A descriptive analysis of quantitative indices for multi-objective block layout

Análisis descriptivo de los indicadores cuantitativos para el problema multiobjetivo del block layout

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ABSTRACT

Layout generation methods provide alternative solutions whose feasibility and quality must be evaluated. Indices must be used to distinguish the feasible solutions (involving different criteria) obtained for block layout to identify s solution's suitability, according to set objectives. This paper provides an accurate and descriptive analysis of the geometric indices used in designing facility layout (during block layout phase). The indices studied here have advantages and disadvantages which should be considered by an analyst before attempting to resolve the facility layout problem. New equations are proposed for measuring geometric indices. The analysis revealed redundant indices and that a minimum number of indices covering overall quality criteria may be used when selecting alternative solutions.

Keywords: Plant layout index, quantitative index, block layout, multi-objective evaluation.

RESUMEN

Los métodos de generación de layouts proporcionan soluciones alternativas que deben ser evaluadas para verificar su factibilidad y calidad. Para poder tomar decisiones sobre su idoneidad, atendiendo a diferentes objetivos, se hace necesario el uso de indicadores que permitan distinguir, las mejores soluciones obtenidas en la fase de diagrama de bloques. En la presente contribución, se realiza una labor de síntesis y análisis descriptivo de los diferentes indicadores aportados por numerosos autores. El análisis deja en evidencia las ventajas y desventajas de cada uno de los indicadores, que deben ser consideradas por el analista antes de la aplicación. Adicionalmente se plantean nuevas formas de cálculo para indicadores de configuración de tipo geométrico. Los resultados del análisis muestran que existen indicadores redundantes y es posible seleccionar un conjunto de indicadores independientes y suficientes, de tal forma que se cumpla con los criterios generales de calidad para la selección de alternativas de solución.

Palabras clave: Indicadores de distribución en planta, indicadores cualitativos, layout de bloques, evaluación multiobjetivo.

Received: July 5th 2011
Accepted: March 6th 2012

Introduction

The facility layout problem (FLP) represents a current issue from the multi-objective point of view for some researchers around the world. This perspective includes fresh solutions which must be compared to obtain the optimum one; such indices should thus be considered in the objective function to be used in optimisation.

Muther (1968) set out systematic layout planning objectives for good industrial plant layout design covering seven principles: overall integration, minimum distance moved, minimum flow, satisfaction and safety, cubic space and flexibility. These principles have been generally accepted and have been reiterated by Apple (1968), Moore (1971) and Francis and White (1974). Some quantitative indices can be found in the literature for each principle (except movement, safety and satisfaction) (Table 1). Redundancy must thus be detected and the range of options must be analysed for suitable indices to become selected.

The indices used by authors for block layout can be classified into qualitative (obtained through expert judgment) and quantitative (measured physically or geometrically).

Although qualitative indices are relevant, this article focuses on quantitative indices as these can be directly obtained from alternative solutions' spatial characteristics, thereby allowing a solution's quality to become known and current solutions improved through heuristics.

The material handling cost (MHC - a flow index) has been most used in FLP optimisation; it provides a measure of solution quality in terms of cost and is obtained through flow and distance

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How to cite: Medina-Palomera, A., Montalvá-Subirats, J. M. and Hospitaler-Pérez, A., A descriptive analysis of quantitative indices for multi-objective block layout, Ingeniería e Investigación. Vol. 33, No. 1. April 2013, pp. 71 – 75.

matrices. Such cost may vary when placing great emphasis on activities or if a plant's environmental issues are considered, such as lighting and ventilation, thereby including an additional term (bi) to consider each activity's installation cost. Other authors have resorted to less-used flow indices such as material movement time (MMT) reflecting plant productivity resulting from the speed at which material moves and is calculated as time per unit distance travelled between activities. Lin and Sharp (1999) considered an extensive classification of flow indices, including clearness, space sufficiency, aisle, distance, robustness of equipment and building expansion; however, this is mostly applied to layout analysis regarding specific projects.

Table 1. Quantitative indices established and mentioned by authors used in resolving FLP regarding the block layout phase

Objective	Quantitative index
Flexibility	MHC Perimeter Compactness Robustness Shape Inertia
Minimum distance	MHC MMT
Cubic space	Perimeter Robustness Compactness Shape Inertia

Geometrical measurement is important in assessing solution quality as it must be verified whether a solution is really feasible, even if it has shown excellent flow indices; such deviations have been described by Contero (1995), having "sandwich" and "target" settings. Solutions involving some regular shaped (square) activities are those having greater geometrical flexibility as they allow better two dimensional distribution.

Geometrical index analysis

The indices involving geometrical formulation shown in Table 2 are an essential tool for providing efficient solutions; some of the most relevant indices for a discrete domain, with n cells, are listed. Index scope covers activity indices (A) describing a value representing a specific activity's individual quality and configuration indices (C) representing a distribution function's quality regarding all its component activities.

A perimeter index for an activity applied to a plant layout problem (PLP) appeared for the first time in Bozer and Meller (1994); they expressed it as Ω_i, and it was based on the fact that the larger an activity's perimeter, the less formal quality it would have. This index appeared later on called shape ratio (SR) in Wang, Hu and Ku's work (2005) as a basic part of an overall configuration index. An δ index appeared in Lin and Sharp (1999) and was formulated as the ratio between activity perimeter and the perimeter of a boundary rectangle covering it completely. Some C indices have been documented. The shape ratio factor index (*SRFwhole*) appeared as an overall C index in Wang, Hu and Ku (2005); this is actually the geometric al mean of Ω_i indices for each activity.

The activity C index has been defined and used in many forms: in Liggett and Mitchell (1981) as coherence ratio, in Moon and McRoberts (1989) as shape rate, in Raoot and Rakshit (1993) as shape ratio, in Contero (1995) as Ω_2, in Lin and Sharp (1999) as area ratio and in Gonzalez (2005) as compactness γi. Compact ness suggested that the more compact an activity were, the easier would its practical implementation be.

Table 2. PLP indices for discrete filling activities

Index	Type	Equation	
Flow			
MHC	C	$MHC=\sum_{i=1}^{n} b_i + \sum_{i=1}^{n}\sum_{j=1}^{n} W_{ij}\cdot d_{ij}$	b_i = cost of installing the i-th activity in its current position W_{ij}= related intensity between activities i and j d_{ij} =distance between activities i and j
Geometric			
Perimeter	A	$\Omega_i = \frac{P_i}{4\sqrt{A_i}}$	P_i= perimeter of activity i A_i= area of activity i
	A	$\delta_i = \frac{P_i}{P_{rectangulo\ inscribe\ i}}$	P_i= perimeter of activity i
	C	$SRF_w = \left(\prod_{i=1}^{n}\frac{P_i}{4\sqrt{A_i}}\right)^{i/n}$	P_i= perimeter of activity i A_i= area of activity i
Compactness	A	$\gamma_i = \frac{A_i}{a_i\cdot h_i}$	A_i= area of activity i a_i= shortest side of rectangle-inscribed activity i h_i= longest side of rectangle-inscribed activity i
Robustness	A	$\rho_i = \frac{\min(a_i,h_i)}{\max(a_i,h_i)}$	a_i= shortest side of rectangle-inscribed activity i h_i=longest side of rectangle-inscribed activity i
Shape	A	$\varphi_i = \gamma_i\cdot\rho_i$	γ_i= compactness of activity i ρ_i= robustness of activity i
	A	$ratio\ k_i = \frac{\varphi_i}{\delta_i}$	φ_i= Shape index for activity i δ_i= perimeter index of activity i
	C	$r_k = \sum u_{pk}^2$	u_{pk}= Manhattan distance between cell p within activity k domain and activity k's centre of gravity
	C	$s = \frac{\sum_{k=1}^{n} r_k}{\sum_{i=1}^{a}\sum_{j=1}^{h}\sum_{k=1}^{n} a_{ijk}}$	r_k= shape factor for each activity a_{ijk}= value 1 if cell i, j[th] is occupied by activity k, otherwise 0 a = number of domain discretisation rows h = number of domain discretisation columns
	C	$\varphi_{MP} = \frac{\sum_{i=1}^{n} A_i\cdot\varphi_i}{\sum_{i=1}^{n} A_i}$	A_i= area of activity I φ_i= shape index for activity i n = number of activities involved in the problem
Inertia	A	$I_{xy} = \frac{1}{12}\cdot(a\cdot b^3 + b\cdot a^3)$	a = activity height b= activity width $\Omega_{4i} = \frac{I_i - I_{min}}{I_{max} - I_{min}}$

The robust (R) activity index has been defined and used in Liggett and Mitchell (1981) as proportion ratio, in Contero (1995) as Ω_3, in Gonzalez (2005) as robustness (ρ_i) and in Aiello, Enea and Galante (2006) as aspect ratio (γ_i).

Neither compactness or robustness separately guarantee a solution having high formal quality as some solutions may have good index values but which are not really so. Gonzalez (2005) expressed this by defining a new index combining C and R to maximise the benefits of both, called form of activity (φ_i), resulting in a measurement overcoming some of the disadvantages of the separate indices. Another form index is the k ratio proposed by Lin and Sharp (1999) combining form index and perimeter index. An original contribution was made by Islier (1998) for whom the configuration form/shape factor was obtained from each activity's r_k value. Some form/shape

C indices have been documented, such as the s factor introduced by Islier (1998) (Table 1) and φ_{MP} proposed by Gonzalez (2005) measuring overall C, this being the weighted sum of all forms/shapes of a configuration's activities.

Contero (1995) proposed the inertia index based on an activity's polar moment of inertia as a measurement of the dispersion of the area associated with it. He used the expression polar moment of inertia regarding the centre of gravity for activity I_{xy} , the normalised value then being calculated to give the geometric quality of activity Ω_4.

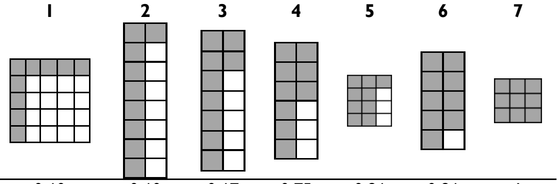
Development

The large variety of indices mentioned above led to them being studied to identify which were significant, self-sufficient and independent. An analysis was thus made of each index's geometrical characteristics regarding a discrete domain to determine their applicability, alternative ways of calculating some indices being proposed.

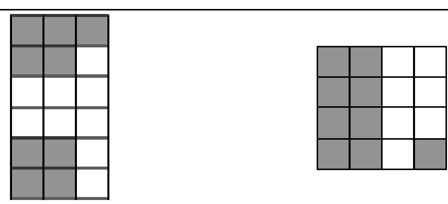
Index pattern regarding different scenarios

This study focused on discrete domains involving different activities in several shapes/forms. Table 3 shows each index's current effect for varying the shape of an activity involving nine cells; it shows that perimeter index δ_i had very low reliability, although solutions to the right of the table were better (δ=1). It would thus be useful in cases of highly degenerate forms of activity. The index was not able to determine whether an activity was accidentally unconnected (Figure 1).

Table 3. Different index values for activities with n cells



	Activity						
Index	1	2	3	4	5	6	7
Ω_i^{-1}	0.60	0.60	0.67	0.75	0.86	0.86	1
δ_i	1	1	1	1	1	1	1
γ_i	0.36	0.56	0.64	0.75	0.75	0.9	1
ρ_i	1	0.25	0.28	0.33	0.75	0.40	1
φ_i	0.36	0.14	0.18	0.25	0.56	0.40	1
$k_{i\ ratio}$	0.36	0.14	0.18	0.25	0.56	0.40	1
r_{ki}	65.9	62.8	51.4	34.8	27.5	23.5	20
Ω_{4i}	0.35	0.87	0.58	0.28	0.13	0.16	0



a) δ=18/18=1 b) δ=16/16=1

Figure 1. Evaluation of index δ in activities involving breakage

Table 3 shows that the best C index values were those giving γ=1; however, activity involving maximum C value may have had an undesirable shape, i.e. fully-elongated. Bozer and Meller (1994) found the C index less effective than the perimeter index in cases of degenerated geometries obtained by complicated filling curves.

R ρ_i gave results equal to the optimal index for disrupted distribution (Table 3), suggesting special care when using filling techniques to form complex shapes. Table 3 shows an incongruity; the $\varphi_1$and φ_6 values were very close; however, their geometrical figures were significantly different. To avoid this, the index was reformulated, penalising configurations in which C was significantly negative by using the following expression: $\varphi_i(k) = \rho \cdot \gamma^k$ where *k* was the penalty value. For example, if *k* = 2, then index values for φ_1 and φ_6 would have been 0.13 and 0.36, respectively.

k ratio index values were identical to those of the φ index, as a result of dividing the latter by the δ perimeter measurement, which is why the index was significant only for situations where the space filling system led to the formation of extremely degenerated activities.

Index area-dependence

Analysis was aimed at detecting dependence regarding an index and an activity area (number of cells) when a discrete domain was completely filled. The minimum and maximum measurement obtained by an index for activities different to n number of cells (up to 500 cells) had to be met for this study. The minimum value concerned the configuration of an activity having a single column of length equal to the total number (n) of the activity cells. The maximum value of the index due to the number of cells would have been the one coming closest to the square of such activity.

A change in the minimum and maximum limit values, based on the area of activity (n cells), was perceived for perimeterΩ_i. It could thus be said that that Ω_i was an area-dependent index, and its values were not comparable between activities. Given the above, an index independent of activity area was proposed by equation 1, using inverse Ω^{-1} to maximise the index, 1 being the optimum value:

$$\Omega_i^{-1}(n) = \frac{\Omega^{-1}(n) - \Omega_{min}^{-1}(n)}{\Omega_{max}^{-1}(n) - \Omega_{min}^{-1}(n)} \quad \text{Eq. 1}$$

where:

$$\Omega_i^{-1}(n) = \frac{2\sqrt{A_i}}{n+1}$$

$\Omega_{max}^{-1}(n)$= index value for the best square shape of an activity having n cells

$\Omega_{min}^{-1}(n) = \frac{2\sqrt{n}}{n+1}$= index value for the most elongated of the activi ties involving n cells

The C index γ was also dependent on activity area and thus indices φ and r_k; this pattern would be considered if such indices were selected for a multi-objective study, as activities needing to be located in an enclosure may have different areas.

After examining the activity indices of interest to identify additional configuration indices which could be used to measure solution quality, an additional option for measuring perimeter configuration involved taking an activity index's minimum value in the configuration as its value-index, and thus consider a worst-case scenario $\Omega_{mini}^{-1}(n)$ from $\Omega_i^{-1}(n)$ so obtained. This measurement would thus have indicated that any other perimeter regarding said configuration would have been better; the quality of a configuration being evaluated would thereby be determined by a representative value. This logic may be used with other indices: the smaller C γ_i, the smaller R ρ_i, the smaller φi and maximum inertia Ω_{4i} of a particular configuration.

No information was found in the pertinent literature regarding a C, R and inertia configuration index; accordingly, compound configuration indices for the aforementioned ones and the rest of the activity indices (i.e. γ_{MP}, ρ_{MP}, φ_{MP}, Ω_{4MP}) were proposed (Table 4), representing activity indices' weighted average value.

Analysis of dependence between indices

Whereas Muther's concept of integration (1968) clearly showed the multi-objective nature of the problem to be solved, it has been represented here by a set of quantitative indices. It was thus evaluated by two or more of the indices listed in Table 2 and 4.

The setting indices were analysed to test each one's self-sufficiency, identifying redundant ones by evaluating different accommodations for different scenarios. A comparative analysis was made between indices resulting from random walking of 25,000 runs for the FW13 benchmark problem in Francis and White (1974). Pairwise comparison of the run results was made using the indices obtained from the 20-activity benchmark problem in Armour and Buffa (1963) for the same number of interactions. The problems mentioned above provided limited information about the issues and only the indices available for calculation were considered.

The FW13 problem involved a problem of locating 13 activities in a 2D space where one of the activities had to remain fixed. Whereas the fixed position of such activity could have been in two preset positions, the results were available for comparing the indices. Pearson's product moment correlation was used to quantify the degree to which to indices were related, i.e. how much one index tended to change when the other one also did so. The following comparative results between indices were obtained. The first one between k_{min} and φ_{min} had r^2=0.997 (Figure 2) and p=0, the second one between K_{MP} and φ_{MP} had r^2=0.999 and p=0, the third one between ρ_{MP} and φ_{MP} gave r^2 = 0.916 and p=0 and the fourth between ρ_{MP} and K_{MP} gave r^2 = 0.914 and p=0. There was high correlation in all cases between each pair of indices. The p-value determined the appropriateness of rejecting the null hypothesis; a p-value of less than 0.05 in this analysis meant that the indices were related.

An additional observation concerns the result of identifying the best solutions for each index. The δ_{max}, Ω^{-1}, *SRFw*, K_{MP}, k_{min}, ρ_{MP}, φ_{min} and φ_{MP} indices had the same plant layout as shown in Table 3 for their first two best solutions.

From the foregoing and in view of the high correlation between indices k_{min} - φ_{min}, K_{MP} - φ_{MP}, ρ_{MP}- φ_{MP} and ρ_{MP} - K_{MP}, it would have been excessive to use them all (i.e. many would have been redundant). Therefore, φ_{min} was used in the first pair index because of existing limitations regarding the variable perimeter in index k_{min}. There was equality amongst the rest of the pairs (K_{MP}, ρ_{MP}, and (φMP), φ_{MP} being preferred because it further penalised disintegrated forms.

The measurements representing the weighted average of activity indices concerned an activity's configuration index pattern for maximising the minimum value or vice versa. The difference lay in weighted values involving less dispersion concerning the visual perception of comparisons, thereby making them more suitable.

Table 3. PLP indices proposed for discrete filling activities

Index	Type	Equation	
Perimeter	A	$\Omega_i^{-1}(n) = \frac{2\sqrt{A_i}}{n+1}$	A_i= area of activity i; n = number of cells occupied by activity i
	C	$\Omega_{mini}^{-1}(n)$	The smaller $\Omega_i^{-1}(n)$ of configuration
Compactness	C	$\gamma_{MP} = \frac{\sum_{i=1}^{n} A_i \cdot \gamma_i}{\sum_{i=1}^{n} A_i}$	A_i= area of each activity; n = number activities involved in the problem; γ_i= compactness of activity i
	C	γ_{min}	The smaller compactness γ_i of configuration
Robustness	C	$\rho_{MP} = \frac{\sum_{i=1}^{n} A_i \cdot \rho_i}{\sum_{i=1}^{n} A_i}$	ρ_i= robustness of activity i; A_i= area of activity i; n = number of problem activities
	C	ρ_{min}	The lowest configuration robustness ρ_i
Form/shape	C	φ_{min}	The smallest tion φ_i
	C	k_{min}	The smallest **configuration k_i ratio**
	C	$K_{MP} = \frac{\sum_{i=1}^{n} A_i \cdot ratio\ k_i}{\sum_{i=1}^{n} A_i}$	A_i= area of activity i; k_i= k ratio indicated by activity i; n = number of activities involved in a problem
Inertia	C	$\Omega_{4MP} = \frac{\sum_{i=1}^{n} A_i \cdot \Omega_{4i}}{\sum_{i=1}^{n} A_i}$	A_i= area of activity i; Ω_{4i}= inertia index; n= number of activities involved in a problem
	C	Ω_{4max}	The largest configuration Ω_{4i}

Table 4. Number of iterations representing the best layout

Index	The number of iterations	
	First layout option	Second layout option
δ_{max}	7,037	1,0619
Ω^{-1}	22,132	7,037
SRFw	7,037	21,132
K_{MP}	7,037	299
k_{min}	7,037	767
ρ_{MP}	299	7,037
φ_{min}	7,037	767
φ_{MP}	7,037	299

Additional random walking (again using space filling curves) for the 20-activity PLP proposed by Armour and Buffa agreed with the comments made by Francis and White; it would thus seem probable that such remarks might be generalised.

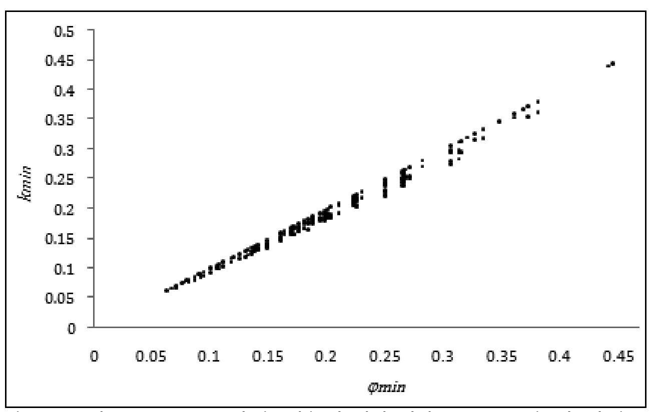


Figure 2. The φ_{min}- k_{min} relationship; both had the same optimal solution at the Pareto frontier

Conclusions

A great deal of independence or self-sufficiency was found between most indices, including all those proposed here; however, each has its own advantages and disadvantages as outlined in this document. They can guide an analyst in selecting appropriate indices for resolving PLP. An analyst must consider index aspects when selecting them, such as a standardised index's requirements, index variation regarding the size of the areas of the activities to be located, index sensitivity in detecting disjointed or very degenerated areas regarding their shape, besides reliability in detecting unsuitable shapes.

Significant correlation was found for geometrical indices, leading to the reduction of options for indices to determine solution quality from a geometrical perspective.

Acknowledgements

We are indebted to the Mexican Public Education Department's Teacher Improvement Programme (PROMEP) for providing the financial support received for this research.

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