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Universidad Nacional de Colombia  
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Centeno González, Luz María; Castillo Castañeda, Eduardo; Becerra Santiago, Luis Omar; Rochín  
García, Alberto

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# A magnetic suspension system for measuring liquid density

## Desarrollo de un sistema de suspensión magnética para la determinación de densidad de líquidos

L. M. Centeno<sup>1</sup>, E. Castillo<sup>2</sup>, L. O. Becerra<sup>3</sup>, A. Rochin<sup>4</sup>

### ABSTRACT

Density is a derived quantity of mass and length; it is defined as mass per volume unit and its SI unit is  $\text{kg/m}^3$ . National metrology institutes have been designing and building their own magnetic suspension systems during the last 5 decades for making fluid density measurements; this has allowed them to carry out research into liquids and gases' physical characteristics. This paper was aimed at designing and developing a magnetic suspension system for a magnetic balance used in determining liquid density to be used in CENAM's metrology density laboratories.

**Keywords:** Hydrostatic weighing, density, magnetic suspension.

### RESUMEN

La densidad es una magnitud derivada de las magnitudes de base masa y longitud, está definida como masa por unidad de volumen y su unidad en el Sistema Internacional de Unidades (SI) es el  $\text{kg/m}^3$ . En las últimas cinco décadas, en el ámbito de la metrología primaria, se han desarrollado sistemas de suspensión magnética para la determinación de densidad de fluidos, lo que ha permitido realizar investigaciones sobre las características físicas de líquidos y gases. El presente trabajo tiene como objetivo el desarrollo y diseño de un sistema de suspensión magnética que será utilizado en una balanza de suspensión magnética para determinar la densidad de líquidos en los laboratorios de metrología de densidad, en el Centro Nacional de Metrología (Cenam).

**Palabras clave:** densidad, suspensión magnética, fuerza magnética.

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### Introduction

The magnetic suspension system has been studied and developed by authors such as Hurley, Ziehlmann and Mizuno. The common factor in their works is that they have followed a basic type of system consisting of an electromagnet, a ferromagnetic body, a position detection system and a closed loop control, as shown in Figure 1 (Hurley, 1997; Ziehlmann, 2000).

A variant of the previous system may be obtained by placing a permanent magnet which interacts with the electromagnet; the force needed by the electromagnet to levitate an object thus becomes reduced, not necessarily being a ferromagnetic one [Kleinrahm, 1986]. This variant minimises electromagnet power

consumption, which is reflected in the heat so generated and, thereby, in low variation of its current. The latter system configuration is widely used in the field of density metrology.

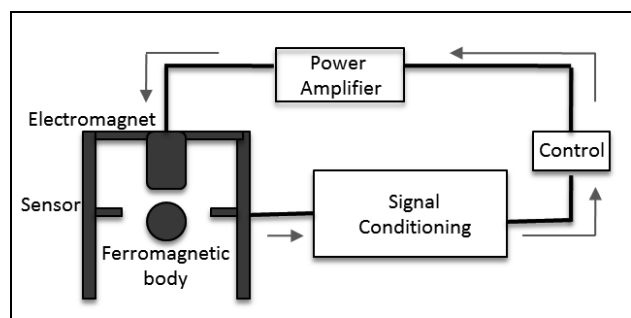


Figure 1. Basic magnetic suspension system

There are many ways of using the magnetic levitation in different kinds of measurement systems; another configuration proposed for density metrology has two permanent magnets and capillary tubes, and sample density is obtained as a function of a levitated object's height (Mirica K. A., 2009).

This paper presents a design for and stability analysis of a magnetic levitation system; it will become part of a complete measurement system for measuring liquid density. Stability regarding mass measurement is thus very important, varying by no more than 1 mg, since the standard deviation of measurements repre-

<sup>1</sup> Luz María Centeno González. M.Sc. Tecnología Avanzada, CICATA, Instituto Politécnico Nacional. Affiliation: Centro Nacional de Metrología, México. E-mail: lcenteno@cenam.mx.

<sup>2</sup> Dr. Eduardo Castillo Castañeda. Dr. in Automatic Control, Institut National Polytechnique de Grenoble. France. Affiliation: CICATA, Instituto Politécnico Nacional, México. E-mail: ecastillo@ipn.mx.

<sup>3</sup> Luis Omar Becerra Santiago. M.Sc. Diseño Mecánico, Instituto Tecnológico de Celaya, Gto. México. Affiliation: Centro Nacional de Metrología, México. E-mail: lbecerra@cenam.mx.

<sup>4</sup> Alberto Rochin García. M. en C. en Electrónica, INAOE. Puebla. México. Affiliation: Facultad de Filosofía, UNAM, México. E-mail: rochin\_alberto@hotmail.com.

sents the second largest contribution to liquid density uncertainty budget (Becerra L, 1998), i.e. on the proposed system, determined by the levitated object's position stability.

## Magnetic suspension systems in determining liquid density

The reference object (solid density standard) in a hydrostatic weighing system is connected to the balance by a wire (through a thread passing over the surface of the liquid to be measured). The reference object's apparent weight loss indicates displaced liquid mass (liquid buoyancy). The force acting on the balance is a measure of the mass of the reference object. If the connecting wire were removed, then the reference object would float or sink freely in the liquid, depending on its density. If a magnetic field were generated to maintain the reference object at a defined position (height), a magnetic suspension system could be obtained (Figure 2).

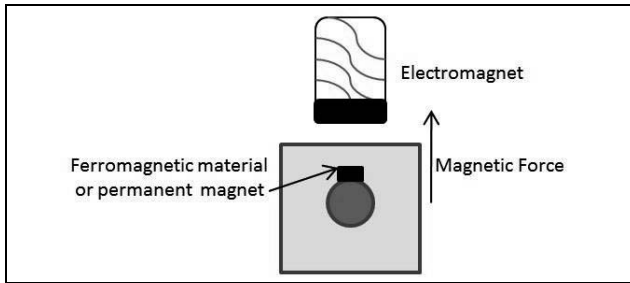


Fig. 2 Basic diagram of thrust by magnetic force

where, if just the vertical forces are considered, and a magnetic force is acting, the balance condition is (Bignell, 2006):

$$g\rho_L V_s - M_s g + F_{mag} = 0 \quad (1)$$

( $V_s$ ) is reference object volume, ( $\rho_L$ ) is the density of the liquid to be measured, ( $M_s$ ) is reference object mass, ( $g$ ) is local gravity acceleration and ( $F_{mag}$ ) is the magnetic force needed to levitate the reference object.

As  $\rho_s$  is reference object density (Bignell, 2006), the magnetic force could be obtained from:

$$F_{mag} = V_s g (\rho_s - \rho_L) \quad (2)$$

A magnetic coupling is used in such magnetic systems consisting of an electromagnet and a permanent magnet (Kleinrahn, 1986), where the first is hung directly from the balance and the second one is located inside a load cell. The reference object is fixed to the permanent magnet by means of a coupling and decoupling device. Figure 3 gives a general scheme of a liquid density measurement system involving magnetic suspension.

## Design requirements

The main uncertainty contributions for density determination of liquids in a conventional hydrostatic weighing system are solid standard volume and measurement variability (Becerra L, 1998; Gupta, 2002). The latter is represented by standard deviation of the measurements taken directly from the balance.

Liquid density, using a magnetic levitation density meter, is measured by using the Archimedes' principle; however, a comparison is made with and without reference object levitation instead of establishing two equations between a reference and a submerged object (Kano, 2006).

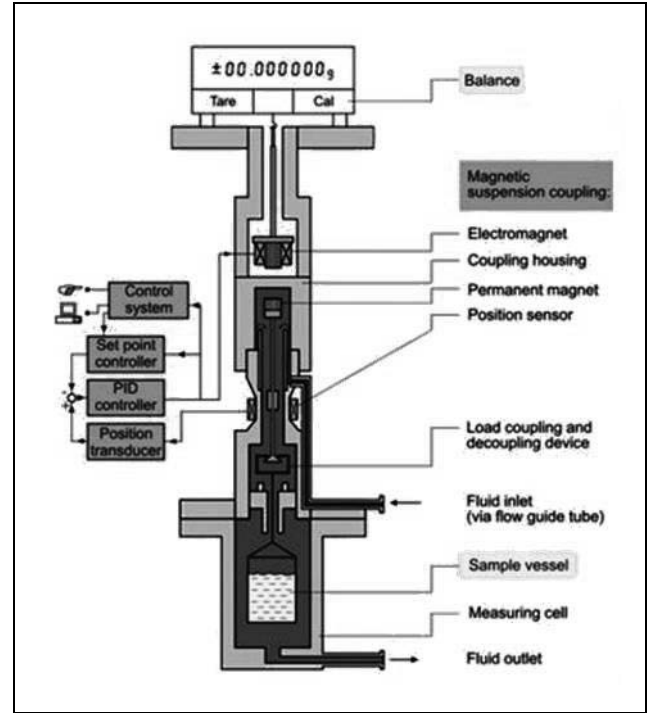


Fig. 3 Basic diagram of a magnetic suspension balance system<sup>5</sup>

The following equation can be established using forces equilibrium for the tare weighting:

$$M_{TP} = M_{sus} - \rho_L V_{sus} \quad (3)$$

where  $M_{sus}$  and  $V_{sus}$  are the subjection systems' mass and reference object volume and  $\rho_L$  is liquid density.

A second equation is established taking into account the hanging object by means of the suspension system and the magnetic coupling:

$$M_{MP} = (M_{sus} + M_s) - \rho_L (V_{sus} + V_s) \quad (4)$$

The density of the liquid being tested can be obtained by the next formula obtained from equations 3 and 4:

$$\rho_L = \frac{M_s}{V_s} - \frac{M_{MP} - M_{TP}}{V_s} \quad (5)$$

There are two possibilities for evaluating the degree of dependency between input quantities regarding measured liquid density; a numerical simulation (JCGM: 101, 2008) may be done or partial derivatives of liquid density may be calculated regarding each input quantity (JCGM:100, 2008) and the data then being organised in a table as the uncertainty budget. The input quantities diagram is shown in Figure 4, where  $\rho_L$  is liquid density,  $M_s$  and  $V_s$  are reference object mass and volume (solid density standard),  $\Delta L$  is the difference between balance readings for  $M_{TP}$  and  $M_{uss}$ ,  $M_{TP}$  is the balance reading in tare position,  $M_{sus}$  is the balance reading where the reference object is hung up,  $F_{mag}$  is the necessary magnetic force for hanging up the solid,  $i$  is the current applied to the coil and  $h$  is the height difference between

<sup>5</sup> Image obtained from de Rubotherm, www.rubotherm.com, manufacturer of magnetic suspension balances

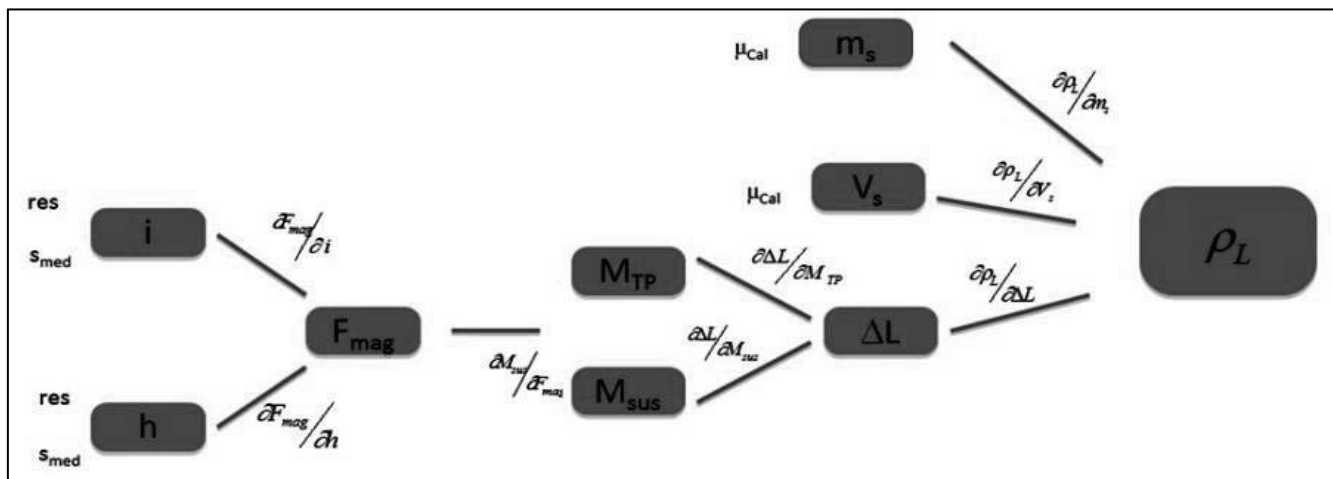


Figure 4. Input quantity diagram

Table 1 Uncertainty budget for the density of a liquid

	Nominal value	Variability	Coefficient $\frac{\partial \rho_L}{\partial x_i}$	Standard Uncertainty	Product $\frac{\partial \rho_L}{\partial x_i} u_{x_i}$	Variance $\left(\frac{\partial \rho_L}{\partial x_i} u_{x_i}\right)^2$	%
<b>Ms</b> (g)	25	0.000000044	0.9999949	0.000000044	4.39998E-08	1.936E-15	1.51E-06
<b>Vs</b> (cm <sup>3</sup> )	10.000515	0.0006	0.1	0.00060	6E-05	3.6E-09	2.80
<b>ΔL</b> (g)	15.000164	0.001, 8 med.	-0.999995	3.54E-04	-0.000353552	1.25E-07	97.20
<b>Inc ρ<sub>L</sub> 0.000359g/cm<sup>3</sup></b>							

the coil core and the hanging reference object when submerged in the liquid.

Table 1 gives the uncertainty budget (JCGM: 100, 2008) for determining liquid density.

Table 1 shows that  $\Delta L$  represents 97.20% of total uncertainty regarding liquid density, this being due to the standard deviation of the balance readings. Levitation system design should thus be focused so that balance reading standard deviation does not affect liquid density uncertainty by more than  $2 \times 10^{-4}$  g/cm<sup>3</sup>.

$\Delta L$  is influenced by two input quantities: balance resolution and balance reading repeatability. Furthermore, repeatability depends on the stability of the magnetic force generated by the electromagnet and the stability of levitation determined by control position performance. The magnetic force could be calculated by the following formula (Moudgalya, 2007):

$$F_{mag} = K \frac{i^2}{h}$$

Assuming 1 A maximum current ( $i$ ), the object to be hung up positioned 1 cm from the coil core ( $h$ ) and 10  $\mu$ m variability at this height, then the uncertainty budget will be as follows:

Table 2. Magnetic force uncertainty budget

	Nominal value	Variability	Inc	Sensitivity Coefficient	Product	Variance	%
<b>i</b> (A)	1	100x10 <sup>-6</sup> , rectangular	2.887 x10 <sup>-5</sup>	0.202	5.831 x10 <sup>-6</sup>	3.4 x10 <sup>-11</sup>	4
<b>h</b> (m)	0.01	10x10 <sup>-6</sup> , rectangular	2.887 x10 <sup>-6</sup>	-10.201	-2.945 x10 <sup>-5</sup>	8.672 x10 <sup>-10</sup>	9
<b>Inc (k=1) 3.002x10<sup>-05</sup>N</b>							

Table 2 shows that taking 100  $\mu$ A current variability and 10  $\mu$ m height, height variability is the magnetic force's largest contribution to final uncertainty, so it can be stated that the greatest height variability will be magnetic force variability and therefore the largest uncertainty in measurement repeatability. Height ( $h$ ) would thus need to be measured with at least 10  $\mu$ m resolution, assuming the largest contribution of uncertainty in this kind of measurement would be the resolution of the instrument used to measure the height. The levitation system controller should provide better than or equal to 10  $\mu$ m height position stability.

## Prototype

Figure 3 shows a complete magnetic suspension balance system. The suspension system would consist of an electromagnet, a permanent magnet, a positioning system and its corresponding controller.

## Coil design

The number of coil turns would be given by (Resnick, 2004; Eisberg, 1982):

$$N^2 = \frac{F_{mag}}{I \mu_0 2\pi r^2} \quad (6)$$

The behaviour of the magnetic field generated by the coil must be understood to complete the coil design; the magnetic field may be described mathematically by means of Biot-Savart's law (Eisberg, 1982):

$$B = \frac{\mu_0 I r^2}{2} \frac{1}{(z^2 + r^2)^{3/2}} \quad (7)$$

$z$  is the vertical distance above the  $z$  axis from the base of the coil and  $r$  is coil radius. Figure 5 gives a simulation of the field's behaviour with different currents according to equation (7).

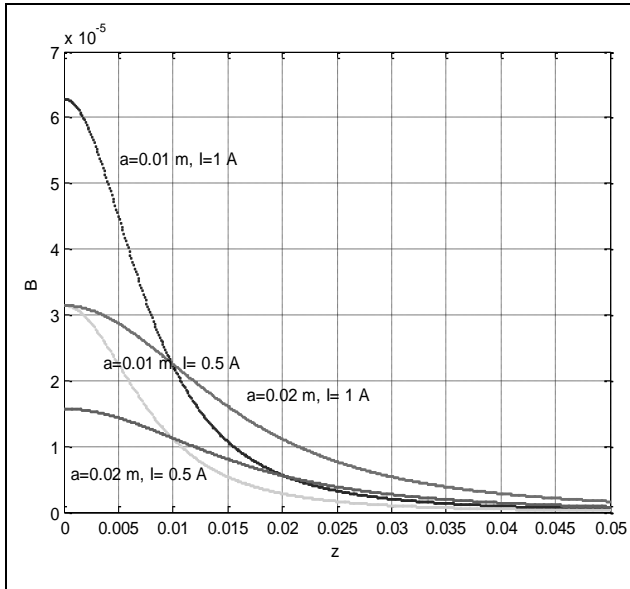


Figure 5. Magnetic field behavior regarding a distance in  $z$

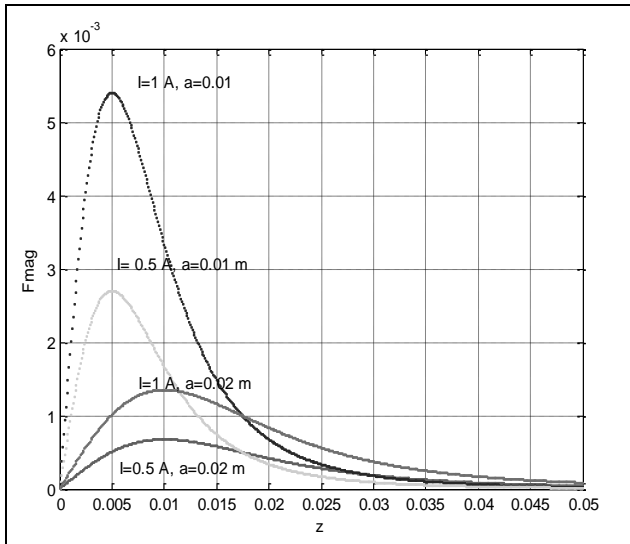


Figure 6. Magnetic force regarding a distance in  $z$

The force acts on magnet moment  $m$ , in the presence of a magnetic field  $B$ ; this force is proportional to  $m$  times  $B$ . For a core of air in a coil,  $B$  is proportional to current  $I$  in the coil and for radius  $r$ , the force is given by (Bignell, 2006):

$$F_{mag} = mB(z, r) = \frac{-3/2 \mu_0 I z r^2}{(z^2 + r^2)^{5/2}} m \quad (8)$$

where,  $B(z, r)$  is the partial derivative regarding  $z$  from equation (7). Figure 6 shows a simulation of magnetic force with different current values and coil radius.

Figure 6 shows that the largest magnetic force would be obtained 5 mm from the origin of the field by applying 1 A current to a coil having 0.01 m radius. Applying equation (6), a 1,600 turn coil can be calculated for hanging up a mass of around 20 g.

### Positioning sensor

Sensor selection is important since it provides feedback for the control system; it thus needs resolution having the desired position stability, as well as a suitable response time. The sensor should be contactless with around 10  $\mu\text{m}$  resolution.

The above led to a PSD sensor being proposed because it is an economical choice, having up to 1  $\mu\text{m}$  resolution and excellent time response (UDT Sensors Inc, 1982).

### Characterising the positioning sensor

Positional detectors (PSD) are silicon photodiodes providing a direct analogue output proportional to the position of the light over an active area (Figure 7):

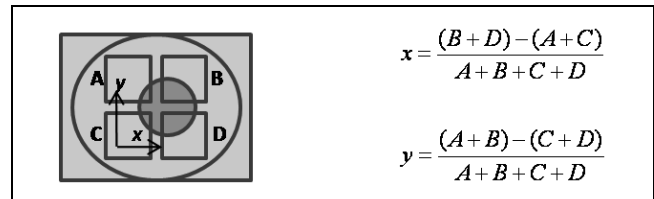


Fig. 7 Characteristics of a four-quadrant PSD sensor

The position would be measured by total cell illumination and a light beam interrupting means involving an anodised board having a 0.5 mm wide slot. Figure 8 shows optical system assembly. Scanning every 50  $\mu\text{m}$  (680 nm wave length laser) from top to bottom and vice versa characterised the PSD's optical area (Figure 9); the response was not totally linear.

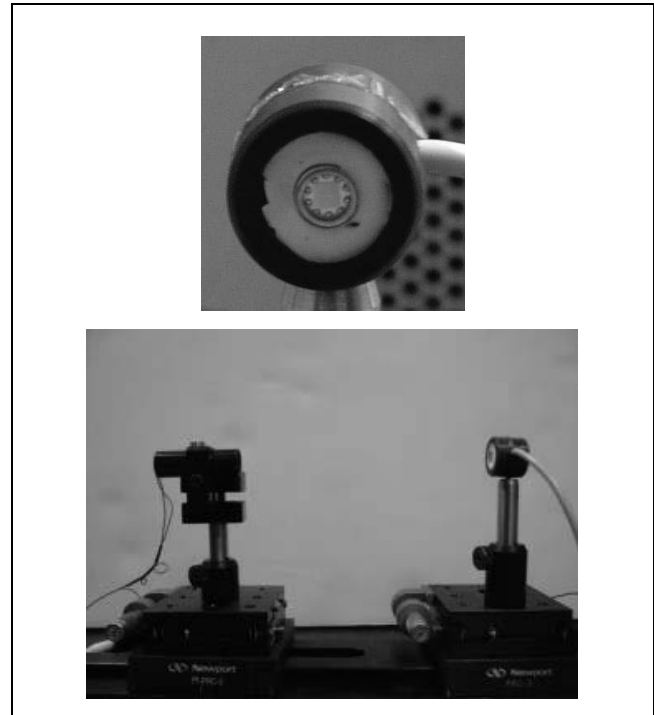


Figure 8. Optical sensor PSD

A fitting curve was calculated to determine the working area, changing the working interval and thus finding the highest factor ( $R^2 = 1$ ), besides trying to exploit the largest work interval.

Table 3 shows that the best fitting linear area came within the - 500  $\mu\text{m}$  to 400  $\mu\text{m}$  interval; a 1 mV/10  $\mu\text{m}$  resolution was ob-

tained in this interval. This resolution was appropriate according to the results of the sensitivity stability study for the system.

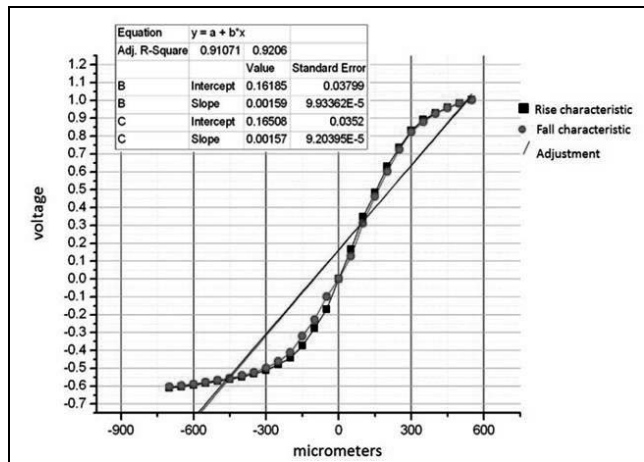


Figure 9. PSD sensor response

Table 3 R factor for different fitting curves

Interval ( $\mu\text{m}$ )	R <sup>2</sup>
-1,500 a 1,500	0.88307
-1,300 a 1,300	0.88831
-450 a 550	0.92060
-500 a 400	0.94366

### Dynamic system and PID control

The electric representation of a magnetic suspension system is shown in Figure 10; the position of an object to be levitated was identified by an optical system where the signal was used as feedback for the control system. Coil inductance varied with the object's position; the current passing through the coil could change the magnetic force generated by the coil, thus allowing the object to fluctuate when being levitated. In Figure 10,  $i(t)$  is the circuit's current,  $x(t)$  is the object's displacement produced by means of the electromagnet and  $x_0$  is the desired position.

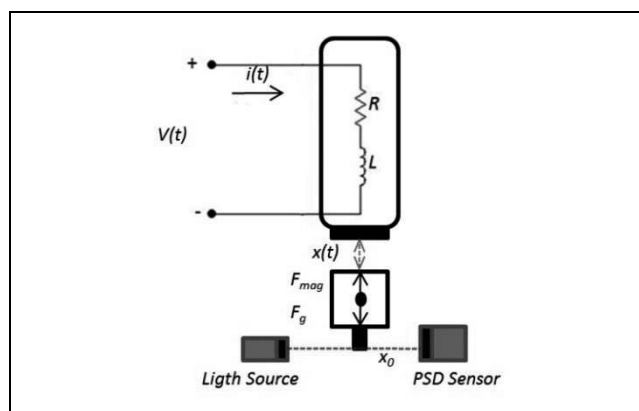


Figure 10. Electro-mechanical system for magnetic suspension

If the electric system was energised by a step voltage in an open loop, the response would be immediate and the permanent magnet absolutely attracted to the coil. So, a Ziegler Nichols method for tuning the PID control was used as a first approach to the control system.

A value of 6 and 25 ms  $P_{cr}$  was obtained by varying  $K_p$  until obtaining a response with sustained oscillations (like that shown in Figure 11).  $K_{cr}$  was then calculated by applying the Ziegler-Nichols rules [Ogata, 2002] and  $K_{pr} = 3.6$  and  $T_d = 3.125 \times 10^{-5}$  were deduced. Plant stability was around 0.03 seconds by applying constants  $K_p$ ,  $K_d$  and  $K_i$  to the control system (Figure 12).

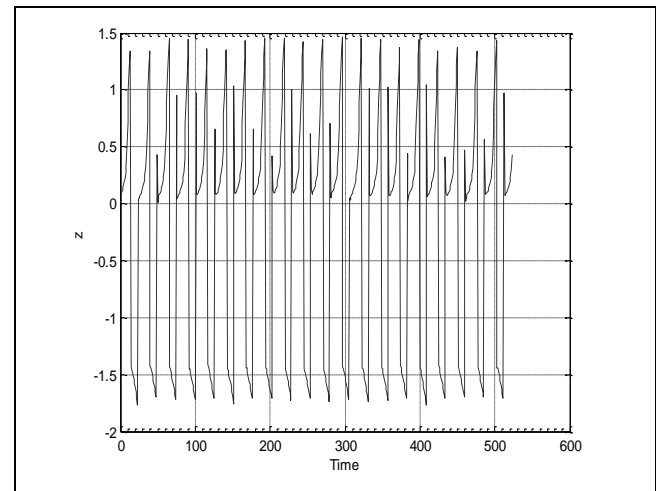


Figure 11. Real system response,  $K_{cr}=6$

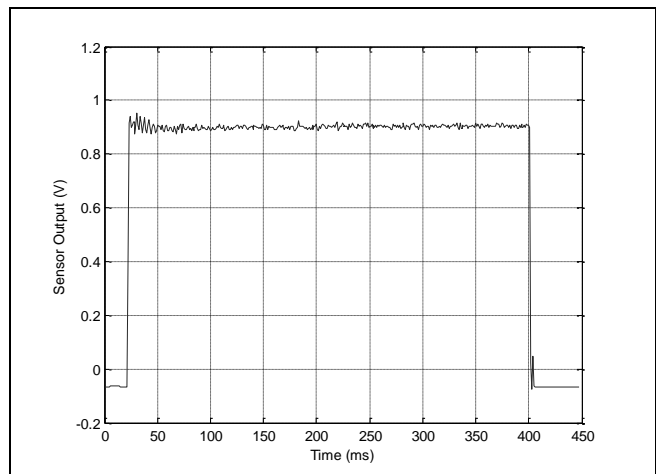


Figure 12. Tuned system response

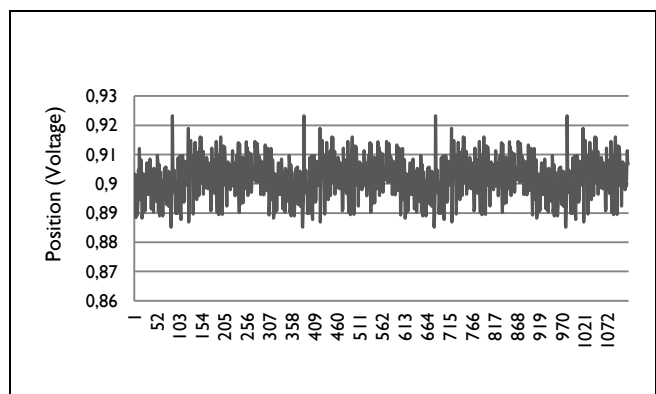


Figure 13. Magnetic suspension system stability

A 10 mV standard deviation was calculated after this response time. Figure 13 shows 3 to 6 second sensor exit after system stability.

A  $\pm 10 \mu\text{m}$  variation led to a 10 mV standard deviation which, according to the system sensitivity study, was enough to reach  $2 \times 10^{-4}$  relative uncertainty when measuring liquid density. However, such uncertainty was not enough for the required uncertainty for density reference materials.

Knowing the magnetic suspension system's transfer function led to optimising the control loop. The system's transfer function was obtained by the least minimum squares' technique.

The magnetic suspension system's second order equation was stated as:

$$\begin{aligned} y(k) - 1.5104y(k-1) + 0.6936y(k-2) \\ = -0.0111u(k-1) \\ + 0.0221u(k-2) \end{aligned} \quad (9)$$

Control loop design will form part of future work in this field and such design will involve using zero power control and the aforementioned function, seeking better stability.

## Conclusions

CENAM's design for a magnetic suspension system has been presented here. This magnetic suspension system was developed in the Density Laboratory as part of a project for designing and constructing a liquid density measurement system involving magnetic coupling (Figure 14). This design led to  $\pm 10 \mu\text{m}$  suspension stability which could be reached along the z axis, representing  $\pm 5 \text{ mg}$  mass stability and up to  $5 \times 10^{-4}$  relative uncertainty when measuring liquid density (without taking possible corrections into account due to magnetic fields in the system's materials). The magnetic suspension system had all the requirements for designing a liquid cell, which is the next step to continue developing a liquid density measurement system by means of magnetic suspension. Controlling the magnetic suspension system must be improved by using with the transfer function so obtained.

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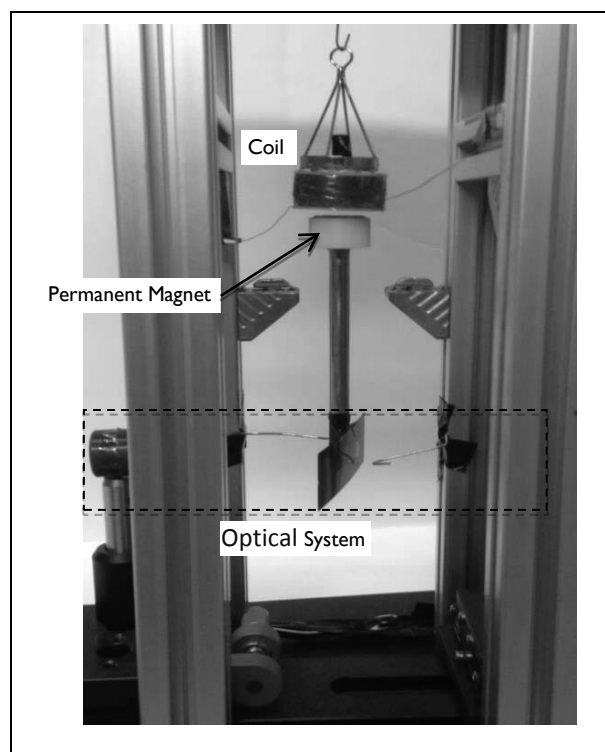


Fig. 14 Magnetic suspension system

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