for different initial states

<table>
<thead>
<tr>
<th>Input</th>
<th>Kosko FCM</th>
<th>RFCM</th>
<th>DRFCM</th>
<th>Iteration</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 0 0 1 0</td>
<td>1 1 1 0</td>
<td>0.8 0.6 0.2 0.2 0.8</td>
<td>0.6 0.6 0.2 0.2 0.6</td>
<td>1</td>
</tr>
<tr>
<td>1 0 1 1 0</td>
<td>1 1 1 0</td>
<td>0.8 0.6 0.8 0.0</td>
<td>0.7 0.7 0.8 0.0 0.2</td>
<td>2</td>
</tr>
<tr>
<td>1 0 0 1 0</td>
<td>1 1 0 0</td>
<td>0.8 0.6 0.8 0.0</td>
<td>0.7 0.8 0.8 0.0 0.0</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 3. The results for the first experiment

Clamping two antithetical concepts allows to test the implications of one or more competing concepts. To illustrate, we begin by clamping $C_1$ and $C_4$ ($S_0=(1 0 0 1 0)$) – a strong foreign inversion can generate more employment. Despite of the foreign inversion, we have an unstable government due to the social problems (the system reaches an equilibrium state of (1 1 0 0 1)). With $S_0=(1 0 1 1 0)$ foreign inversion and social problems remain clamped, but we also clamp the ability to have a good law system. The system reaches an equilibrium state of 1 1 1 0 0 – A peaceful country at the social level but one unstable government. Next, we test for $S_0=(1 1 1 1 0)$. In our model, the inference process is:
In this example, we could take advantage of the ability to study the inference process during execution of the simulation. That means, we could study the different states of the concepts during the inference process ($S_1$, $S_2$). This example suggests the social problem is the main factor to have an unstable government. Obviously, our goal in analyzing this model was not to determine policy choices for a country. Rather, we tried to illustrate the advantages of the DRFCM for/in this sort of analysis. The nature of the domain is such that a quantitative model is difficult to construct, if not impossible. Resorting to qualitative measures permitted us to rapidly construct a model and analyze a variety of alternative policy options. Our results indicate that DRFCMs quickly come to an equilibrium regardless of the complexity of the model. DRFCM gives similar results than RFCM.

4.2. Second Experiment: Virtual Worlds

Dickerson and Kosko proposed a novel use for FCMs [7, 8, 14]. They employed a system of three interacting FCMs to create a virtual reality environment populated by dolphins, fish, and sharks. The use of FCMs proved to be a computationally inexpensive means of encoding behavior. [15] refines the Dickerson and Kosko’s approach to be used the FCM to model the “soft” elements of an environment in concert with an expert system capturing the procedural or doctrinal – “hard” elements of the environment. In their paper, they present a FCM modeling a squad of soldiers in combat. This map is shown in Figure 2. This is a good example where we can use a dynamic model to caught ideas like: an army needs several battles to know the strength of its enemy before a decisive battle. We introduce these aspects in this experiment (previous experiences) during the runtime (for this reason we do not use RFCM in this example). The concepts in this map are:

- Cluster ($C_1$): the tendency of individual soldiers to close with their peers for support.
- Proximity of enemy ($C_2$): the observed presence of hostile forces within firing range.
- Receive fire ($C_3$): taking fire from hostile forces.
- Presence of authority ($C_4$): command and control inputs from the squad leader.
- Fire weapons ($C_5$): the state in which the squad fires on the enemy.
- Peer visibility ($C_6$): the ability of any given soldier to observe his peers.
- Spread out ($C_7$): dispersion of the squad.
- Take cover ($C_8$): the squad seeking shelter from hostile fire.
- Advance ($C_9$): the squad proceeding in the planned direction of travel with the intent of engaging any encountered enemy forces.
- Fatigue ($C_{10}$): physical weakness of the squad members.
In the hybrid system we suggest, the presence of authority concept would be replaced by an input from an expert system programmed with the enemy’s small unit infantry doctrine and prevailing conditions. Similarly, the proximity of the enemy would be an input based on the battlefield map and programmed enemy locations. Here, however, we give them initial inputs and allow them to vary according to operation of the FCM. In addition, during the runtime we introduce results of previous battles.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
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<td>1</td>
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<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
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<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
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<td>0</td>
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<td>0</td>
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<td>0</td>
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<td>0</td>
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<td>0</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<td>-0.5</td>
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<td>0</td>
<td>0</td>
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<td>0</td>
<td>0</td>
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Table 4. The edge connection initial matrix for the virtual word experiment
The table 5 presents the results for the initial states 0 0 0 1 0 1 1 0 1 0.

<table>
<thead>
<tr>
<th>Input</th>
<th>Kosko FCM</th>
<th>DFRCM</th>
<th>Iteration</th>
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<tbody>
<tr>
<td>0 0 0 1 0 1 1 0 1 0</td>
<td>0 0 0 1 0 1 1 0 1 0</td>
<td>0.2 0.4 0.7 0.6 0.5 0.6 0.6 0.4 0.6 0.4</td>
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</tr>
<tr>
<td>1 1 1 0 1 0 1 0 1</td>
<td>0.6 0.6 0.6 0.6 0.5 0.1 0.4 0.6 0.6 0.8</td>
<td>2 *</td>
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<tr>
<td>1 0 1 1 0 1 0 1 1 0</td>
<td>0.6 0.6 0.6 0.6 0.5 0.1 0.4 0.6 0.8 0.8</td>
<td>3 *</td>
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<tr>
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<td>0.8 0.8 0.6 0.6 0.8 0.1 0.2 0.8 1 0 8</td>
<td>4 *</td>
<td></td>
</tr>
<tr>
<td>0 1 1 0 1 1 0 0 1 1</td>
<td>1 0 8 0 1 0 8 0 0 0 8 1 0 8</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>0 1 1 1 0 0 0 0 1 1</td>
<td>0 1 1 0 0 0 0 1 1</td>
<td>6</td>
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<tr>
<td>1 1 1 1 0 0 1 1 1 1</td>
<td>1 1 1 1 0 0 1 1 1 1</td>
<td>7</td>
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</tr>
</tbody>
</table>

Table 5. The results for the virtual word experiment (* means that we introduce in this iteration results of new battles)

We define the starting state $S_0=(0 0 0 1 0 1 1 0 1 0)$ i.e., presence of authority, peer visibility, spread out and advance are present, but all other concepts are inactive. We then obtain the discrete time series show on the second and third columns of the table 5. The system stabilizes to the state $S_7$ (Kosko model) or state $S_5$ (DRFCM). The introduction of new information during the runtime doesn't affect the convergence of our system (we obtain the same result of Kosko). The first * consist of clamping the first concept (Cluster) because the soldiers are closed with their peers. The second * clamps proximity of enemy, receive fire, and fatigue because that are new conditions that are observed from the environment. This is reasonable system operation and suggests the feasibility of FCMs as simple mechanisms for modeling inexact and dynamic behavior that is difficult to capture with formal methods.

4.3. Stability

Stability in dynamic systems is typically analyzed through the use of Lyapunov functions. Kosko [14, 15] finds that a simpler form of fuzzy systems, the standard additive model, may be checked for stability in terms of the eigenvalues of the edge connection matrix. The standard additive model processes inputs in exactly the same way as FCMs, but does not iterate through multiple feedback cycles. It is strictly an “if-then” model of fuzzy systems. If all the eigenvalues of a system have negative real parts, it will achieve some form of equilibrium. Positive real parts indicate instability, and the form of the eigenvalues may classify the types of instability. Unfortunately, complex feedback dynamics prohibit this sort of analysis for FCMs. But, with our feedback mechanism inside of the DRFCM we can solve this problem.

![Fig. 3: Simple FCM for stability analysis](image)
José Aguilar

Table 6. Simple FCM edge connection initial matrix

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th></th>
<th>D</th>
<th></th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
<td>0</td>
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<tr>
<td>D</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The edge connection matrix has the eigenvalues $1.255, -0.084\pm1.408i, -0.543\pm0.325i$, suggesting instability. Instead, using our approach given $C_0 = (0, 1, 1, 0, 0)$, it reaches the equilibrium state $C_3 = (0.9, 0, 1, 1, 0.9)$.

5. Conclusions

FCMs are an interesting yet isolated decision support tool. Their implicitly qualitative nature is at odds with general practice in the automation of decision support tools. In this paper, we have proposed a dynamic/adaptive FCM based on the RNN, the DRFCM. We show fusing the RFCM with a traditional reinforced learning algorithm can yield excellent results. The DRFCM may be rapidly adapted to changes in the modeled behavior. It is a useful method in complex dynamic system modeling. We do not observe any inconsistent behavior of our DRFCM with respect to the previous FCMs. Our DRFCM exhibit a number of desirable properties that make it attractive:

- Provide qualitative information about the inferences in complex dynamic models.
- Can represent an unlimited number of reciprocal relationships.
- Is based in a unsupervised learning (based on a reinforced learning procedure).
- Can model both mediator and moderator relationships.
- Facility the modeling of dynamic, time evolving phenomena and process.
- Has a high adaptability to any inference with feedback.

Another important characteristic is its simplicity, the result of each DRFCM’s cycles is computed from the equation (3). Most of the computations are intrinsically parallel and can be implemented on SIMD or MIMD architectures. The ease of construction and low computational costs of the DRFCM permits wide dissemination of low-cost training aids. In addition, the ability to easily model uncertain systems at low cost and with adaptive behavior would be of extraordinary value in a variety of domains. We hope we have demonstrated their promise in a variety of areas.

Acknowledgment

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References

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