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ОРИГИНАЛНИ НАУЧНИ ЧЛАНЦИ

ОРИГИНАЛЫЕ НАУЧНЫЕ СТАТЬИ

ORIGINAL SCIENTIFIC PAPERS

CONTRACTIVE CONDITIONS IN *b*-METRIC SPACES

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Abstract:

The purpose of this paper is to consider various contractive conditions in b-metric spaces which have been recently published. Our results improve and complement many recent results from this field. Using the recently obtained result by R. Miculescu and A. Mihail (Miculescu & Mihail, 2017, pp.1-11) the authors of this article show that the proofs of the majority of known results in the context of b-metric spaces can be shortened.

Keywords: metric space, common fixed point, altering distance function, point of coincidence, weak compatibility.

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Introduction

It is well known that the Banach Contraction Principle (Banach, 1922, pp.133-181) states that, if a self-mapping T of a complete metric space (M,d) is a contraction mapping, then T has a unique fixed point (say u) and for each $v \in M$ the corresponding Picard sequence $\{T^n(v)\}$ converges to this fixed point u. In general, this principle has been generalized in two directions. On the one hand, the usual contractive condition is replaced by a weakly contractive condition. On the other hand, the action spaces are replaced by metric spaces endowed with an ordered or partially ordered structure or with some kind of generalized metric space (like cone metric space, G-metric space, partial metric space, fuzzy metric space, etc.).

In 1989 I. A. Bakhtin (Bakhtin, 1989, pp.26-37) and in 1993 S. Czerwik (Czerwik, 1993, pp.5-11) introduced a new distance on a non-empty set which is called a *b*-metric. A *b*-metric space is an attempt to generalize the metric space by replacing only the triangle inequality introducing one real constant. Their definition of this new kind of generalized metric space is the following.

Definition 1 (Bakhtin, 1989, pp.26-37), (Czerwik, 1993, pp.5-11) Let M be a (non-empty) set and $K \geq 1$ a given real number. A function $d_1 : M \times M \to [0,\infty)$ is called a *b*-metric on M if, for all $p,q,r \in M$, the following conditions hold:

(b1) $d_1(p,q) = 0$ if and only if $p = q$;

(b2) $d_1(p,q) = d_1(q,p)$;

(b3) $d_1(p,r) \leq K(d_1(p,q) + d_1(q,r))$.

In this case, (M,d_1,K) is called a *b*-metric space. If (M, $\preccurlyeq$) is still a partially ordered set, then $(M, \preccurlyeq, d_1, K)$ is called an ordered *b*-metric space.

Otherwise, for all other definitions of the notions in *b*-metric spaces such as *b*-convergence, *b*-Cauchy sequence, *b*-completeness, see (Abbas et al, 2016, pp.1413-1429), (Ansari et al, 2017, pp.315-329), (Bakhtin, 1989, pp.26-37), (Huang et al, 2015), (Jovanović, 2016), (Radenović et al, 2017a, 2017b), (Roshan et al, 2013), (Zhang et al, 2017, pp.1334-1344) and the reference therein.

Definition 2 (Khan et al, 1984, pp.1-9) A function $\varphi : [0,\infty) \to [0,\infty)$ is called an altering distance function if the following properties hold:

(1) φ is continuous and nondecreasing;

(2) $\varphi(t)=0$ if and only if $t=0$.

First, a very known (important) result from a *b*-metric space is the following:

Theorem 1 (Czerwik, 1993, pp.5-11, Theorem 1) Let (M,d_1,K) be a b-complete *b*-metric space and let $T:M\to M$ satisfy

$$d_1(T(p),T(q))\le\varphi(d_1(p,q)),\ p,q\in M, \tag{1}$$

where $\varphi:[0,\infty)\to[0,\infty)$ is an increasing function such that $\lim_{n\to\infty}\varphi^n(t)=0$ for each fixed $t>0$. Then T has an exactly one fixed point v and

$$\lim_{n\to\infty}d_1(T^n(p),v)=0 \tag{2}$$

for each $p\in M$.

Lemma 1 (Miculescu & Mihail, 2017, Lemma 2.2.) Let $\{t_n\}$ be a sequence in a *b*-metric space (M,d_1,K) such that

$$d_1(t_n,t_{n+1})\le\mu\cdot d_1(t_{n-1},t_n) \tag{3}$$

for some $\mu\in[0,1)$, and each $n=1,2,\ldots$. Then $\{t_n\}$ is a *b*-Cauchy sequence in (M,d_1,K).

Remark 1 In several published papers based on the *b*-metric concept, the authors assume that $\mu\in[0,\frac{1}{K})$ instead of $\mu\in[0,1)$, which is obviously weaker. Then under this weaker condition they show that the Picard sequence $\{t_n=T(t_{n-1})\}_{n=1,2,\ldots}$, $t_0\in M$ is a b-Cauchy. For the proof, the authors used the following clear inequality:

$$d_1(t_m,t_n)\le Kd_1(t_m,t_{m+1})+K^2d_1(t_{m+1},t_{m+2})+\ldots+K^{n-m-1}d_1(t_{n-2},t_{n-1})+K^{n-m}d_1(t_{n-1},t_n), \tag{4}$$

where $n,m\in N$ and $n>m$.

However, putting $\varphi(r)=\mu\cdot r$, $r\in[0,\infty)$, $\mu\in(0,1)$ in (1), the proof of Theorem 1 from (Czervik, 1993, pp.5-11) follows that Picard sequence $\{t_n=T(t_{n-1})\}_{n=1,2,\ldots}$, $t_0\in M$ is a b-Cauchy.

Now, we can show that Lemma 2.2. from (Miculescu & Mihail, 2017) is an immediate consequence of the one part of Theorem 1 from (Czerwik, 1993, pp.5-11).

First of all, we give the next result:

Lemma 2 If $\{t_n\}_{n\in N}$ is an arbitrary sequence in the nonempy set M, then there exists at least one mapping $T: M \to M$ such that it is Picard sequence of T with t_1 as the beginning point.

Proof. We define $T: M \to M$ as $T(t_k) = t_{k+1}$ for $k = 1,2,3,...$ as well as $T(t) = v_0$ in case $t \in M \setminus \{t_1, t_2, ..., t_n, ...\}$ and $v_0 \notin \{t_1, t_2, ..., t_n, ...\}$. The last one is possible if $\{t_1, t_2, ..., t_n, ...\} \subseteq M$ and $\{t_1, t_2, ..., t_n, ...\} \neq M$.

Proposition 1 Lemma 2.2. from (Miculescu & Mihail, 2017) is an immediate consequence of (Czerwik, 1993, pp.5-11, Theorem 1).

Proof. Indeed, the $\{t_n\}$ is a Picard sequence of the mapping defined in Lemma 2. It is obvious that the mapping T satisfies the condition (1) where $\varphi(r) = \mu \cdot r$, $r \in [0, \infty)$, $\mu \in (0,1)$. Further (3) becomes $d_1(T(t_{n-1}), T(t_n)) \le \mu d(t_{n-1}, t_n)$, $n = 2,3,4,..$ i.e. the sequence $\{t_n\}$ is a *b*-Cauchy according to the proof of (Czerwik, 1993, pp.5-11, Theorem 1).

Now, by (Czerwik, 1993, pp.5-11, Theorem 1) that is, by (Miculescu, Mihail, *2017*, Lemma 2.2.), the majority of already known results can be improved. Also, by using the same argument some known results can be made significantly shorter and nicer.

The first such result is the following:

Proposition 2 Let T be a self-map on a b-complete *b*-metric space (M, d_1, K) satisfying

$$d_1(T(p), T^2(p)) \le \mu d_1(p, T(p)) \text{ for some } \mu \in (0,1), \tag{5}$$

either (**i**) for all $p \in M$, or (**ii**) for all $p \in M, p \neq T(p)$, and suppose that T has a fixed point. Then T has a property P.

Otherwise, if T is a map which has a fixed point v, then v is a also a fixed point of T^n for every natural number *n*. However, the converse is false. For, consider $M = [0,1]$, T is defined by $T(p) = 1 - p$. Then T has a unique fixed point at $\frac{1}{2}$, but $T^n = I$ for each $n > 1$, which has every point of $[0,1]$ as a fixed point. On the other hand, if $M = [0, \pi]$, $T(p) = \cos p$, then T is nonexpansive and every iterate of T has the same fixed point as T. Involutions are also examples where $F(T) \neq F(T^n)$. See, e.g. (Jeong & Rhoades, 2005, pp.71-105) and the references therein.

We shall say that a map T has a property P if $F(T) = F(T^n)$ for every $n \in N$.

Proof (of Proposition 2). The statement for $n=1$ is trivial. Therefore, we shall assume that $n>1$ is a given (fixed natural number). It is clear that $F(T)\subseteq F(T^n)$. Let $v\in F(T^n)$.

Case 1. Suppose that T satisfies (i). Then, using (5),

$$\begin{aligned} d_1(v,T(v)) &= d_1(T^n(v),T^{n+1}(v)) \\ &= d_1(T(T^{n-1}(v)),T^2(T^{n-1}(v)))\le \mu d_1(T^{n-1}(v),T(T^{n-1}(v))) \\ &= \mu d_1(T(T^{n-2}(v)),T^2(T^{n-2}(v))) \\ &\le \mu^2 d_1(T^{n-2}(v),T(T^{n-2}(v)))\le \ldots \le \mu^n d_1(v,T(v)), \end{aligned}$$

which implies that $v=T(v)$.

Case 2. Let now T satisfy (ii).

If $v=T(v)$, then there is nothing to prove. Suppose, if possible, that $v\ne T(v)$. Then a repetition of the argument for Case 1 again leads to $d_1(v,T(v))\le \mu^n d_1(v,T(v))$, which implies that $v=T(v)$ and $F(T^n)=F(T)$.

Remark 2 Proposition 1.8. obviously generalize the corresponding result, Theorem 1.1. from (Jeong & Rhoades, 2005, pp.71-105), for standard metric spaces.

Corollary 1 Let T be a selfmap of a b-complete *b*-metric space (M,d_1,K) satisfying

$$d_1(T(p),T(q))\le \mu d_1(p,q) \text{ for all } p,q\in M \text{ and for some } \mu\in(0,1). \tag{6}$$

Then T has a property P.

Proof. Indeed, condition (6) implies (5). Also, by (Czerwik, 1993, pp.5-11, Theorem 1) $F(T)\ne\emptyset$. Then the result follows according to Proposition 2.

The next is also generalization of one result from a metric to a *b*-metric space.

Proposition 3 Let T be a selfmap of a b-complete *b*-metric space (M,d_1) satisfying

$$d_1(T(p),T^2(p))\le \mu d_1(p,T(p)) \text{ for all } p\in M \text{ and some } \mu\in(0,1). \tag{7}$$

Then $F(T)\ne\emptyset$, if T is a $b-$continuous.

Proof. Let $p_0\in M$ be an arbitrary point and let $\{p_n\}$ be a corresponding Picard sequence. For each $n\in\{0\}\cup N$ we have

$$d_1(p_{n+1},p_{n+2})=d_1(T(p_n),T^2(p_n))\le \mu d_1(p_n,T(p_n))=\mu d_1(p_n,p_{n+1}). \tag{8}$$

Further, according to (Miculescu & Mihail, 2017, Lemma 2.2.) (see also (6)) follows that $\{p_n\}$ is a b-Cauchy sequence. Since (M, d_1) is a *b*-complete *b*-metric space there is $v \in M$ such that $p_n \to v$ as $n \to \infty$. The continuity of T implies that $T(v) = v$, i.e., $F(T) \neq \varnothing$.

Jungck's result in the concept of *b*-metric spaces:

Theorem 2 Let (M, d_1, K) be a *b*-metric space and $T, S : M \to M$, $T(M) \subseteq S(M)$ be self mappings such that for all $p, q \in M$.

$$d_1(T(p), T(q)) \leq \mu d_1(S(p), S(q)), \text{ where } \mu \in (0,1). \tag{9}$$

Also, assume that, at least one of the following conditions hold:

(i) $(T(M), d_1)$ or $(S(M), d_1)$ is *b*-complete;

(ii) (M, d_1, K) is *b*-complete, S is *b*-continuous and T and S are commuting.

Then T and S have a unique point of coincidence. Moreover, if T and S are weakly compatible (for case (i)) then they have a unique common fixed point in M.

Proof. First, we notice that if a point of coincidence of T and S exists, then it is unique. Indeed, if w_1 and w_2 are two distinct points of coincidence of T and S, then there exist two points $u_1, u_2 \in M, u_1 \neq u_2$, such that $T(u_1) = S(u_1) = w_1 \neq w_2 = S(u_2) = T(u_2)$. Now, by (9) we have $d_1(w_1, w_2) = d(T(v_1), T(v_2)) \leq \mu d_1(S(v_1), S(v_2)) = \mu d_1(w_1, w_2) < d_1(w_1, w_2)$, which is a contradiction.

Further, the condition $T(M) \subseteq S(M)$ implies that there exists Jungck's sequence $j_n = T(v_n) = S(v_{n+1})$, where $\{v_n\}$ is a sequence in $M, v_0 \in M$ is an arbitrary point. We shall prove that the sequence $\{j_n\}$ is a *b*-Cauchy. Indeed, for each $n \in \{0\} \cup N$ we have that

$$d_1(j_{n+1}, j_{n+2}) = d_1(T(v_{n+1}), T(v_{n+2})) \leq \mu d_1(S(v_{n+1}), S(v_{n+2})) = \mu d_1(j_n, j_{n+1}),$$

i.e., for all $n \in \{0\} \cup N$ the sequence $\{j_n\}$ satisfies condition (3). This means that it is *b*-Cauchy.

Now, let (i) holds. Therefore, since $(S(M), d_1)$ is a *b*-complete *b*-metric space, it follows that there exists $v \in M$ such that $T(v_n) = S(v_{n+1}) = j_n \to Sv$ as $n \to \infty$. We will prove that $T(v) = S(v)$. In order to prove this equality, we have

$$\frac{1}{K}d_1(T(v),S(v)) \le d_1(T(v),T(v_n)) + d_1(T(v_n),S(v)) \le \mu d_1(S(v),S(v_n)) + d_1(j_n,S(v))$$
$$= \mu d_1(S(v), j_{n-1}) + d_1(j_n,S(v)) \to \mu \cdot 0 + 0 = 0.$$

Hence, $T(v)=S(v)=w$ is a point of coincidence (unique) of the pair (T,S).

If $(T(M),d_1)$ is a *b*-complete the proof is very similar.

If (ii) holds, then since (M,d_1) is *b*-complete, there exists $v \in M$ such that $T(v_n)=S(v_{n+1})=j_n \to v,$ as $n \to \infty$. Since both self-mappings T and S are *b*-continuous, we have when $n \to \infty$:

$$S(T(v_n)) \to S(v) \text{ and } T(S(v_n)) \to T(v) \text{ when } n \to \infty.$$

Since T and S are commuting, we again obtain that $T(v)=S(v)=w$ is a point of coincidence (unique) of the pair (T,S).

For both cases (i) and (ii), according to the known Jungck's result, it follows that w is a unique common fixed point of T and S.

The next is a common fixed point theorem of the Zamfirescu type in $b-$metric spaces.

Theorem 3 (Jovanović, 2016), (Khan et al, 1984, pp.1-9), (Rhoades, 1977, pp.257-290, Theorem 4.3.) Let (M,d_1,K) be a *b*-complete *b*-metric space and let $T: M \to M$ be a mapping and let there exist nonnegative numbers a,b,c such that for all $p,q \in M$ at least one of the following conditions:

1^0 $d_1(T(p),T(q)) \le a d_1(p,q)$;

2^0 $d_1(T(p),T(q)) \le b[d_1(p,T(p)) + d_1(q,T(q))]$;

3^0 $d_1(T(p),T(q)) \le c[d_1(p,T(q)) + d_1(q,T(p))]$

holds.

If $a < \frac{1}{K}, b < \frac{1}{2K^2}, c < \frac{1}{2K^2}$ then T has a unique fixed point.

Remark 3 By using (Miculescu & Mihail, 2017, Lemma 2.2) the conditions for a,b,c can be relaxing, that is., we get $a < 1, b < \frac{1}{2}$ i $c < \frac{1}{2K}$ (for details see Theorem 2.2. below).

Main results

In this section, we shall consider several important as well as significant contractive conditions announced in the existing literature. Readers can compare all these conditions to the corresponding ones in the context of standard metric spaces, for more details see (Rhoades, 1977, pp.257-290).

Let Ψ_1 be the family of all nondecreasing functions $\psi_1 : [0,\infty) \to [0,\infty)$ such that $\lim_{n\to\infty} \psi_1^n(t) = 0, \text{ for all } t > 0.$

It is well known that if $\psi_1 \in \Psi_1$ then $\psi_1(t) < t$ if $t > 0$ as well as $\psi_1(0) = 0.$

Our first result is the improvement of the proof in (Abbas et al, 2016, pp.1413-1429, Theorem 2.2.)

Theorem 4 Let $(M, \preccurlyeq, d_1, K > 1)$ be a partially ordered b-complete b-metric space and let T: M → M be an increasing mapping with respect to $\preccurlyeq$ such that there exists an element $p_0 \in M$ with $p_0 \preccurlyeq T(v_0)$. Assume that

$$K \cdot \frac{1 + K \cdot d_1(p,q)}{1 + \frac{1}{2} d_1(p, T(p))} \cdot d(T(p), T(q)) \le \psi_1(M_1(p,q)) + L_1 \cdot N_1(p,q) \tag{10}$$

for all comparable elements $p, q \in M$, where $L_1 \ge 0, \psi_1 \in \Psi_1$,

$$M_1(p,q) = \max\left\{ d_1(p,q), \frac{d_1(p,T(p)) d_1(q,T(q))}{1 + d_1(T(p),T(q))} \right\} \tag{11}$$

and

$$N_1(p,q) = \min\{d_1(p,T(p)), d_1(p,T(q)), d_1(q,T(p)), d_1(q,T(q))\}. \tag{12}$$

If T is continuous, then T has a fixed point.

Proof. If $p_0 \ne T(p_0)$ then $p_0 \prec T(p_0)$. Further, for the Picard sequence we can assume that $d_1(p_n, p_{n+1}) > 0$ for all $n \in \{0\} \cup N$. Now, we will prove that

$$d_1(p_n, p_{n+1}) \le \frac{1}{K} d_1(p_{n-1}, p_n), \text{for all } n \in N. \tag{13}$$

Indeed, since

$$\frac{1+Kd_1(p_{n-1},p_n)}{1+\frac{1}{2}d_1(p_{n-1},T(p_{n-1}))}=\frac{1+Kd_1(p_{n-1},p_n)}{1+\frac{1}{2}d_1(p_{n-1},p_n)}>\frac{1+d_1(p_{n-1},p_n)}{1+\frac{1}{2}d_1(p_{n-1},p_n)}>1,$$

then by using (10) with $p=p_{n-1},q=p_n$, we obtain

$$Kd_1(p_{n+1},p_n)=Kd_1(T(p_n),T(p_{n-1}))\le\psi_1(d_1(p_n,p_{n-1}))+L_1\cdot N_1(p_n,p_{n-1}).$$

Because $M_1(p_n,p_{n-1})=d_1(p_{n-1},p_n),\psi_1(d_1(p_n,p_{n-1}))<d_1(p_n,p_{n-1})$ and $N_1(p_n,p_{n-1})=0$ we obtain that (13) holds.

This means that the sequence $\{p_n\}$ is a *b*-Cauchy, according to Lemma 2.2. from (Miculescu & Mihail, 2017) which then converges to some $u\in M$. The continuity of T implies that u is a fixed point of T.

Remark 4 *All that shows that our approach gives a much shorter and nicer proof than the ones in (Ansari et al, 2017, pp.315-329). Also, by the same method, the proofs of all results in (Ansari et al, 2017, pp.315-329) can be improved.*

In fact, the main (important) question is the following: Does some given contractive condition in the framework of any class of generalized metric spaces imply (give) that the corresponding Picard sequence is a Cauchy (in this class)? The previously contractive condition is such. We proved that for it holds $d_1(p_n,p_{n+1})\le\mu d_1(p_{n-1},p_n)$ for all $n\in N$ and some $\mu\in(0,1)$. Since $K>1$ and $\mu=\frac{1}{K}$ then the result follows by (Miculescu & Mihail, 2017, Lemma 2.2.).

In the framework of $b-$metric spaces, the following two results are specific.

Theorem 5 *Let* (M,d_1,K) *be a b-complete b-metric space and let* $T:M\to M$ *be a b-continuous mapping. Also let*

$$d_1(Tp,Tq)\le ad_1(p,Tp)+bd_1(q,Tq),\ \text{for all } p,q\in M, a,b\ge 0, a+b<1$$

that is

$$d_1(Tp,Tq)\le ad_1(p,Tq)+bd_1(Tp,q),\ \text{for all } p,q\in M, a,b\ge 0, a+b<\frac{1}{K}$$

In each of the given cases, T has a unique fixed point (say v) and for any $u\in M$ the sequence $\{T^n(u)\}\to v$ as $n\to\infty$.

Proof. In the first (Kannan) case, we obtain that $d_1(p_{n+1}, p_n) \leq \frac{a}{1-b} \cdot d_1(p_n, p_{n-1})$, while in the second one (Chatterjea), we have $d_1(p_{n+1}, p_n) \leq \frac{(a+b)K}{2-(a+b)K} \cdot d_1(p_n, p_{n-1})$. According to (Miculescu, & Mihail, 2017, Lemma 2.2.) it follows that in both cases that the Picard sequence $\{T^n p_0\}_{n\in\{0\}\cup N}, p_0 \in M$ is a *b*-Cauchy. Since T is *b*-continuous, the result follows.

Conclusion

Based on the previous discussion, we can conclude that the proofs of the majority results in the existing literature for the concept of b-metric spaces can be significantly shortened by using (Miculescu & Mihail, 2017, Lemma 2.2.).

All these results are in the following papers (Aghajani et al. 2014, pp. 941-960), (Allahyar et al, 2014), (Chandok et al, 2017a), (Chandok et al, 2017b, pp.331-345), (Demma & Vetro, 2015), (Ding et al, 2016, pp. 151-164), (Dung & Hang, 2016, pp. 267-284), (Harandi, 2014, pp. 351-358), (Kaushik et al, 2017), (Khamsi & Husain, 2010, pp. 3123-3129), (Kir & Kiziltunc, 2013, pp. 13-16), (Kumam et al, 2015), (Latif et al, 20915, pp. 363-377), (Liu & Gu, 2016, pp. 5909-5930), (Ozturk & Ansari, 2017, pp. 45-52), (Parvaneh et al, 2013), (Petrusel et al, 2017, pp. 199-215), (Piri & Kumam, 2016), (Roshan et al, 2014a, pp. 725-737), (Roshan, et al, 2015), (Roshan et al, 2014b, pp. 613-624), (Sarwar et al, 2017, pp. 3719-3731), (Sarwar & Rahman, 2015, pp. 70-78), (Sintunavarat, 2016, pp. 397-416), (Zabihi & Razani, 2014).

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УСЛОВИЯ СЖАТИЯ В *b*-МЕТРИЧЕСКИХ ПРОСТРАНСТВАХ

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Резюме:

В данной работе представлен анализ различных условий сжатия в b-метрических пространствах, которые недавно были опубликованы. На основании исследований настоящих результатов мы дополнили и откорректировали многие аспекты результатов в данной области. Так, например, исследовав недавние результаты, полученные Р. Микулеску и А. Михаилом (Miculescu & Mihail, 2017, pp.1-11), авторы настоящей статьи доказали, что многие известные результаты в контексте b-метрических пространств могут быть значительно сокращены.

Ключевые слова: метрическое пространство, общая фиксированная точка, функция изменения расстояния, точка совпадения, низкая совместимость.

КОНТРАКТИВНИ УСЛОВИ У *b*-МЕТРИЧКИМ ПРОСТОРИМА

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Сажетак:

Циљ овог рада јесте да размотри разне контрактивне услове у b-метричким просторима који су недавно објављени. Наши резултати поправљају и допуњују многе недавне резултате из *овог контекста. Користећи недавно добијени резултат Р. Микулескуа и А. Михаила, (Miculescu & Mihail, 2017, pp.1-11) аутори овог чланка показују да докази многих познатих резултата у контексту b-метричких простора могу бити доста скраћени.*

Кључне речи: метрички простор, заједничка фиксна тачка, функција промене раздаљине, тачка коинциденције, слаба компатибилност.

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