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EFFECT OF A VISCOSITY REDUCER IN A LIQUID-LIQUID FLOW: II UNSTEADY STATE ANNULAR MODEL IN A PIPELINE

EFECTO DE UN REDUCTOR DE VISCOSIDAD EN UN FLUJO LÍQUIDO-LÍQUIDO: II MODELO ANULAR NO ESTACIONARIO PARA UN OLEODUCTO

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Abstract

The study of extra heavy crude oil flow has increased in recent years, due to the challenges relating to higher viscosity that limits the installed capacity of pipelines and the pumping infrastructure used. This has required the development and implementation of new technologies and chemical formulations to enhance the transport of heavy oil. When flow improvers are injected in a crude stream, it generally does not mix due to its different density and viscosity properties, and the laminar flow regime in the pipe. However, depending on the characteristics of the jet (momentum and turbulence), a mixing might take effect downstream, or a liquid-liquid stratified flow may occur in the fully developed region. Although this behavior can affect the corresponding pressure drop, the transition that occurs from the injection point to the fully developed region has been little studied. Based on conservation of momentum, a mathematical model has been developed to describe the temporal behavior of the velocity profile, where there is no mixing between the oil and the improver, i.e. the fluids are considered immiscible. The model solution shows that the injection into a point very near the tube wall is the best option to reduce pressure drop.

Keywords: two-phase flow, non-steady profile, extra heavy crude, high viscosity.

Resumen

El estudio del flujo de crudo extrapesado ha ganado importancia en los últimos años, debido a los desafíos relativos a la limitación y reducción de la capacidad instalada y la infraestructura utilizada, que implica el desarrollo de nuevas tecnologías y formulaciones para incrementar el transporte de líquidos. Cuando los mejoradores de flujo se inyectan en una corriente de crudo extrapesado, puede suceder que no se mezclen debido tanto a sus propiedades físicas como al régimen de flujo de laminar. Sin embargo, dependiendo de las características del chorro inyectado (momentum y turbulencia), cierto mezclado puede producirse aguas abajo o generarse un flujo estratificado líquido-líquido en la región completamente desarrollada. Aunque este comportamiento puede afectar las correspondientes caídas de presión, la transición que ocurre desde el punto de inyección hasta la zona completamente desarrollada ha sido poco estudiada. Basado en las ecuaciones de cantidad de movimiento, se desarrolló en este trabajo un modelo de flujo en estado no permanente, donde no hay mezcla entre el aceite y el mejorador, es decir, son inmiscibles. El modelo predice que la inyección en un punto cercano a la pared del tubo es la mejor opción para reducir la caída de presión del flujo.

Palabras clave: flujo bifásico, perfil no estacionario, crudo extrapesado, alta viscosidad.

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1 Introduction

The oil industry is currently interested in the study of extra heavy crude flow due to the challenges of transport that it entails (Santos *et al*., 2014; Huang *et al*., 2011 and Tian *et al*., 2014). The study of velocity profiles is very important because these effects are directly related to the pumping power and transport costs (Martínez-Palou *et al*., 2011), but few studies have explored the effects related to the injection system of the flow improver, that may enhance or limit the mixing process with the crude oil.

The study of the transport mechanism of a liquid flow improver can demonstrate its efficiency in terms of the rate (Toms, 1948) and the interaction effects that occur at a molecular level that generally changes the rheological properties of the main flow (Sai Ravindra, Panuganti, 2013; Tharanivasan, 2012) i.e. polymers dosed in crude (Edomwonyi-Otu, *et al*., 2015; Abubakar *et al*., 2016)

There are different methods to improve pipeline transport for heavy and extra heavy oil crude. Viscosity modification can be achieved by heating, dilution, addition of chemical products and emulsion formation (Martínez-Palou *et al*., 2011). Stratified flow has been amply studied but considering only the use of water (Mukhaimer *et al*., 2015; Ismail *et al*., 2015). Although viscosity reduction impacts pressure losses (Suárez-Domínguez *et al*., 2014), mechanical mixing homogenization is required mainly with high viscosity fluids (Gonzalez-Davila *et al*., 2015; Argillier, 2005; Gateau *et al*., 2004); absent a premixing process, it is very likely to find a liquid-liquid flow in the pipe.

Although it has been found that injection into the annular region adjacent to the tube wall may increase the flow (Bensakhria *et al*., 2004; Brauner *et al*., 1996), the usual practice is to inject the fluid perpendicular to the flow, be it at the top or bottom of the tube. There have been studies (Manning, Lind, 2014; Silva *et al*., 2014) focused on the improver effect on the interfacial tension and the changes in its viscosity, but in this case the fluids are not miscible, then the less viscous one will "adhere" to the wall, reducing the overall drag (Joseph *et al*., 1997; Loh and Premanadhan, 2016). Some effects in the core near to the wall in annular flow have been described (Imbert-Gonzalez, 2016) but focused in turbulence and vortex formation. Nevertheless, there are few reported studies about the effects of the procedure of injection of these products. For the case of highly viscous fluids, such as heavy oil and extra heavy oil, that usually flow in a laminar regime, it has been determined experimentally that a stratified liquid-liquid flow occurs in a horizontal pipe if a mechanical mixture of the oil with the drag reducing agent (DRA) is not carried out (Palacio-Pérez *et al*., 2014).

In a previous publication (Suarez-Dominguez *et al*., 2016), the authors presented the effect of a DRA when it is injected in a single-phase pre-mixed fluid, and also when it is injected into a two-phase stratified flow in steady state. It was found that the injection of DRA near the pipe wall is the best way to transport high-viscosity oil crude.

The present work aims to analyze how the injection of the less viscous fluid (i.e, the flow improver) influences the transient behavior of flow and pressure drop when a stratified flow regime is considered for two immiscible liquids. A mathematical model that describes the temporal behavior of the velocity profile when a liquid is injected into the center, and into the annular section adjacent to the walls of the tube, is proposed in the second section. The third section describes the transient flow behavior with constant pressure drop, as well as the transient pressure drop behavior for a constant flow rate. For each case, both forms of injection were taken into account: near the pipeline center and near the tube wall. The fourth section presents and discusses the results of the mathematical model.

2 Method

In order to obtain the transient velocity profile of the two-phase flow, the following considerations were taken into account: i) both flows are Newtonian liquids; ii) the flow moves along a tube where one of the fluids is in contact with the wall and the other moves through the core (Figure 1); iii) the density of both incompressible fluids is constant; iv) laminar flow regime; v) isothermal system, where each fluid comprises a single component and there is no mass exchange.

2.1 Theoretical model

Taking into account the established considerations and geometrical characteristics of the system, the following differential partial equations were obtained (Bird *et al*., 2002) (1 and 2 represent each fluid):

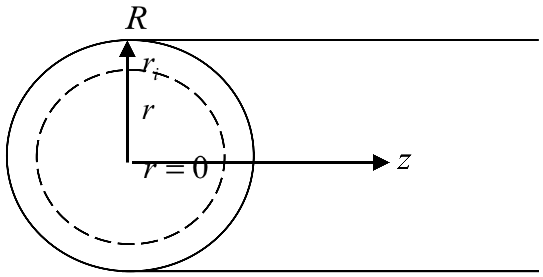

Fig. 1. Graphical representation of two-fluid annular flow in a tube. R. tube radius; z longitudinal coordinate

$$\rho_1 \frac{\partial v_1}{\partial t} = \frac{\partial p}{\partial z} + \mu_1 \frac{1}{r}\frac{\partial v_1}{\partial r} + \mu_1 \frac{\partial^2 v_1}{\partial r^2}, \text{ for } r_i < r < R \quad (1)$$

$$\rho_2 \frac{\partial v_2}{\partial t} = \frac{\partial p}{\partial z} + \mu_2 \frac{1}{r}\frac{\partial v_2}{\partial r} + \mu_2 \frac{\partial^2 v_2}{\partial r^2}, \text{ for } 0 < r < r_i \quad (2)$$

where r_i represents the interphase position between both fluid, ρ, μ, p, r, v are density, viscosity, pressure, radius and velocity, respectively.

Defining the following dimensionless variables:

$$\alpha = \frac{r_i}{R}; \quad \gamma = \frac{r}{R}$$

And the parameters:

$$\beta = R^2 \rho; \quad \Phi = \frac{\partial p}{\partial z} R^2$$

equations (1) and (2) can be rewritten as:

$$\beta_1 \frac{\partial v_1}{\partial t} = \Phi + \mu_1 \frac{1}{\gamma}\frac{\partial v_1}{\partial \gamma} + \mu_1 \frac{\partial^2 v_1}{\partial \gamma^2}, \text{ for } \alpha < \gamma < 1 \quad (3)$$

$$\beta_2 \frac{\partial v_2}{\partial t} = \Phi + \mu_2 \frac{1}{\gamma}\frac{\partial v_2}{\partial \gamma} + \mu_2 \frac{\partial^2 v_2}{\partial \gamma^2}, \text{ for } 0 < \gamma < \alpha \quad (4)$$

Here it is obtained a linear partial differential equation system, which can be solved in several ways. In this paper, in order to obtain an exact solution of the system given by equations (3) and (4), the velocity v is expressed as the sum of the velocity in steady state V and a velocity in non steady state q in such way that:

$$0 = \Phi + \mu_1 \frac{1}{\gamma}\frac{\partial V_1}{\partial \gamma} + \mu_1 \frac{\partial^2 V_1}{\partial \gamma^2} \quad (5)$$

$$0 = \Phi + \mu_2 \frac{1}{\gamma}\frac{\partial V_2}{\partial \gamma} + \mu_2 \frac{\partial^2 V_2}{\partial \gamma^2} \quad (6)$$

$$\beta_1 \frac{\partial q_1}{\partial t} = \mu_1 \frac{1}{\gamma}\frac{\partial q_1}{\partial \gamma} + \mu_1 \frac{\partial^2 q_1}{\partial \gamma^2} \quad (7)$$

$$\beta_2 \frac{\partial q_2}{\partial t} = \mu_2 \frac{2}{\gamma}\frac{\partial q_2}{\partial \gamma} + \mu_2 \frac{\partial^2 q_2}{\partial \gamma^2} \quad (8)$$

The ordinary differential equation system (5-6) has the analytical solution:

$$V_1(\gamma) = C_{11} + C_{12}\ln\gamma - \frac{1}{4}\Phi\frac{\gamma^2}{\mu_1} \quad (9)$$

$$V_2(\gamma) = C_{21} + C_{22}\ln\gamma - \frac{1}{4}\Phi\frac{\gamma^2}{\mu_2} \quad (10)$$

where components of shear stress between the two fluids are given by:

$$\tau_1(\gamma) = \frac{1}{2}\Phi\gamma - \frac{1}{\gamma}\mu_1 C_{12} \quad (11)$$

$$\tau_2(\gamma) = \frac{1}{2}\Phi\gamma - \frac{1}{\gamma}\mu_2 C_{22} \quad (12)$$

and the following boundary conditions apply:

$$V_1(1) = 0, V_2(0) = cte., V_1(\alpha) = V_2(\alpha), \tau_1(\alpha) = \tau_2(\alpha)$$

The steady state velocity profiles were obtained as:

$$V_1(\gamma) = \frac{1}{4}\frac{\Phi}{\mu_1} - \frac{1}{4}\Phi\frac{\gamma^2}{\mu_1}, \text{ for } \gamma > \alpha \quad (13)$$

$$V_2(\gamma) = \frac{1}{4}\frac{\Phi}{\mu_1}\left(1 - \alpha^2\left(1 - \frac{\mu_1}{\mu_2}\right)\right) - \frac{1}{4}\Phi\frac{\gamma^2}{\mu_2}, \text{ for } \gamma < \alpha \quad (14)$$

The analytical solution of partial differential equations (7) and (8) is:

$$q_1(\gamma,t) = \exp\left(-\frac{t}{\beta_1}C_1\right)\left(C_3 BesselI_0\left(\frac{\gamma}{\mu_1}C_1\sqrt{-\frac{\mu_1}{C_1}}\right) + C_4 BesselK_0\left(\frac{\gamma}{\mu_1}C_1\sqrt{-\frac{\mu_1}{C_1}}\right)\right), \quad \text{for } \gamma > \alpha(t) \quad (15)$$

$$q_2(\gamma,t) = \exp\left(-\frac{t}{\beta_2}C_2\right)\left(C_5 BesselI_0\left(\frac{\gamma}{\mu_2}C_2\sqrt{-\frac{\mu_2}{C_2}}\right) + C_6 BesselK_0\left(\frac{\gamma}{\mu_2}C_2\sqrt{-\frac{\mu_2}{C_2}}\right)\right), \quad \text{for } \gamma < \alpha(t) \quad (16)$$

Where $\alpha(t)$ describes the interface transient behavior. Taking into account the following boundary conditions, which implies that fluid velocity at the inner pipe wall is zero that is $\alpha = 1$:

$$q_1(1,t) = 0, \quad q_2(1,t) = 0$$

and the initial conditions:

$$v_1(\gamma, 0) = \frac{1}{4}\frac{\Phi}{\mu_1} - \frac{1}{4}\Phi\frac{\gamma^2}{\mu_1}, \quad \text{for } \gamma > \alpha_0 \tag{17}$$

$$v_2(\gamma, 0) = \frac{1}{4}\frac{\Phi}{\mu_1}\left(1 - \alpha_0^2\left(1 - \frac{\mu_1}{\mu_2}\right)\right) - \frac{1}{4}\Phi\frac{\gamma^2}{\mu_2}, \quad \text{for } \gamma < \alpha_0 \tag{18}$$

the temporal velocity profile is given by:

$$v_1(\gamma, t) = \frac{\Phi}{4\mu_1}\left(1 - \gamma^2\right), \quad \text{for } \gamma > \alpha(t) \tag{19}$$

$$\begin{aligned} v_2(\gamma, t) = {} & \frac{\Phi}{4\mu_2}\left(\frac{\mu_2}{\mu_1}\left(1 - \alpha^2\left(1 - \frac{\mu_1}{\mu_2}\right)\right) - \gamma^2\right)\left(1 - \exp\left(-5.784\frac{\mu_2}{\beta_2}t\right)\right) \\ & + \frac{\Phi}{4\mu_2}\left(\frac{\mu_2}{\mu_1}\left(1 - (\alpha_0)^2\left(1 - \frac{\mu_1}{\mu_2}\right)\right) - \gamma^2\right)\exp\left(-5.784\frac{\mu_2}{\beta_2}t\right), \quad \text{for } \gamma < \alpha(t) \end{aligned} \tag{20}$$

In equations (17)-(20) α_0 represents the interface position for a time equal to zero. So, if the flow improver is injected at the tube center then $\alpha_0 = 0$, whereas if the injection occurs very near the tube wall $\alpha_0 = 1$. $\alpha(t \to \infty)$ depends on the two-phase flow composition, which is defined as the volumetric fraction of the fluid 2:

$$y_2 = \frac{Q_2}{Q_1 + Q_2} \tag{21}$$

where:

$$Q_1 = \int_0^{2\pi}\int_\alpha^1 V_1(\gamma)\,\gamma d\gamma d\theta \tag{22}$$

$$Q_2 = \int_0^{2\pi}\int_0^\alpha V_2(\gamma)\,\gamma d\gamma d\theta \tag{23}$$

The solution of the system of equations (17) to (20), which is included in the annex, together with equations (21) to (23) result in the following expression for α:

$$\alpha = \left(\frac{\mu_2 - \sqrt{\mu_2(1 - y_2)(\mu_1 y_2 + \mu_2(1 - y_2))}}{(\mu_2(2 - y_2) - \mu_1(1 - y_2))}\right)^{0.5} \tag{24}$$

The temporal interface behavior was determined taking into account the boundary condition

$$v_1(\alpha(t), t) = v_2(\alpha(t), t)$$

resulting:

$$\alpha(t) = \left(\alpha^2\left(1 - \exp\left(-5.784\frac{\mu_2}{\beta_2}t\right)\right) + \alpha_0^2\exp\left(-5.784\frac{\mu_2}{\beta_2}t\right)\right)^{0.5} \tag{25}$$

2.2 *Temporal behavior of flow and pressure*

Constant pressure drop. Considering a system in which the pressure drop is constant, the expression for the two-phase flow is determined as

$$Q(t) = Q_1(t) + Q_2(t)$$

$$Q(t) = \int_0^{2\pi}\int_{\alpha(t)}^1 v_1(\gamma, t)\,\gamma d\gamma d\theta + \int_0^{2\pi}\int_0^{\alpha(t)} v_2(\gamma, t)\,\gamma d\gamma d\theta \tag{26}$$

where:

$$Q(t) = \frac{\pi R^4 \Delta P}{8L}\left(\frac{\varphi_1(t)}{\mu_1} + \frac{\varphi_2(t)}{\mu_2}\right)$$

$$\varphi_1(t) = (b(t) - 1)^2$$

$$\begin{aligned} \varphi_2(t) = {} & 2b(t)\left(\frac{\mu_2}{\mu_1}\left(1 - \alpha^2\right) + \left(\alpha^2 - \frac{b(t)}{2}\right)\right)\left(1 - \exp\left(-\frac{5.784\mu_2}{R^2\rho_2}t\right)\right) \\ & + 2b(t)\left(\frac{\mu_2}{\mu_1}\left(1 - \alpha_0^2\right) + \left(\alpha_0^2 - \frac{b(t)}{2}\right)\right)\exp\left(-\frac{5.784\mu_2}{R^2\rho_2}t\right) \end{aligned}$$

$$b(t) = \left(\alpha^2 - \left(\alpha^2 - \alpha_0^2\right)\exp\left(-\frac{5.784\mu_2}{R^2\rho_2}t\right)\right)$$

and ΔP is the frictional pressure loss, R is the pipe radius and L is its length. Equations (25) and (26) allow to determine the temporal behavior in the flow rate when the improver injection in a system occurs, while the pressure drop remains constant. For the steady state condition, where the value of the total flow will depend on the flow and the pressure drop of the system, the expressions are:

$$Q = \frac{\pi R^4 \Delta P}{8L}\left(\frac{\psi_1(\alpha)}{\mu_1} + \frac{\psi_2(\alpha)}{\mu_2}\right) \tag{27}$$

where:

$$\psi_1(\alpha) = \left(\alpha^2 - 1\right)^2$$

$$\psi_2(\alpha) = \alpha^2\left(\left(1 - 2\frac{\mu_2}{\mu_1}\right)\alpha^2 + 2\frac{\mu_2}{\mu_1}\right)$$

$$\alpha^2 = \frac{\mu_2 - \sqrt{\mu_2(1-y_2)(\mu_1 y_2 + \mu_2(1-y_2))}}{\mu_2(2-y_2) - \mu_1(1-y_2)}$$

Constant flow rate. For a system in which it is considered that the flow rate is constant, the behavior of the pressure drop with respect to the composition of the liquid-liquid flow is given by:

$$\Delta P = \frac{\mu_1\mu_2}{\mu_1\psi_2(\alpha) + \mu_2\psi_1(\alpha)}\frac{8LQ}{\pi R^2} \tag{28}$$

Similarly for unsteady-state it results:

$$\Delta P = \frac{\mu_1\mu_2}{\mu_1\varphi_2(t) + \mu_2\varphi_1(t)}\frac{8LQ}{\pi R^2} \tag{29}$$

From equation (29) it is possible to estimate the transient behavior of pressure at a given point in the system for an initial fixed pressure, in this case:

$$P_L = P_0 - \frac{\mu_1\mu_2}{\mu_1\varphi_2(t) + \mu_2\varphi_1(t)}\frac{8LQ}{\pi R^2} \tag{30}$$

3 Results and discussion

To analyze the results predicted by the model, the following property data was considered for a crude typical of production in northern Mexico. For this case: $\mu_1 = 0.007$ Pa·s, $\rho_1 = 870$ kg m^{-3}, $\mu_2 = 35$ Pa·s, $\rho_2 = 950$ kg m^{-3}, where the liquid 1 is the improver liquid (which is a biodiesel derived from the Jatropha plant that flows next to the walls) and the liquid 2 is the oil crude; $R = 0.0254$m, $L = 10^3$m, $\Delta P = 10^4$Pa. The predicted results are shown in Figure 2 in terms of the transient velocity profiles.

For the given pressure drop, the differences in the velocity profiles resulting from injection at the center of the pipe (figure 2a) and injection into the annular region (figure 2b) can be clearly perceived, finding a more uniform and higher velocity for the second case by two orders of magnitude.

Figure 3 presents the values of alpha as a function of time, assuming that the injected fluid remains along the wall tube. This result suggests that in practice it will suffice to employ lower dosages of flow improver to attain a required oil crude flow.

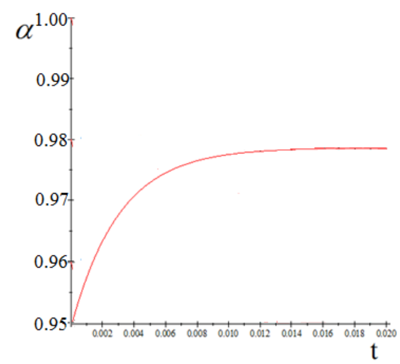

Fig. 3. Evolution of interphase in time for the case of injection in the wall tube periphery for a constant flow.

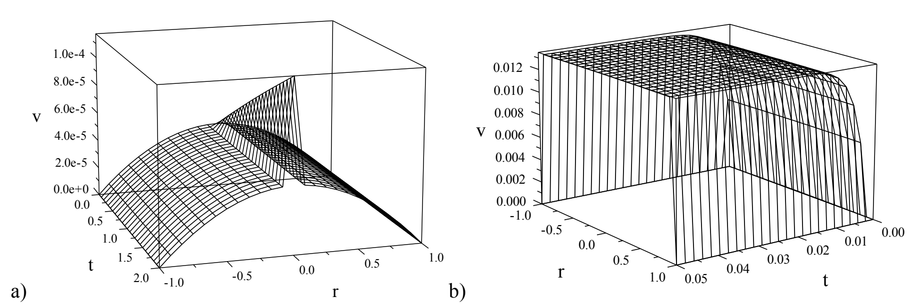

Fig. 2. Behavior of the predicted velocity profile, with a constant pressure, when the injected flow improver is in a) the tube center and b) at the tube wall. The velocity (v), time (t) and radius (r) are presented. Radius (r) is in dimensionless units.

Figures 4 and 5 show the transient behavior of increase in flow rate and decrease in pressure drop respectively, that occur when the flow improver is injected at the center line or at the pipe wall. As can be observed, the proposed model predicts that when the flow improver is injected into a point very near the tube wall, the increase of flow at constant pressure (fig 4b) and the decrease in pressure drop (fig 5b) at constant flow rate, are both remarkable compared to the injection into the tube centerline, which would be expected due to the interaction with the wall pipeline reducing friction effects.

It can be noticed that the pressure drop changes almost instantly when fluid is injected near the wall periphery, unlike the case of injection into tube centerline, where it takes much longer to stabilize. In this case, it is evident the enhanced effect of the flow improver when it is injected into a point near the tube wall. These results agree with the observed experimental behavior obtained with water (Abarasi, H. 2014; Livinus, A. *et al*. 2016). Likewise, the time response to the increase in flow rate is much faster for the case of injection at the pipe wall.

When injection is effected in the annular region, the time required to reach the steady state is practically instantaneous which can be expected in a system with a higher velocity. On the other hand, for the case of injection at the center of the pipe, the velocity increases slowly with time at the injection point, and therefore the flow rate increases slowly.

These results allow to establish that for the transport of high viscosity fluids, such as heavy and extra heavy crude oils, the use of non-mixing diluents with the crude leads to flow improvement, mainly if they are injected at the pipe wall.

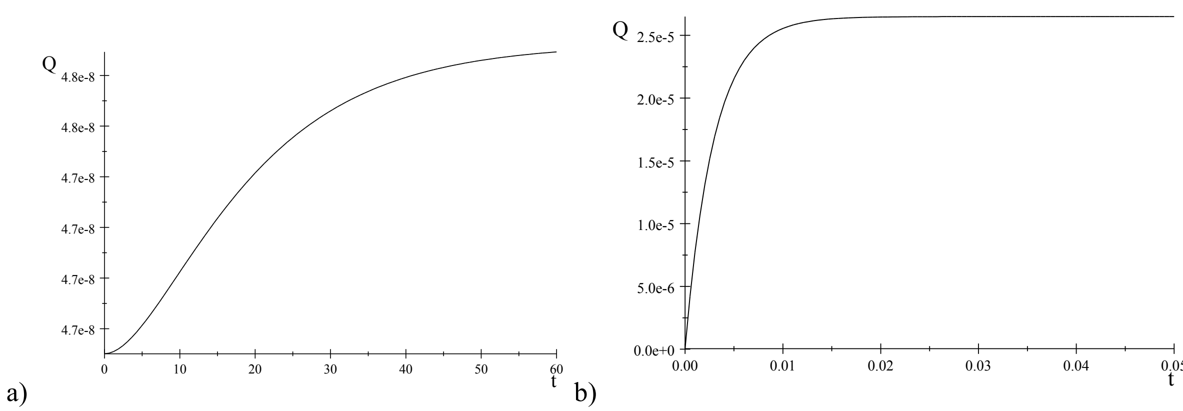

Fig. 4. Flow temporal behavior when the fluid is injected at constant pressure into the a) center tube and b)near the tube wall.

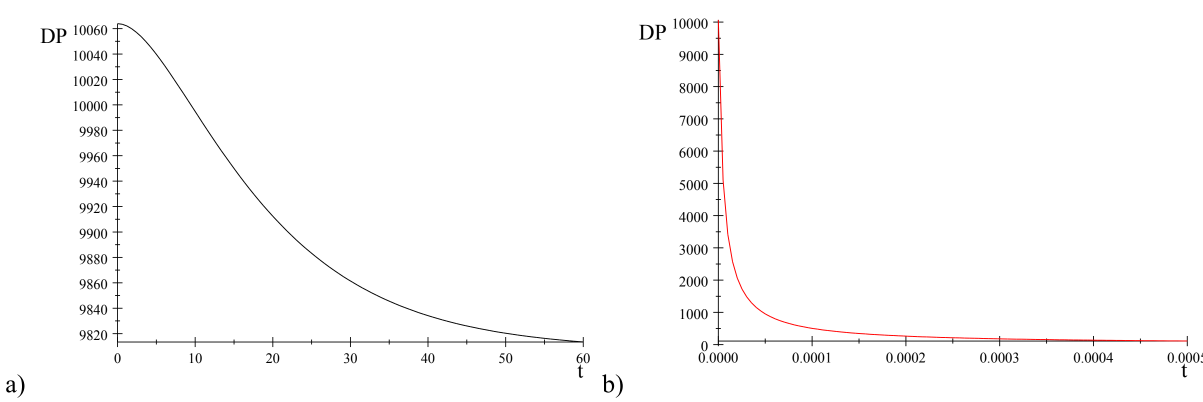

Fig. 5.a. Temporal Behavior of the pressure drop at constant flow ($Q = 4.7 \times 10^{-8}$ $m^3 \cdot s^{-1}$) for a) injection in tube center and b) tube wall.

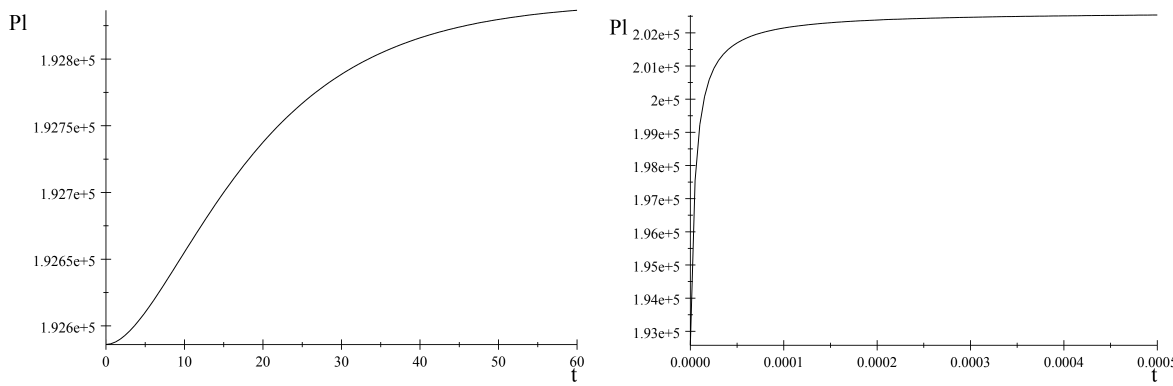

Fig. 5.b. Behavior of the discharge pressure with respect to time when the fluid is injected into the tube center (left) and in the pipe wall (right).

Conclusions

A mathematical model to predict both the pressure drop at constant flow rate and the flow evolution at constant pressure drop of a liquid-liquid flow in non-steady state conditions is proposed in this paper. This model was applied to study the optimal injection point of a flow improver for heavy oil pipeline transport. Two cases were analyzed, one where the injection is effected near the tube wall and another where the injection point is at the tube centerline. The predicted results showed the enhanced effect of the flow improver when it is injected into a point near the tube wall, both quantitatively as well as in terms of time response. These results suggest that in practice it will suffice to employ lower dosages of flow improver to attain a required oil crude flow, when it is injected through the annular region.

One limitation of the present model is that the flow improver remains along the injection position, that is through the pipe centerline or through the annular region, which means that it cannot predict the progression of the flow improver from the centerline to the wall of the pipe or vice versa.

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Symbols

L length of the pipe, m
p pressure, Pa
P pressure in a time t, Pa
ΔP pressure drop, Pa
q velocity in unsteady state, ms^{-1}
Q flow rate, m^3s^{-1}
r radial distance of the pipe, m
R total radius of the pipe, m
t time, s
v velocity, ms^{-1}
V velocity at steady state, ms^{-1}
y volumetric fraction
z longitudinal coordinate
α dimensionless interface position between both flows
β, Φ parameters
γ dimensionless radius
ρ density, kgm^{-3}
τ component of shear stress

References

Abarasi, H. (2014). A review of technologies for transporting heavy crude oil and bitumen via pipelines. *Journal of Petroleum Exploration and Production Technology 4*, 327-336.

Abubakar, A., Al-Wahaibi, Y., Al-Wahaibi, T., Al-Hashmi, A. A., Al-Ajmi, A., and Eshrati, M. (2016). Effect of water-soluble drag-reducing polymer on flow patterns and pressure gradients of oil/water flow in horizontal and upward-inclined pipes. *SPE Journal*, 1-14.

Argillier, J., Henaut, I., Heraud, J.P. and Glenat, P. (2005). Heavy oil dilution. In *SPE International Thermal Operations and Heavy Oil Symposium*. Society of Petroleum Engineers.

Bensakhria, A., Peysson, Y. and Antonini, G. (2004) Experimental study of the pipeline lubrication for heavy oil transport. *Oil & Gas Science Technology 59*, 523-533

Bird, R.B., Stewart, W.E. and Lightfoot, E.N. (2002). *Transport Phenomena*. Jonh Wiley & Sons, 2 Ed. New York.

Brauner, N., Rovinsky, J. and Maron, D. M. (1996). Analytical solution for laminar-laminar two-phase stratified flow in circular conduits. *Chemical Engineering Communications 141*, 103-143.

Edomwonyi-Otu, L. C., Chinaud, M., and Angeli, P. (2015). Effect of drag reducing polymer on horizontal liquid-liquid flows. *Experimental Thermal and Fluid Science 64*, 164-174.

Gateau, P., Hénaut, I., Barré, L., and Argillier, J. F. (2004). Heavy oil dilution. *Oil & Gas Science and Technology 59*, 503-509

González-Dávila, V. ,Flores-López, J., González-Dávila, J.L., Palacio-Pérez, A., Izquierdo-Kulich, E.F. and Suárez-Domínguez, E.J. (2015). Optimizing oil production through use of a viscosity bio reducer. Presentation number PAMFC15-115 January, 22. Veracruz, Ver.: *Pan American Mature Field Congress*.

Hewitt, G. F., Delhaye, J. M. and Zuber, N. (2013). *Multiphase Science and Technology*. V.2. Springer Science & Business Media.

Huang, Z., Lee, H.S., Senra, M. and Fogler, H. S. (2011). A fundamental model of wax deposition in subsea oil pipelines. *AlChE Journal 57*, 2955-2964.

Imbert-Gonzalez, J., Garcia-Valladares, O. and Guillen-Gordin, R. (2016). Hydrodynamic analysis of a turbulent flow in an annular space with inserted wire spiral. *Revista Mexicana de Ingenieria Química 15*, 149-159.

Ismail, A. S., Ismail, I., Zoveidavianpoor, M., Mohsin, R., Piroozian, A., Misnan, M. S., and Sariman, M. Z. (2015). Experimental investigation of oil-water two-phase flow in horizontal pipes: Pressure losses, liquid holdup and flow patterns. *Journal of Petroleum Science and Engineering 127*, 409-420.

Joseph, D. D., Bai, R., Chen, K. P., and Renardy, Y. Y. (1997). Core-annular flows. *Annual Review of Fluid Mechanics 29*, 65-90.

Livinus, A., Yeung, H. and Lao, L. (2016). Restart time correlation for core annular flow in pipeline lubrication of high-viscous oil. *Journal of Petroleum Exploration and Production Technology*, 1-10.

Loh, W. L. and Premanadhan, V. K. (2016). Experimental investigation of viscous oil-water flows in pipeline. *Journal of Petroleum Science and Engineering 147*, 87-97.

Manning, W. and Lind, G. (2014). Data analysis and Discussion of Product Interface Size on a Batched Crude Oil Pipeline, conference, 6-9 May, Baltimore, Maryland, USA: *PSIG Annual Meeting Pipeline Simulation Interest Group*.

Martínez-Palou, R., De Lourdes Mosqueira, M., Zapata-Rendón, B., Mar-Juárez, E., Bernal-Huicochea, C., De la Cruz Clavel-López, J and Aburto, J. (2011). Transportation of heavy and extra-heavy crude oil by pipeline: A review. *Journal of Petroleum Science and Engineering 75*, 274-282.

Mukhaimer, A., Al-Sarkhi, A., El Nakla, M., Ahmed, W. H. and Al-Hadhrami, L. (2015). Effect of water salinity on flow pattern and pressure drop in oil-water flow. *Journal of Petroleum Science and Engineering 128*, 145-149.

Oliemans RVA and Ooms G. (1986). Core-annular flow of oil and water through a pipeline. *Multiphase Science and Technology 2*, 1-4.

Palacio-Pérez, A. and González-Dávila, V. (2014) Reporte de proyecto SENER-CONACYT. Demanda específica D2/CH2011-01 Proyecto No 166923. México, D.F.

Panuganti, S.R. (2013). Asphaltene Behavior in Crude Oil Systems. Engineering thesis, director: Walter G. Chapman. Rice University. Houston, Tx.

Santos, R.G., Loh, W., Bannwart, A.C. and Trevisan, O.V. (2014). An overview of heavy oil properties and its recovery and transportation methods. *Brazilian Journal of Chemical Engineering 3*, 571-590.

Silva, R. de C., Almeida, D.G., Rufino, R.D. Luna, J.M., Santos, V. A. and Sarubbo, L. A. (2014). Applications of biosurfactants in the petroleum industry and the remediation of oil spills. *International Journal of Molecular Science 15*, 12523-12542

Suárez-Domínguez, E.J., Palacio-Pérez, A., Rodríguez Contreras, A. and Izquierdo-Kulich (2014). Influencia del biorreductor de viscosidad en el gradiente de presión en un ducto horizontal que transporta fluido no newtoniano. *Revista Cubana de Ingeniería 5*, 45-50.

Suárez-Domínguez, E.J., Palacio-Pérez, A., Rodríguez Contreras, A. and Izquierdo-Kulich (2016). Análisis de difusividad molecular de un reductor de viscosidad en crudo extrapesado. La Habana, Cuba: *Cuban Chemical Society Congress*.

Subramanian, D., Wu, K., and Firoozabadi, A. (2015). Ionic liquids as viscosity modifiers for heavy and extra-heavy crude oils. *Fuel 143*, 519-526.

Tharanivasan, A.K. (2012). Asphaltene Precipitation from Crude Oil Blends, Conventional Oils, and Oils with Emulsified Water. Doctoral Thesis. University of Calgary: Faculty of Graduate Studies. 211pp.

Tian, Z., Jin, W., Wang, L. and Jin, Z. (2014). The study of temperature profile inside wax deposition layer of waxy crude oil in pipeline. *Frontiers in Heat and Mass Transfer 5*, 1-8.

Toms, B.A. (1948). Some observation on the flow of linear polymer solution through straight tubes at large Reynolds number. In: *Proceedings of the 1st International Congress on Rheology*, vol.2. North Holland, Amsterdam, The Netherlands, 135-141.

Annex

Solution of equation system in steady state:

$$\begin{aligned} 0 &= \Phi + \mu_1 \frac{1}{\gamma}\frac{dV_1}{d\gamma} + \mu_1 \frac{d^2V_1}{d\gamma^2} \\ 0 &= \Phi + \mu_2 \frac{1}{\gamma}\frac{dV_2}{d\gamma} + \mu_2 \frac{d^2V_2}{d\gamma^2} \end{aligned}$$

Solution is given by:

$$\begin{aligned} V_1 &= C_{11} + C_{12}\ln\gamma - \frac{1}{4}\Phi\frac{\gamma^2}{\mu_1} \\ V_2 &= C_{21} + C_{22}\ln\gamma - \frac{1}{4}\Phi\frac{\gamma^2}{\mu_2} \end{aligned}$$

with the boundary conditions and solutions:

$$\begin{aligned} V_2(0) &= finite \\ C_{22} &= 0 \end{aligned}$$

$$\begin{aligned} \mu_1\left(\frac{dV_1}{d\gamma}\right)_{\gamma=\alpha} &= \mu_2\left(\frac{dV_2}{d\gamma}\right)_{\gamma=\alpha} \\ \frac{1}{\alpha}\mu_1 C_{12} - \frac{1}{2}\Phi\alpha &= \frac{1}{\alpha}\mu_2 C_{22} - \frac{1}{2}\Phi\alpha \\ \frac{1}{\alpha}\mu_1 C_{12} - \frac{1}{2}\Phi\alpha &= \frac{1}{\alpha}\mu_2(0) - \frac{1}{2}\Phi\alpha \\ C_{12} &= 0 \end{aligned}$$

$$\begin{aligned} C_{11} + (0)\ln\alpha - \frac{1}{4}\Phi\frac{\alpha^2}{\mu_1} &= C_{21} + (0)\ln\alpha - \frac{1}{4}\Phi\frac{\alpha^2}{\mu_2} \\ C_{11} + (0)\ln 1 - \frac{1}{4}\Phi\frac{1^2}{\mu_1} &= 0 \end{aligned}$$

where:

$$\begin{aligned} C_{11} &= \frac{1}{4}\frac{\Phi}{\mu_1} \\ C_{21} &= \frac{1}{4}\frac{\Phi}{\mu_1} - \frac{1}{4}\Phi\frac{\alpha^2}{\mu_1} + \frac{1}{4}\Phi\frac{\alpha^2}{\mu_2} \end{aligned}$$