



Ingeniería e Investigación

ISSN: 0120-5609

revii\_bog@unal.edu.co

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Colombia

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Ingeniería e Investigación, vol. 33, núm. 2, agosto, 2013, pp. 56-62

Universidad Nacional de Colombia

Bogotá, Colombia

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# An optimal algorithm for estimating angular speed using incremental encoders

## Un algoritmo óptimo para la aproximación de la velocidad angular utilizando encoders

J. D. Rairán Antolines<sup>1</sup>

### ABSTRACT

This paper proposes a new algorithm using signals from an incremental encoder for estimating a rotating shaft's speed. This algorithm eliminated the oscillations appearing in classical fixed-time and fixed-space algorithms, even when speed was constant. A fixed-time algorithm measures angular displacement at fixed-time intervals, while a fixed-space algorithm measures time every fixed-angular displacement. Time and displacement measurements were used to generate estimations for speed. The new algorithm generated a unique value for estimating speed due to synchronising encoder pulses and a signal formed by impulses every time increase ( $\Delta t$ ). A first modification of the proposed algorithm was defined, resulting in the harmonic mean between these two proposed alternatives having the smallest relative error possible. This error was always smaller than half the error with fixed-time and fixed-space algorithms. Experimental setup and algorithms are shown, as well as Simulink results using signals acquired from an incremental encoder.

**Keywords:** Fixed-space, fixed-time, incremental encoder, optimal algorithm, estimating speed, truncation error.

### RESUMEN

En este artículo se propone un algoritmo nuevo, el cual utiliza las señales provenientes de un *encoder* incremental para aproximar la velocidad de giro de un eje. Este algoritmo elimina las oscilaciones que aparecen en los algoritmos clásicos, conocidos como a espacio fijo y a tiempo fijo, las que son visibles incluso a velocidad constante. Un algoritmo a tiempo fijo mide desplazamiento angular a intervalos de tiempo fijo, mientras que un algoritmo a espacio fijo mide tiempo a intervalos de espacio fijo. Las medidas de tiempo y desplazamiento se utilizan para estimar la velocidad. El algoritmo nuevo genera una aproximación sin saltos a velocidad constante, dada la sincronización entre los pulsos del encoder y la señal formada a partir de impulsos cada  $\Delta t$  de tiempo. Se propone una primera modificación del algoritmo, y el resultado es que el promedio armónico entre las dos propuestas de algoritmo tiene el menor error relativo posible. Ese error siempre es menor que la mitad del error que se produciría con los algoritmos a tiempo fijo o a espacio fijo. Finalmente, se presenta un arreglo experimental para probar los algoritmos, y se muestran los resultados de la ejecución en Simulink, usando señales adquiridas de un encoder incremental.

**Palabras clave:** Aproximación de la velocidad, error de truncamiento, espacio fijo, tiempo fijo.

Received: May 31st 2012

Accepted: July 19th 2013

### Introduction

Estimating speed represents an interesting problem, especially regarding control involving the dynamics of a system based on signals given by sensors; i.e. based on an estimations of what actually happens. The algorithm proposed in this paper was aimed at obtaining maximum information from an incremental encoder, thus establishing a fast and stable measurement of a shaft's angular speed. An incremental encoder is an electromechanical instrument. The mechanical part is an evenly spaced slotted disc, whereas the electric part includes a light beam and detector. A high light level (one) is recorded when the light

passes through the slit and reaches the detector, and zero otherwise.

Measuring speed has one physical limitation; estimating speed requires measuring two instants in time and the result necessarily neglects one of them. An estimation of speed has at least two sources of error because it requires two measurements: position and time. Since  $\omega_m$  is  $\Delta\theta/\Delta t$ , error regarding estimation  $\omega_m$  lies in the nature of such deltas (i.e. discrete and digital values). Digital measurement involves an inevitable truncation error. Thus,  $\Delta\theta$  is an integer multiple of the pulses emitted by the encoder ( $pul$ ), and  $dt$  is likewise an integer multiple of clock resolution ( $ts$ ).

The speed at which the time between two consecutive pulses from the encoder lasts exactly  $dt$  defines the so-called *speed limit* given in revolutions per second ( $\omega_{lim} = (1/pul)/dt$ ). Speed values less than  $\omega_{lim}$  are low, whereas greater or equal than this are high. A similar concept (Tsuji et al., 2009) has defined what authors call speed resolution.

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**How to cite:** Rairán, J. D., An optimal algorithm for estimating angular speed using incremental encoders, Ingeniería e Investigación. Vol. 33, No. 2, August 2013, pp. 56 – 62.

Two classical algorithms estimate speed (fixed-time and fixed-space algorithms); other algorithms usually come from these two. If an estimation involved using the fixed-time approach, which means updating the counting of pulses ( $Nep$ ) each  $dt$ , then the algorithm generates an estimation  $\omega_m = Nep\omega_{lim}$ . On the other hand, the fixed-space approach updates  $dt$  counts (called  $Ndt$ ) between every two consecutive pulses, so  $\omega_m = (1/Ndt)\omega_{lim}$ . The fixed-space algorithm is recommended for low speeds, whereas the fixed-time algorithm is useful for high speeds. The truncation error makes  $\omega_m$  oscillate regardless of the approach adopted. The effect of these oscillations requires using low-pass filters to smooth the value of the estimation, thereby implying delay.

Current solutions for avoiding the effect of truncation can be classified mainly into two groups: highly accurate measurements of time and synchronising position and time signal (a third group is also referenced, including other options). Solutions from the first group use the power of computing electronic circuits for high resolution measurement of time, for instance microcontrollers (Petrella and Tursini, 2008), DSP (Boggapu and Kavanagh, 2010) and FPGAs. A more elaborate solution (Merry, Molengraft and Steinbuch, 2007) detects rising edges from the encoder to make a polynomial fitting, thereby smoothing the estimation. Other work (Lygouras *et al.*, 2008) has gone so far as to claim that the truncation error effect disappears when clock resolution reaches 50 ns. The oscillations within the estimation, though small, still remain in this first solution group. On the other hand, the second group looks for synchronising position and time measurements. An algorithm named the S method (Tsuji *et al.*, 2009) has been proposed based on this synchronisation concept, mainly being helpful for high speed. Another proposal (Se-Han, Lasky and Velinsky, 2004) for optimising encoders with few slits uses synchronisation for high speeds, whereas for low speeds it switches the algorithms for another based on its model of the system. The third group of solutions is not exactly focused on the truncation problem, but nonetheless the problem is well referenced and is taken into account in the solution. Some work (Su *et al.*, 2005) has focused on very low speeds, estimation being made by absolute position measurement while the average speed of every rotation (Hachiya and Ohmae, 2007) has been used for developing a simple and effective method for eliminating fluctuations, as well as mitigating the effect of some mechanical errors. Another solution consists of using filters and switching from fixed-time to fixed-space methods or vice versa to ensure obtaining the smallest relative error possible (D'arco, Piegari and Rivo, 2003). It has been suggested that some steps can be traced backwards to control the highest relative error (Liu, 2002).

## Describing the new algorithm

The new algorithm presented in this section eliminated oscillations in estimating speed when they came from discrete and digital measurements, as in an incremental encoder. The core of the proposal consisted of synchronizing position and time pulses. Since there was no control over the time at which the encoder pulse appeared, the algorithm simply started to count  $dt$  when a position pulse appeared. As a result, such estimation would have been constant for constant speeds, without the use of state observers or system models as required in other work (Se-Han, Lasky and Velinsky, 2004).

Guaranteeing the mathematical results in this section involved the following assumptions: the encoder did not lose pulses during counting, measuring time was exact and in multiples of clock

time ( $ts$ ) and the mechanical distance between pulses remained constant. These assumptions idealised an incremental encoder's true performance, although they were necessary for ensuring error bounds on the estimations.

The first step in transforming the encoder signal detected the rising edge from each pulse. This detection generated a pulse train ( $lep$ ) as shown in Figure 1. The next step counted these pulses and led to a stair-type signal named ( $Cep$ ). If counting  $Cep$  went to zero, for instance because the time reached  $dt$ , then a register saved the last value in the counter into a variable  $Nep_I$ . The scheme in the lower part of Figure 1 illustrates an additional series of blocks. These blocks counted  $dt$ s. A pulse  $dt$  resulted when the accumulation of  $ts$  reached the value of  $dt$ . This event reset and started the counting of  $ts$  over again. The counter block held  $dt$  counting in a variable  $Cdt$ . If an  $lep$  pulse reset that counter, then a register held the last value of  $Cdt$  into  $Ndt_I$ .

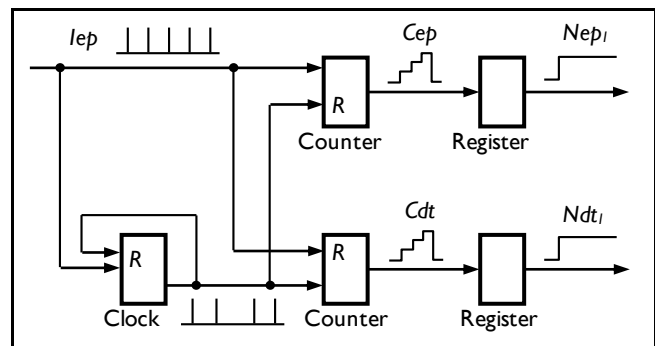


Figure 1. Algorithm scheme and definition of  $Nep_I$  and  $Ndt_I$ .

Speed was estimated as  $\omega_{mI} = n_I\omega_{lim}$ , where  $n_I$  was  $Nep_I/Ndt_I$ . Counting required that  $Nep_I$  should be a whole number (0, 1, 2, ...), while  $Ndt_I$  had to be a natural number. There were more  $lep$  pulses in the count than  $dt$  pulses for speeds higher than  $\omega_{lim}$ , so  $Ndt_I$  stayed fixed at one. On the contrary, there were more  $dt$  pulses than  $lep$  pulses for speeds lower than  $\omega_{lim}$ , so  $Nep_I$  remained fixed at one. Possible values for  $n_I$ , for high speeds were 1, 2, 3, and so on, and  $1/2$ ,  $1/3$ ,  $1/4$ , and so on for low speeds.

Table 1. Pseudocode for the proposed speed estimation algorithm

Input: $ts$ , $dt$ , $lep$ impulses	
Output: $Ndt_I$ , $Nep_I$	
1	<b>Begin</b>
2	$Cep \leftarrow 0$ , $Cdt \leftarrow 0$ , $Nep_I \leftarrow 0$ , $Ndt_I \leftarrow 1$
3	$clock \leftarrow 0$
4	<b>Repeat</b>
5	$clock \leftarrow clock + ts$
6	<b>if</b> $lep = 1$ & $Cdt \neq 0$ <b>then</b>
7	$Ndt_I \leftarrow Cdt$
8	$clock \leftarrow 0$
9	<b>if</b> $clock = dt$ & $Cep \neq 0$ <b>then</b>
10	$Nep_I \leftarrow Cep$
11	<b>if</b> $lep = 1$ <b>then</b>
12	$Cep \leftarrow Cep + 1$ , $Cdt \leftarrow 0$
13	<b>if</b> $clock = dt$ <b>then</b>
14	$Cdt \leftarrow Cdt + 1$ , $Cep \leftarrow 0$
15	$clock \leftarrow 0$
16	<b>return</b> ( $Ndt_I$ , $Nep_I$ )
17	<b>until</b> stop
18	<b>End</b>

Table 1 shows the algorithm's pseudocode. This algorithm generated a new estimation  $\omega_{mI}$  every  $ts$  by computing  $Ndt_I$  and  $Nep_I$ . The core of the algorithm synchronised time and space pulses in line 6 where the clock went to zero. However, this

synchronisation did not mean that  $Ndt_l$  and  $Nep_l$  updated their values synchronously:  $Ndt_l$  updated its value in line 6 (synchronised with  $lep$  pulses) and  $Nep_l$  updated its value in line 9 (synchronised with  $dt$  pulses).

Plots a) and b) in Figure 2 illustrate algorithm operation for low and high speeds, respectively. Event  $a$  started the programme in both examples, and subsequent letters defined other events. For instance, events  $d$  and  $g$  updated  $Nep_l$  in both examples. Estimation  $\omega_{ml}$  remained constant for events later than  $f$  in plot a. On the other hand, estimation after any event later than  $d$  in plot b remained constant.

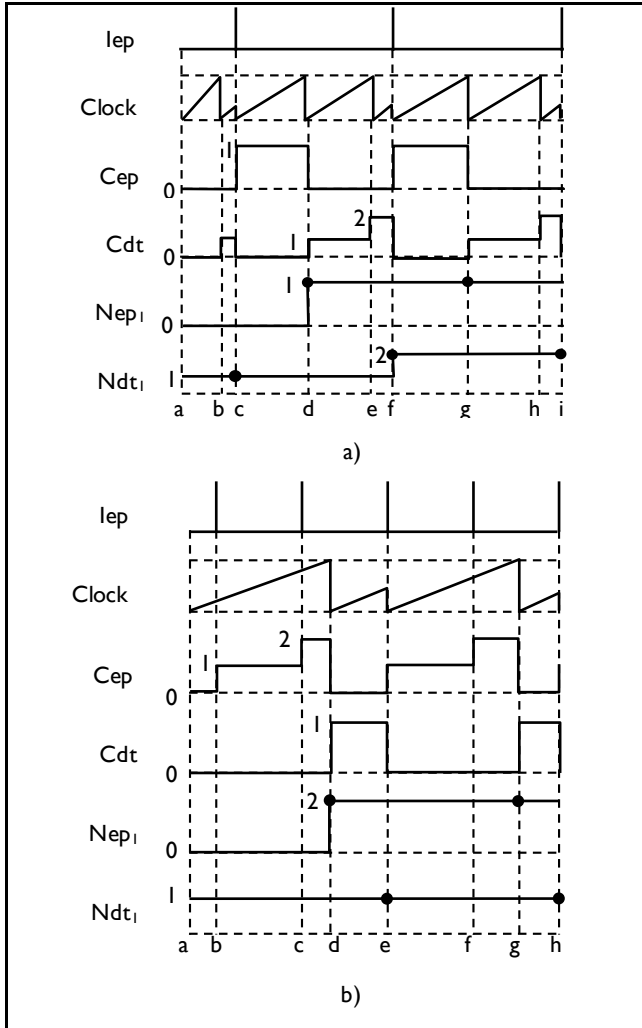


Figure 2. Examples of algorithm use: a)  $\omega \approx 0.25\omega_{lim}$  b)  $\omega \approx 1.5\omega_{lim}$

Solid points in Figure 2 depicted the time when the algorithm updated  $Nep_l$  or  $Ndt_l$ . The time between updates for low speeds was  $dt_a = (\omega_{lim}/\omega) dt$ , and for high speeds  $dt_a$  (see Eq. 1). The maximum value of  $dt_a$  in Equation 1 was twice  $dt$  as speed approached  $\omega_{lim}$ , and delay decreased as  $\omega$  grew, with  $dt$  as its limit:

$$dt_a = \left( \frac{-1}{\left[ \frac{\omega}{\omega_{lim}} \right]} \cdot \frac{\omega}{\omega_{lim}} + 2 + \frac{1}{\left[ \frac{\omega}{\omega_{lim}} \right]} \right) dt \quad (s) \quad (1)$$

The value of  $Nep_l$  represented the ceiling for the number of pulses  $lep$ , as can be seen in Figure 2;  $Ndt_l$  was the floor for counting  $dts$ , so estimation  $n_l$ , defined as  $Nep_l/Ndt_l$ , was always

greater than the actual speed, as shown in Figure 3, where the X and Y axes had been scaled by  $\omega_{lim}$ .

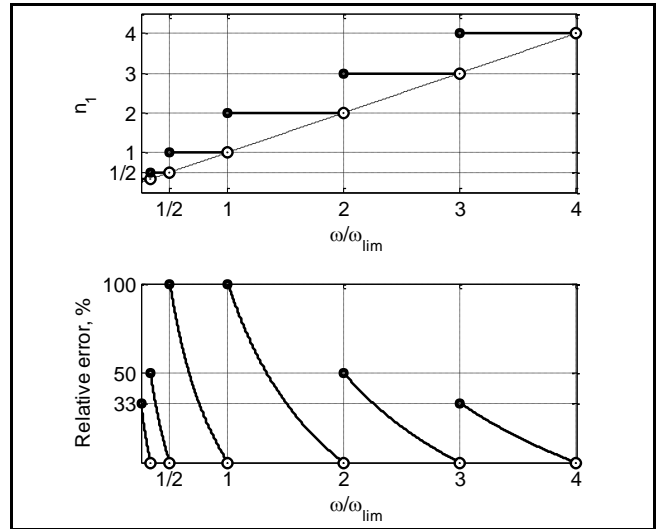


Figure 3. Speed estimation,  $\omega_{ml}$ , and relative error for the proposed algorithm

The first measurement of quality for estimating speed correlated relative error and speed, as shown in the lower part of Figure 3. A second measurement of quality (maximum relative error per interval) took into account that the value of  $\omega$  actually was unknown. An interval covered the whole range of speeds having the same estimation. For instance, all speeds estimated as  $n_l = 3$  as estimation defined an interval. This second measurement of quality depended entirely on algorithm output. Even so, Equation 2 showed the definition of relative error for the second measurement of quality to come up with an equation for this indicator:

$$er_l = \frac{|\omega_{ml} - \omega|}{\omega} 100\% \quad (2)$$

Since  $\omega_{ml} = n_l \omega_{lim}$  then,

$$er_l(\omega, n_l) = \left( n_l \frac{\omega_{lim}}{\omega} - 1 \right) 100\% \quad (3)$$

Relative error  $er_l$  maximised its value at the left of each interval when ratio  $\omega_{lim}/\omega$  was maximum. For high speeds (when  $n_l$  exceeded 1), ratio  $\omega_{lim}/\omega$  was the inverse of  $n_l - 1$ , as can be seen by analysing the upper part of Figure 3. For instance, if  $n_l = 3$ , maximum ratio would have been  $\omega_{lim}/\omega = 1/2$ , so maximum relative error reached 50%. For low speeds the ratio was  $\omega_{lim}/\omega = (1/n_l) + 1$ . For instance, if  $n_l = 1/2$ , then  $\omega_{lim}/\omega = 3$ , so maximum relative error reached 50% again. Previous analysis of ratio  $\omega_{lim}/\omega$  produced the expression for the maximum relative error shown in Equation 4:

$$Er1_{Max}(n_l) = \begin{cases} n_l \times 100\% & , n_l \leq 1 \\ \frac{1}{n_l - 1} \times 100\% & , n_l \geq 2 \end{cases} \quad (4)$$

The relative error in Equation 4 matched the maximum relative error for the traditional fixed-time algorithm if  $\omega \geq \omega_{lim}$  and  $dt = ts$ .  $Er1_{Max}$  matched the maximum relative error for the traditional fixed-space algorithm, when  $\omega < \omega_{lim}$  and  $dt = ts$ . The proposed

algorithm equated its measurement of quality with relative error from traditional algorithms.

### Optimising maximum relative error

The algorithm's optimum minimised error  $Er_{Max}$ , such minimisation coming from an observation about where maximum error happened; according to the result shown in Figure 3, such maximum error took place at the left end from each interval. If the estimations shifted down one place, for example from  $n_1$  equal to 3 to a new  $n_2$  equal to 2, or from  $n_1 = 1/2$  to  $n_2 = 1/3$ , then relative error decreased. This decrease occurred due to the maximum difference between  $\omega$  and the new  $\omega_{m2}$  taking place at the right of each interval, instead of at the left.

The new estimation ( $\omega_{m2} = n_2 \omega_{lim}$ ) requires  $n_2$  as given in Eq. 5.

$$n_2 = \begin{cases} \frac{1}{\frac{1}{n_1} + 1}, & \omega < \omega_{lim} \\ n_1 - 1, & \omega \geq \omega_{lim} \end{cases} \quad (5)$$

$Nep_1$  and  $Ndt_1$  provide an alternate means for calculating  $n_2$ , as shown in Equation 6:

$$n_2 = \begin{cases} \frac{Nep_1}{Ndt_1 + 1}, & \omega < \omega_{lim} \\ \frac{Nep_1 - 1}{Ndt_1}, & \omega \geq \omega_{lim} \end{cases} \quad (6)$$

The relative error for estimation  $\omega_{m2}$  is shown in Equation 7:

$$er_2(\omega, n_2) = \left(1 - n_2 \frac{\omega_{lim}}{\omega}\right) 100\% \quad (7)$$

According to Equation 7, and for high speeds, every integer ratio  $\omega/\omega_{lim} = n_2 + 1$  maximised relative error. For instance if  $n_2$  equal to 2, then  $\omega_{lim}/\omega$  was  $1/3$ , so the maximum relative error reached 33.3%. For low speeds, integer ratio  $\omega/\omega_{lim}$  was the inverse of  $(1/n_2) + 1$ . So, for instance, if  $n_2 = 1/2$ , fraction  $\omega_{lim}/\omega = 3$ . As a result, maximum error reached 33.3% again. Error in terms of  $n_2$  is shown in Equation 8. This equation defined a supremum and not a maximum, because the right end of each interval was open, as shown in Figure 3:

$$Er2_{Sup}(n_2) = \begin{cases} n_2 \times 100\% & , n_2 < 1 \\ \frac{1}{n_2 + 1} \times 100\% & , n_2 \geq 1 \end{cases} \quad (8)$$

Computing estimation  $\omega_{m1}$  and  $\omega_{m2}$  required the actual value of  $\omega$  to classify a speed as low or high (Eq. 5 and 6). The value of the speed was unknown, although there were two approaches for determining whether speed passed speed limit,  $\omega_{lim}$ . Both approaches used an indirect measurement. In the first approach, whereas low speeds had the time between consecutive  $lep$ 's exceeding  $dt$ , that time did not exceed  $dt$  for high speeds. The second approach counted  $n_1$ . Whereas that number exceeded  $n_1 \geq 2$  for high speeds, for low speeds it did not.

Estimation  $\omega_{m2}$  reduced maximum error by up to 50% with estimation  $\omega_{m1}$ , mainly for speeds close to  $\omega_{lim}$ . On the other

hand,  $Er1_{Max}$  and  $Er2_{Sup}$  became closer and closer as speed went farther from  $\omega_{lim}$ , or, in other words, when  $n_1$  and  $n_2$  went either to zero or infinity.

Any estimation greater than  $\omega_{m1}$  generated errors larger than  $Er1_{Max}$ ; any estimation under  $\omega_{m2}$  resulted in errors over  $Er2_{Sup}$ . So if there was a way to reduce maximum error per interval, the resultant estimation would have fallen between  $\omega_{m1}$  and  $\omega_{m2}$ . See, for instance that estimation  $\omega_{m3}$  equalled  $n_3 \omega_{lim}$  in Figure 4. The work then involved finding a value for  $n_3$  that minimised error in region  $\omega_{m2} \leq \omega_{m3} \leq \omega_{m1}$ :

$$er_L = \frac{n_3 - n_2}{n_2} 100\% = \left(\frac{n_3}{n_2} - 1\right) 100\% \quad (9)$$

$$er_R = \frac{n_1 - n_3}{n_1} 100\% = \left(1 - \frac{n_3}{n_1}\right) 100\% \quad (10)$$

The minimum relative error for  $\omega_{m3}$  could have been on the right or left side at each interval. If estimation  $\omega_{m3}$  decreased, relative error on the left also decreased. As a result, relative error on the right increased. Thus, errors on the left opposed errors on the right and optimal value levelled the error on both sides, as shown by Equations 9, 10 and 11:

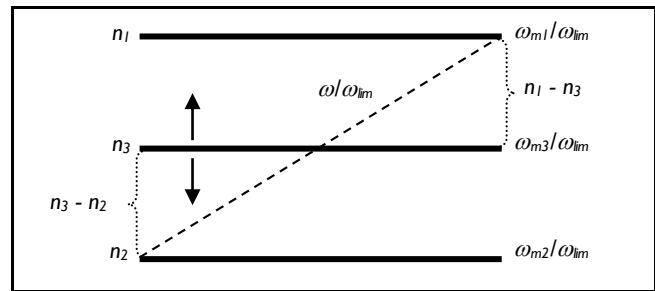


Figure 4. Finding the best estimation,  $n_3$

Error  $er_L$  reached zero when  $n_3 = n_2$  in Equation 9, and according to the representation in Figure 4 the maximum occurred when  $n_3$  equalled  $n_1$ . Equation 10 showed that  $er_R$  reached zero when  $n_3$  equalled  $n_1$ , and the maximum occurred when  $n_3 = n_2$ . Since  $er_R$  and  $er_L$  were linear, then the minimum from combining  $er_R$  and  $er_L$  occurred when  $er_L$  equalled  $er_R$ , as shown in Equation 11. The value of  $n_3$  in Equation 11 defined optimal estimation for  $\omega_{m3}$ : the harmonic mean between  $\omega_{m1}$  and  $\omega_{m2}$ . No other value of  $n_i$  produced a smaller error:

$$\frac{n_3 - n_2}{n_2} = \frac{n_1 - n_3}{n_1} \rightarrow n_3 = \frac{2}{\frac{1}{n_1} + \frac{1}{n_2}} \quad (11)$$

Error  $Er3_{Max}$  was defined in terms of  $n_3$  in Eq. 12, and in terms of  $n_1$  in Eq. 13:

$$Er3_{Max}(n_3) = \begin{cases} \frac{n_3}{2} \times 100\% & , n_3 < 1 \\ \frac{n_3 - \lfloor n_3 \rfloor}{\lfloor n_3 \rfloor} \times 100\% & , n_3 > 1 \end{cases} \quad (12)$$

If  $n_1 = 1$  or 2 in Equation 13 then  $Er3_{Max}$  equalled one third of the error given by Equation 4. On the other hand, when  $n_1 \rightarrow \infty$  or  $n_1 \rightarrow 1/\infty$  then error  $Er3_{Max}$  went below half  $Er1_{Max}$ . Therefore, this third estimation  $\omega_{m3}$  in fact optimised error by setting the smallest possible bound for the relative error at each interval.

$$Er3_{Max}(n_1) = \begin{cases} \frac{1}{1 + \frac{2}{n_1}} \times 100\% & , n_1 \leq 1 \\ \frac{1}{2n_1 - 1} \times 100\% & , n_1 \geq 2 \end{cases} \quad (13)$$

The bounds for relative error were calculated for constant speed and also applied to variable speeds, if acceleration did not go beyond a given bound. The method for finding the maximum acceleration began by supposing constant acceleration and high speeds. Under these assumptions, and according to the result in Equation 1, any estimation  $\omega_{mi}$  updated its value at most every  $2dt$ , so maximum acceleration had an increase in speed equal to  $\omega_{lim}$  in an interval of time equal to  $2dt$ . The same analysis could be made for speeds under  $\omega_{lim}$ , as shown in Equation 14:

$$a_{max} = \begin{cases} \frac{\omega_{lim}}{2dt} \frac{1}{Ndt(Ndt+1)} \cdot \left( \frac{rev}{s^2} \right) & \omega < \omega_{lim} \\ \frac{\omega_{lim}}{2dt} \cdot \left( \frac{rev}{s^2} \right) & \omega \geq \omega_{lim} \end{cases} \quad (14)$$

The solid lines in Figure 5 illustrate estimations  $\omega_{mi}$ , and the dashed line represents the actual speed. Estimation  $\omega_{m1}$  was always greater than  $\omega_{m3}$ , and  $\omega_{m3}$  was always greater than  $\omega_{m2}$ . The simulation parameters in Figure 5 were  $dt = 1$  ms,  $pul = 10,000$  and  $ts = 0.1$   $\mu$ s. These values made  $\omega_{lim} = 0.1$  rev/s, so maximum acceleration was  $a_{max} = 50$  rev/s<sup>2</sup>. The ramp with positive acceleration had 8.33 rev/s<sup>2</sup> constant acceleration whereas negative acceleration was -12.5 rev/s<sup>2</sup>.

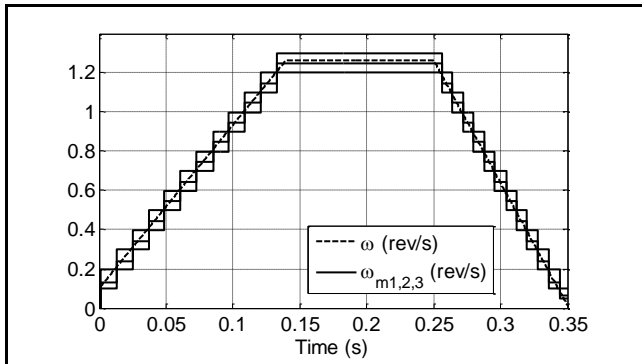


Figure 5. Estimation  $\omega_{mi}$  for variable speed

The estimation technique tested in this section was also used as an incremental encoder, so the results appeared to be limited to measuring angular speed, even though this technique can be equally applied for estimating any variable's rate of change. There was only a single restriction on the data: sampling rate had to be constant. This restriction, along with the discrete nature of the measurements, generated oscillations on traditional estimation algorithms, even when the rate of change remained constant; however, those oscillations disappeared when using the proposed algorithm.

## Experimental results

In this section, the algorithm generated estimations using real signals coming from an encoder. The scheme in Figure 6 illustrates an encoder coupled to a shaft, where the system aimed at estimating the shaft's angular speed. The encoder's output fluctu-

ated from zero to five volts for low and high levels, respectively. A data acquisition card captured this voltage signal, and a computer recorded its values for later use. Data acquisition card PCI-6024E had 100  $\mu$ s  $ts$  sampling time. The 160-slot encoder and the DC motor (Minertia 6GFMED) are shown in the lower part of Figure 8.

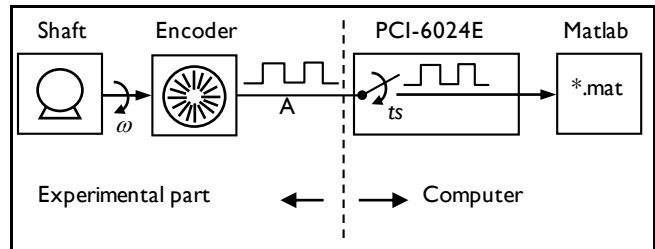


Figure 6. Experimental setup and signal processing scheme

The algorithm's verification required an actual value of  $\omega$  to compute relative error. Instead of using a highly accurate sensor for obtaining the aforementioned value, the proposed algorithm found estimations using the best parameters for the smallest feasible error. For instance, ideal parameters were  $pul = 160$  and  $dt = 2$  s. These parameters guaranteed a boundary for relative error = 0.07%, when  $\omega = 2$  rev/s, as shown in Equation 12. This error decreased to 0.0065%, if  $\omega$  equalled 24 rev/s.

The first experiment had the next parameters:  $pul = 160$  and 1 ms for  $dt$ ,  $dt$  thus overcoming  $ts$  by a factor of ten. In order to capture the data, the motor ran at constant voltage; a computer then saved data for ten seconds starting only when the motor entered its stationary stage. The algorithm obtained estimations using the recorded data as input. The same steps were repeated every 0.5 V for constant voltages from 3 to 24 V.

The solid line in Figure 7 represents the bound for relative error per interval, as indicated in Equations 4, 8 and 13. The dots represent the experimental values for relative errors:  $e_{r1}$  for  $\omega_{m1}$ ,  $e_{r2}$  for  $\omega_{m2}$  and  $e_{r3}$  for  $\omega_{m3}$ . Results in Figure 7 agreed with the analysis given in the previous section. Relative errors always fell within bounds, and estimation  $\omega_{m3}$  consistently had minimum relative error.

Other experiments have used  $dt$  equal to 2, 5, 10, 20, 50 and 100 ms. If  $dt$  increased  $\omega_{lim}$  decreased. For instance if  $dt$  were greater than 5 ms,  $\omega_{lim}$  went below experimental shaft speed. The greater the  $dt$ , the larger the  $n_i$  and, as a result, relative error decreased, but updating time increased. So the relationship between  $dt$  and relative error became a trade-off problem, the solution of which depended on its application.

Relative error also decreased by making  $\omega_{lim}$  higher than any experimental speed; this change in  $\omega_{lim}$  value required the use of an encoder having a lower number of slots, or the definition of a smaller  $dt$ . Thus, instead of changing the encoder to run this experiment, only one of every  $k$  pulses of the encoder was recorded. The experiment used  $k = 40$  and  $dt = 1$  ms. So  $\omega_{lim} = 250$  rev/s, and since all speeds fell under 30 rev/s, maximum relative error fell under 10%, as was experimentally proven.

The algorithm was tested on-line using an embedded system for testing the algorithm's application in an electronic device (i.e. dsPIC33FJ128MC802). dsPIC output was estimated as  $\omega_{m3}$ , closing a control loop in Simulink, as shown in the upper part of Figure 8, where  $u$  was the actuating signal. The filter made the system slow enough to have  $ts = 1$  ms and  $dt = 40$  ms.

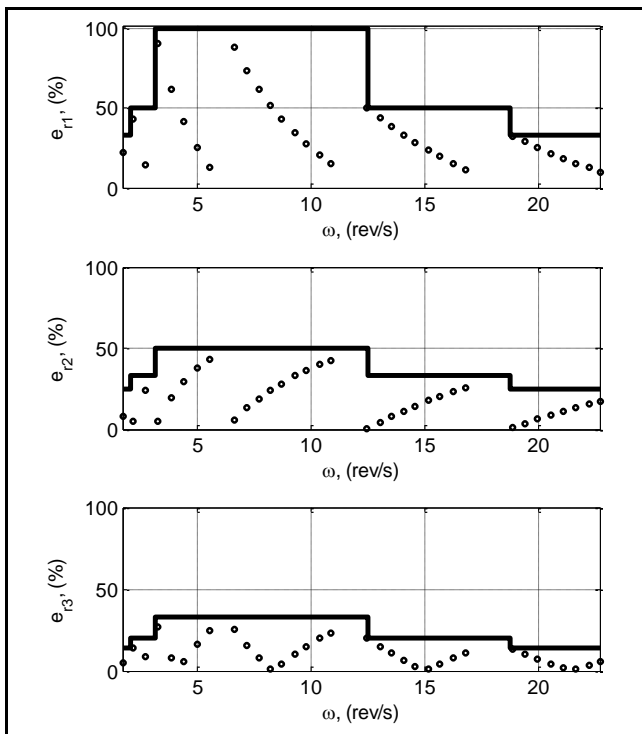


Figure 7. Relative error for  $pul = 160$ ,  $dt = 1$  ms, and  $ts = 100$   $\mu$ s

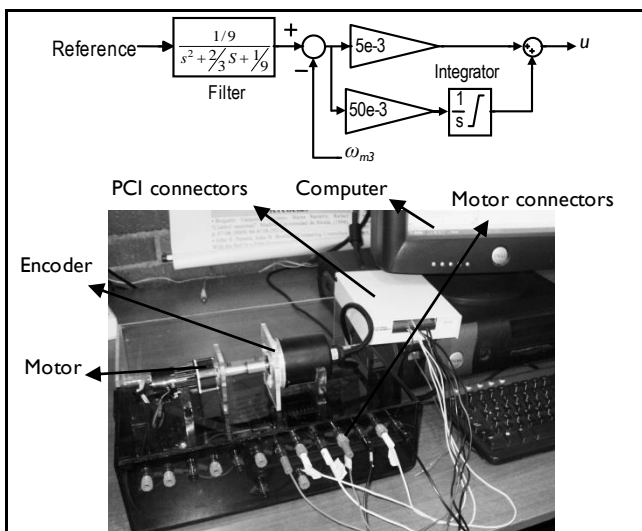


Figure 8. On-line test. Control scheme in Simulink shown in the upper part and experimental setup in the lower part

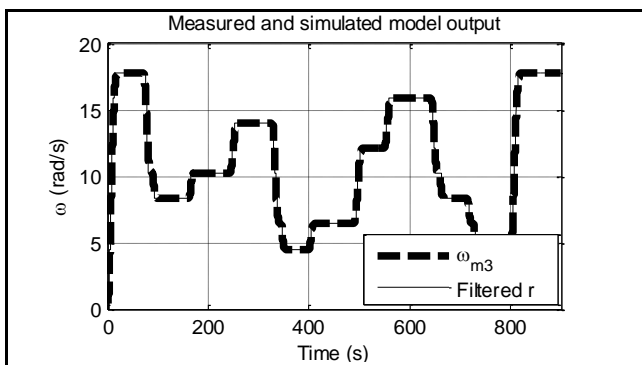


Figure 9. Identification result.

Identification involved using the filtered reference as input and  $\omega_{m3}$  as output. The identification result is shown in Figure 9. The model for the output-input ratio,  $H(S)$ , was  $H(S) = 1/((1+1.512S)(1+1.506S))$ . The model fit 94.41% of the data after 20 iterations, thereby showing the quality of the data and control loop.

## Conclusions

A new algorithm for estimating speed has been proposed proven and experimentally tested (using incremental encoders). One of the the algorithm's advantages was that it had constant output when the speed was constant, contrary to fixed-time and fixed-space algorithms producing two values.

Relative error for the stationary case was computed and compared to fixed-time and fixed-space algorithms, showing that maximum relative error per interval was the same, where an interval was defined as the range of speeds havin the same value as estimation. A modification of the algorithm has been proposed, aimed at reducing maximum relative error by interval. It was proved that the optimal value was the harmonic mean between estimation  $\omega_{m1}$  and  $\omega_{m2}$ . The maximum relative error was always smaller than half of that produced by fixed-time and fixed-space algorithms.

Even when the mathematical proof was made for constant speed, it was shown that the estimation remained valid for acceleration smaller than a given maximum, depending exclusively on  $pul$  and  $dt$  for high speed and  $pul$ ,  $dt$  and  $Ndt$  for lower speed.

Observing experimental data showed the effect of changing parameters  $dt$  and  $pul$  in the encoder; such parameter changes reduced the relative error of the estimation, and could be done using three approaches: increasing  $pul$  and  $dt$  at their highest possible values, reducing  $pul$  and  $dt$  and decreasing  $ts$ .

Some problems of interest remain. For example, it appears that an adaptive algorithm for dynamically changing the value of  $dt$ ,  $ts$  and  $k$  could boost algorithm performance, because making  $\omega_{lim}$  greater or smaller than current speed would reduce error in the estimation. Implementing the algorithm in an embedded system, faster than dsPIC, including additional aspects such as rotation sense (also called sign) would make the algorithm ideal for use in industrial applications.

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