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# FUZZIFICATION OF THE SAATY'S SCALE AND A PRESENTATION OF THE HYBRID FUZZY AHP-TOPSIS MODEL: AN EXAMPLE OF THE SELECTION OF A BRIGADE ARTILLERY GROUP FIRING POSITION IN A DEFENSIVE OPERATION

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*Summary:*

*This paper presents the new way of fuzzification of the Saaty's scale. In this model of fuzzification, the confidence interval of fuzzy numbers that describes the comparison in pairs' degree is not determined before the comparison. It is defined (calculated) during and after the comparison, based on the degree of certainty of decision-makers/experts. Thus, the confidence interval can vary depending on the comparison, no matter if it refers to the same degree of comparison. Also, in group deciding, confidence intervals differ depending on the decision-maker/expert's opinion. Such explanation is supported by the new fuzzyficated Saaty's scale. The application of the new scale is shown in the hybrid fuzzy AHP-TOPSIS model when choosing firing positions of an artillery brigade group in a defensive operation. Finally, the impact of the degree of certainty on the final decision is analyzed, where it is demonstrated that this element affects output results. The impact is reflected in the change of the size of weight vectors, i.e. output results (the distance from the ideal alternative), as well as in the change of the rank of alternatives, but only at limit values.*

Key words: *Fuzzy logic, Firing position, Hybrid model, TOPSIS, AHP.*

# Introduction

Decision making is one of the most important management elements. It has been empirically proven that decision making makes even 92% of a manager's job structure (Čupić, Suknović, 2010, p.xxv). Considerable attention is paid to the decision-making process in military organizations. The reason for this can be found in the fact that a man is in the center of every decision, and it is not expected that all people respond equally to situations in which they find themselves, as expressed particularly in combat operations, where the consequences of wrong decisions can often be irreparable (Pamučar, et al., 2011a, p.3). In the Army of Serbia many decisions are made in the processes of planning, organization and preparation for the execution of missions and tasks. The useful tools which support the decision-making process are the methods of multi criteria decision making.

This paper presents a hybrid model, using the fuzzificated Saaty's scale and the TOPSIS method (Technique for Order Preference by Similarity to Ideal Solution). The paper is focused on the demonstration of a new way of fuzzification of the Saaty's scale used for comparison in pairs with varying a confidence interval depending on the comparison. The scale is used for obtaining criteria weight coefficients, while the TOPSIS method is used for the final ranking. This model is illustrated by an example of decision making during the selection of a brigade artillery group (BrAG) firing position area in a defensive operation. The example presents only one segment from a series of decisions that decision makers face in the preparation and execution of (military) operations.

# Fuzzy logic and fuzzy sets

In conventional logic, belonging of an element to the given set is strictly defined, i.e., an element can belong or not belong to the set. In fuzzy logic, belonging of an element to the specific set is not precisely defined - the element can be more or less part of the set; therefore, it is closer to human perception than conventional logic (Pamučar, et al., 2011b, p.594). Fuzzy logic allows quantification of seemingly imprecise information, which is a very common situation when describing social phenomena.

The first step in designing fuzzy sets is defining the degree of the membership of an element x ($x \in X$) to the set A. This is described with the membership function $\mu_A(x)$, which in the classic theory has a value of 0 (does not belong) or 1 (belongs), while in a fuzzy set the membership function can have any value between 0 and 1. So, it can be said that the closer the $\mu_A(x)$ is to 1, the belonging of the x to A is greater, and vice versa. Every fuzzy set is completely and uniquely defined by its membership function (Zadeh, 1965). A fuzzy set is defined as a set of ordered pairs

$$A\left\{\left(x,\mu_A(x)\right)\middle|x\in X,0\le\mu_A(x)\le 1\right\}. \quad (1)$$

where:
– X is a universal set or a set of considerations based on which the fuzzy set A is defined;
– $\mu_A(x)$ is a membership function of the element x to the set A.

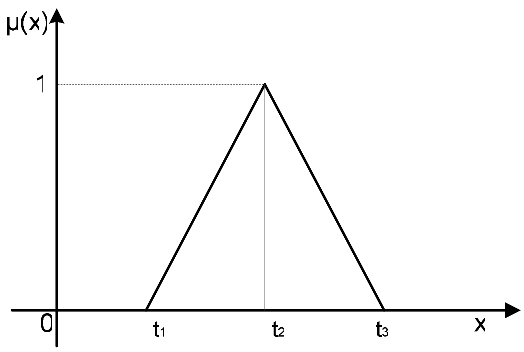
The membership function forms and the width of the confidence interval are usually selected on the basis of subjective assessment or experience, so that they best describe the phenomenon they represent. In practice, a variety of membership functions is used: triangular, trapezoidal, Gaussian, etc.

In this paper, triangular fuzzy numbers will be used. They will be presented in the form T = ($t_1$, $t_2$, $t_3$), where (Figure 1):

– $t_2$ is where the membership function of a fuzzy number has a value of 1;

– $t_1$ is the left distribution of the confidence interval of the fuzzy number T, and

– $t_3$ is the right distribution of the confidence interval of the fuzzy number T (Pamučar, 2011, p.45).




*Picture 1* – Triangular fuzzy number T
*Фигура 1* – Треугольное нечеткое число T
*Slika 1* – Trouglasti fuzzy broj T

The membership function of the fuzzy number T is defined in the following way:

$$\mu_{\tilde{T}}(x)=\begin{cases} 0, & x<t_1 \\ \dfrac{x-t_1}{t_2-t_1}, & t_1\le x\le t_2 \\ 1, & x=t_2 \\ \dfrac{t_3-x}{t_3-t_2}, & t_2\le x\le t_3 \\ 0, & x>t_3 \end{cases} \tag{2}$$

For its final purpose, the fuzzy number $T=(t_1, t_2, t_3)$ is converted into a real number. Different methods are used for this procedure (Herrera, Martínez, 2000).

## Fuzzification of the Saaty's scale

The Analytical Hierarchy Process method belongs to the group of light optimization methods. It is based on the interpretation of complex problems in a hierarchy, with an aim at the top and criteria, sub-criteria and alternatives at the levels and sub-levels of the hierarchy (Saaty, 1980). One of the key phases in the application of this method is the development of the comparison matrix by pairs, corresponding to every level of the hierarchy. A pairwise comparison is performed according to the data collected and by measuring them, as well as based on the beliefs, estimates or experiences of those who carry out the assessment (Čupic, Suknović, 2010). The main problem in the pairwise comparison is to quantify linguistically formulated selections or phrases (Kujačić, 2001). Different evaluation scales are developed for this purpose. Basic approaches in its development are: linear (Ma, Zheng, 1990) and exponential (Lootsma, 1988). Previous analyzes have shown that there is no ideal scale, but their quality varies from case to case (Triantaphyllou, et al., 1988). The standard for the AHP method presents the Saaty's scale, Table 1 (Saaty, 1980).

*Table 1* – Saaty's scale for comparison in pairs
*Таблица 1* – Шкала Саати парных сравнений
*Tabela 1* – Saaty-jeva skala za poređenje u parovima

| Definition | Standard values | Inverse values |
|---|---|---|
| The same importance | 1 | 1 |
| Low dominance | 3 | 1/3 |
| High dominance | 5 | 1/5 |
| Very high dominance | 7 | 1/7 |
| Absolute dominance | 9 | 1/9 |
| Intermediate values | 2, 4, 6, 8 | 1/2, 1/4, 1/6, 1/8 |

The Saaty's scale is applied by decision-makers or analysts performing comparisons in pairs on the basis of semantic preferences from the left column of the Saaty's scale, or by direct association. Number values in columns two or three of Table 1, which correspond to the semantic preferences in the left column, are entered in the square comparison matrix.

$$A = \begin{matrix} & C_1 & C_2 & \dots & C_n \\ C_1 & a_{11} & a_{12} & \dots & a_{1n} \\ C_2 & a_{12} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ C_n & a_{n1} & a_{n2} & \dots & a_{nn} \end{matrix} \qquad (3)$$

Since it is true that $a_{ji} = 1/a_{ij}$ and $a_{ii} = 1$ for every $i, j = 1, 2, ..., n$, the matrix A is positive, symmetrical and reciprocal. The essential information on elements of preferences is located only in the upper triangle of the matrix, but all methods for its further analysis use the reciprocal value from the lower triangle (Bozanic, et al., 2013).

When applying the classic Saaty's scale, relations in the pairwise comparison are strictly defined. However, very often, when defining these values, one cannot be completely sure of the relations of the pairs compared. Therefore, in the literature there is an increasing number of papers which approach the fuzzification of the Saaty's scale in various ways ((Ray, Triantaphyllou, 1999), (Zhu, et al., 1999), (Chen, 2007), (Srđević, et al., 2008), (Gardašević-Filipovic, Šaletić, 2010), (Janacković, et al., 2013), (Rezaei, et al. 2014), (Janjić, et al., 2014) and others). Most authors use a predetermined interval of a fuzzy number in the fuzzification, i.e., preset the left and the right distribution of the most commonly used triangular fuzzy number T = ($t_1$, $t_2$, $t_3$). Some authors have recognized the necessity to leave the possibility of some uncertainty, such as in (Božanić, et al., 2011), (Božanić, et al., 2013), (Pamučar, et al., 2011c) (Pamučar, et al., 2012), (Pamučar, 2013) (Pamučar, et al., 2015), (Božanić, et al., 2015b), where the level of uncertainty for the whole scale is pre-defined, based on which the left and right distribution of the fuzzy number T are calculated. This level of uncertainty, i.e., the confidence interval, changes depending on the case or depending on the decision-maker.

On the basis of the Saaty's scale and the idea that the confidence interval of the fuzzy number does not always have to be identical, as shown in (Božanić, et al., 2011), (Božanić, et al., 2013), (Pamučar, et al., 2011c) (Pamučar, et al., 2012), (Pamučar, 2013) (Pamučar, et al., 2015), (Božanić, et al., 2015b), a new scale is defined, Table 2 (Božanić, Pamučar), (Božanić, et al., 2015a), (Božanić, et al., 2016). Defining this new fuzzyficated Saaty's scale has started from the assumption that decision makers and analysts have a different degree of certainty $\gamma_{ji}$ concerning the accuracy of comparisons in pairs. This degree of certainty differs from one comparative pair to the other. The value of the degree of certainty belongs to the interval $\gamma_{ji} \in [0, 1]$. In the cases when $\gamma_{ji}=0$, it is considered that the decision-maker/analyst has no data about this relationship, so it should not be used in the decision-making process, because it points to the absolute ignorance of the decision-making subject. The value of the degree of certainty where $\gamma_{ji}=1$ describes the absolute certainty of decision-makers and analysts in the defined comparison. The lower the certainty in the performed comparison is, the lower the element $\gamma_{ji}$.

*Table 2* – Fuzzified Saaty's scale for comparison in pairs (Božanić, Pamučar), (Božanić, et al., 2015a), (Božanić, et al., 2016)
*Таблица 2* – Фаззифицированная шкала Саати парных сравнений (Božanić, Pamučar), (Božanić, et al., 2015a), (Božanić, et al., 2016)
*Tabela 2* – Fazifikovana Saaty-jeva skala za poređenje u parovima (Božanić, Pamučar), (Božanić, et al., 2015a), (Božanić, et al., 2016)

| Definition | Standard values | Fuzzy number | Inverse values of the fuzzy number |
|---|---|---|---|
| The same importance | *1* | (1, 1, 1) | (1, 1, 1) |
| Low dominance | *3* | $(3\gamma_{ji},3,(2-\gamma_{ji})3)$ | $(1/(2-\gamma_{ji})3,1/3,1/3\gamma_{ji})$ |
| High dominance | *5* | $(5\gamma_{ji},5,(2-\gamma_{ji})5)$ | $(1/(2-\gamma_{ji})5,1/5,1/5\gamma_{ji})$ |
| Very high dominance | *7* | $(7\gamma_{ji},7,(2-\gamma_{ji})7)$ | $(1/(2-\gamma_{ji})7,1/7,1/7\gamma_{ji})$ |
| Absolute dominance | *9* | $(9\gamma_{ji},9,(2-\gamma_{ji})9)$ | $(1/(2-\gamma_{ji})9,1/9,1/9\gamma_{ji})$ |
| Intermediate values | *2, 4, 6, 8* | $(x\gamma_{ji},x,(2-\gamma_{ji})x)$, | $(1/(2-\gamma_{ji})x,1/x,1/x\gamma_{ji})$ $x=2,\ 4,\ 6,\ 8$ |

By defining different values of the parameter $\gamma_{ji}$, the left and the right distribution of fuzzy numbers change from comparison to comparison, according to the expression:

$$T=(t_1,t_2,t_3)=\begin{cases} t_1=\gamma t_2, & t_1\le t_2, \quad t_1,t_2\in[1/9,9] \\ t_2=t_2, & t_2\in[1/9,9] \\ t_3=(2-\gamma)t_2, & t_3\le t_2, \quad t_2,t_3\in[1/9,9] \end{cases} \tag{4}$$

the value of $t_2$ represents the value of linguistic expressions from the classic Saaty's scale, which in a fuzzy number has a maximum membership $t_2 = 1$.

The fuzzy number $T=(t_1,t_2,t_3)=\left(x\gamma_{ji},x,(2-\gamma_{ji})x\right)$, $x\in[1,9]$ is defined by the expressions:

$$t_1=x\gamma_{ji}=\begin{cases}x\gamma_{ji}, \ \forall\ 1\le x\gamma_{ji}\le x\\ 1, \quad \forall\ x\gamma_{ji}<1\end{cases} \tag{5}$$

$$t_2=x,\ \forall\ x\in[1,9] \tag{6}$$

$$t_3=(2-\gamma_{ji})x, \forall x\in[1,9] \tag{7}$$

The inverse fuzzy number $T^{-1}=(1/t_3,1/t_2,1/t_1)=\left(1/(2-\gamma_{ji})x,1/x,1/\gamma_{ji}x\right)$, $x\in[1,9]$ is defined as follows:

$$1/t_3=1/(2-\gamma_{ji})x=\begin{cases}1/(2-\gamma_{ji})x,\ \forall\ 1/(2-\gamma_{ji})x<1\\ 1,\ \forall\ 1/(2-\gamma_{ji})x\ge 1\end{cases} \tag{8}$$

$$1/t_2=1/x,\ \forall\ 1/x\in[1,9] \tag{9}$$

$$1/t_1=1/\gamma_{ji}x, \forall 1/x\in[1,9] \tag{10}$$

The defined scale is further used in standard steps of the AHP method, which is described in a number of papers (Saaty, 1980), (Lootsma, 1988), (Nikolić, Borović, 1996), (Srđević, Srđević, 2004) (Čupić, Suknović, 2010), (Karović, Pušara, 2010), (Devetak, Terzić, 2011), (Inđić, et al., 2014) and others.

Based on the pre-defined scale, decision-makers and analysts fill in the new, modified matrix:

$$A=\begin{array}{c} \\ C_1\\ C_2\\ \vdots\\ C_n\end{array}\begin{array}{c}\begin{array}{cccc}C_1 & C_2 & \dots & C_n\end{array}\\ \begin{bmatrix}a_{11};\gamma_{11} & a_{12};\gamma_{12} & \dots & a_{1n};\gamma_{1n}\\ a_{21};\gamma_{21} & a_{22};\gamma_{22} & \dots & a_{2n};\gamma_{2n}\\ \vdots & \vdots & \ddots & \vdots\\ a_{n1};\gamma_{n1} & a_{n2};\gamma_{n2} & \dots & a_{nn};\gamma_{nn}\end{bmatrix}\end{array} \tag{11}$$

As it can be seen, the matrix is extended with the degree of certainty in the comparison made, whereby $\gamma_{ji}=\gamma_{ij}$, where $\gamma_{ji}\in[0,1]$. After the calculation is finished, the defuzzification can be performed using one of well-known methods. Some of well-known expressions for defuzzification (Seiford, 1996) are the following ones:

$$A=((t_3-t_1)+(t_2-t_1))/3+t_1 \tag{12}$$

$$A=\left[\lambda t_3+t_2+(1-\lambda)t_1\right]/2 \tag{13}$$

where λ represents the degree of optimism (Božanić, et al., 2015b). Also, it is possible to perform further calculations using fuzzy weight coefficients without defuzzification, which would be done at the end of the calculation of the criteria functions of alternatives.

The scale presented can be implemented in the classic application of the AHP method, where weight coefficients are calculated first, and then criteria functions are evaluated for every alternative studied. The scale is also suitable for the evaluation of criteria weights with the aim of later implementation of some other method.

The defined scale is also suitable for group decision making, which has recently become more and more popular. Involving experts greatly improves the quality of decisions made because knowledge and experience are collected and consolidated into a single unit. The most widely used approach in data collecting by experts is the Delphi method (Lootsma, 1988). The scale defined in this paper is applied in group decision making, as well as in the standard AHP method (more information in (Srđević, Zoranović, 2003) (Zoranović, Srđević, 2003)).

## The TOPSIS method

The TOPSIS method was developed by Hwang and Joon (1981). This method consists in ranking alternatives by multiple criteria comparisons based on the distance from the ideal solution and the negative ideal solution. The ideal solution minimizes the cost type criteria and maximizes the benefit type criteria, while the negative ideal solution is reverse (Srđević, et al., 2002). The optimum alternative is the one that is the closest in a geometrical sense to the ideal solution, i.e., the farthest from the negative ideal solution (Srđević, et al., 2002).

The starting point of this method is the initial decision-making matrix.

$$
\begin{array}{c} \\ \\ \\ \\ P = \end{array}
\begin{array}{c} \\ \\ A_1 \\ A_2 \\ A_3 \\ \cdot \\ \cdot \\ \cdot \\ A_n \end{array}
\begin{array}{c}
\begin{array}{cccccc} C_1 & C_2 & C_3 & \cdot & \cdot & C_m \\ w_1 & w_2 & w_3 & \cdot & \cdot & w_m \end{array} \\
\begin{bmatrix} r_{11} & r_{12} & r_{13} & \cdot & \cdot & \cdot & r_{1m} \\ r_{21} & r_{22} & r_{23} & \cdot & \cdot & \cdot & r_{2m} \\ r_{31} & r_{32} & r_{33} & \cdot & \cdot & \cdot & r_{3m} \\ \cdot & \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & \cdot & & & & \cdot \\ r_{n1} & r_{n2} & r_{n3} & \cdot & \cdot & \cdot & r_{nm} \end{bmatrix}
\end{array} \tag{14}
$$

With the decision-making matrix, the *n* alternatives and the *m* criteria are defined. The weight of the criteria $w_i$ is joined to each criterion. Criteria weights should meet the following requirement:

$$\sum_{i=1}^{n} w_i = 1 \tag{15}$$

After defining the decision-making matrix, the TOPSIS method can be implemented. The application of the method can be divided into six steps:

– *First step*: normalization of the decision-matrix values:

$$x_{ij} = r_{ij}\left[\sqrt{\sum_{i=1}^{n} r_{ij}^{2}}\right]^{-1} \tag{16}$$

After the application of expression (16), a new dimensionless matrix is obtained:

$$P' = \begin{array}{c} \\ \\ A_1 \\ A_2 \\ A_3 \\ \vdots \\ A_n \end{array} \begin{array}{c} \begin{matrix} C_1 & C_2 & C_3 & \cdots & C_m \\ w_1 & w_2 & w_3 & \cdots & w_m \end{matrix} \\ \begin{bmatrix} x_{11} & x_{12} & x_{13} & \cdots & x_{1m} \\ x_{21} & x_{22} & x_{23} & \cdots & x_{2m} \\ x_{31} & x_{32} & x_{33} & \cdots & x_{3m} \\ \vdots & \vdots & \vdots & & \vdots \\ x_{n1} & x_{n2} & x_{n3} & \cdots & x_{nm} \end{bmatrix} \end{array} \tag{17}$$

– *Second step*: multiplication of the normalized values and the criteria weight coefficients:

$$v_{ij} = x_{ij} w_j ; j = 1, 2, ..., m \tag{18}$$

After the application of expression (18), a new matrix is obtained:

$$P'' = \begin{array}{c} \\ \\ A_1 \\ A_2 \\ A_3 \\ \vdots \\ A_n \end{array} \begin{array}{c} \begin{matrix} C_1 & C_2 & C_3 & \cdots & C_m \\ w_1 & w_2 & w_3 & \cdots & w_m \end{matrix} \\ \begin{bmatrix} v_{11} & v_{12} & v_{13} & \cdots & v_{1m} \\ v_{21} & v_{22} & v_{23} & \cdots & v_{2m} \\ v_{31} & v_{32} & v_{33} & \cdots & v_{3m} \\ \vdots & \vdots & \vdots & & \vdots \\ v_{n1} & v_{n2} & v_{n3} & \cdots & v_{nm} \end{bmatrix} \end{array} \tag{19}$$

– *Third step*: determination of ideal solutions. The ideal and negative ideal solution are obtained using the following expression:

$$A^{*} = \left\{ \left( \max v_{ij} \mid j \in C \right) \left( \min v_{ij} \mid j \in C' \right), i = 1, 2, ..., n \right\} = \left\{ v_1^{*}, v_2^{*}, ..., v_m^{*} \right\} \tag{20}$$

$$A^{-} = \left\{ \left( \min v_{ij} \mid j \in C \right) \left( \max v_{ij} \mid j \in C' \right), i = 1, 2, ..., n \right\} = \left\{ v_1^{-}, v_2^{-}, ..., v_m^{-} \right\} \tag{21}$$

where: C is the set of benefit type criteria, and C' is the set of cost type criteria.

– *Fourth step*: determination of the distance of alternatives from the ideal solution.

$$S_i^* = \sqrt{\sum_{j=1}^{m} \left(v_{ij} - v_j^*\right)^2}, i = 1, 2, ..., n \tag{22}$$

$$S_i^- = \sqrt{\sum_{j=1}^{m} \left(v_{ij} - v_j^-\right)^2}, i = 1, 2, ..., n \tag{23}$$

– *Fifth step*: determination of the relative proximity of alternatives to the ideal solution. Determination of the distance from the ideal alternative is performed by applying the expression:

$$Q_i^* = \frac{S_i^-}{S_i^* + S_i^-}, i = 1, 2, ..., n \tag{24}$$

– *Sixth step*: ranking alternatives. Alternatives are ranked on the basis of the results obtained by applying expression (24). The best alternative is considered the alternative whose value $Q_i^*$ is the highest, and vice versa.

# Selection of a BrAG firing position area by using the hybrid FAHP-TOPSIS model

## *Description of the problem and defining the criteria*

A brigade artillery group is a temporary formation of artillery units, formed at the tactical level. It is intended for artillery firing support of one's own forces in combat operations of a land forces brigade and territorial brigades (Vojni leksikon, 1981, p.72). A BrAG performs its tasks from firing positions (primary, suplementary, alternate, temporary). A firing position (FP) is a part of the land in the area of the operation, prepared and occupied or intended to be occupied by the artillery units for the execution of firing support (Vojni leksikon, 1981, p.658). An FP area of a BrAG is determined depending on its purpose, type of weapons and ammunition, targets of artillery firing support, combat, space, time and other conditions.

There are rules and regulations governing fundamental criteria of grouping artillery and its deployment into action, as well as a part of the conditions an FP area should meet in order to be selected as an alternative. However, in the available literature, this problem has not been fully developed, systematized, nor the method of selection of an FP area has been elaborated (neither criteria are precisely defined, nor their weight values and their relationship, nor the particularities of various combat operations). Consequently, decision-makers usually have to select an FP of BrAG area relying on the acquired theoretical knowledge, experience and assessment in the specific situation. A number of criteria that influence the ranking and selection of alternatives indicate a possibility of applying multiple criteria methods.

For the purposes of the selection of the best firing position for a BrAG in a defensive operation, the following six criteria are defined (Kurtov, et al., 2014, p.708):

$C_1$ - "*distance from the ideal location for action*" - the ideal location is generally defined as 1/3 of the rank of artillery weapon from the front end of one's own forces;

$C_2$ - "*sheltering height*" - represents the height of the obstacles that allow hiding or masking combat effects from the survey instruments, electronic effects and enemy fire;

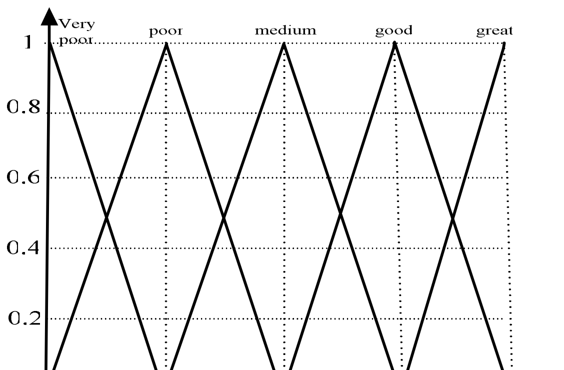
$C_3$ - "*masking conditions*" - terrain features that enable successful masking of the BrAG and movement of parts or the whole BrAG;

$C_4$ - "fortification *conditions*" - terrain features that allow successful fortification of artillery to enhance force protection;

$C_5$ - "c*onditions for maneuver*" – terrain features based on which the assessment of the possibility of fast moving to the following firing position is performed;

$C_6$ - "*average height difference between individual instruments*" - for successful actions, it is necessary for all individual instruments in BrAG artillery batteries to be located on the height difference less than 20m.

The values of the criteria $C_1$, $C_2$ and $C_6$ are described numerically and the values of the criteria $C_3$, $C_4$ and $C_5$ are described with fuzzy linguistic descriptors, Figure 2.




*Picture 2* – Graphic display of the fuzzy linguistic descriptors
*Фигура 2* – Графическое изображение нечеткого лингвистического дескриптора
*Slika 2* – Grafički prikaz fuzzy lingvističkih deskriptora

The membership functions of the fuzzy linguistic descriptors are defined through the expressions:

$$\mu_{I_{VL}} = \begin{Bmatrix} 1, & 1 \geq x \\ 2-x, & 1 \leq x \leq 2 \end{Bmatrix} \tag{25}$$

$$\mu_{I_L} = \begin{Bmatrix} x-1, & 1 \leq x \leq 2 \\ 3-x, & 2 \leq x \leq 3 \end{Bmatrix} \tag{26}$$

$$\mu_{I_S} = \begin{Bmatrix} x-2, & 2 \leq x \leq 3 \\ 4-x, & 3 \leq x \leq 4 \end{Bmatrix} \tag{27}$$

$$\mu_{I_D} = \begin{Bmatrix} x-3, & 3 \leq x \leq 4 \\ 5-x, & 4 \leq x \leq 5 \end{Bmatrix} \tag{28}$$

$$\mu_{I_O} = \begin{Bmatrix} x-4, & 4 \leq x \leq 5 \\ 1, & x \geq 5 \end{Bmatrix} \tag{29}$$

## *Calculation of the criteria weight coefficients*

The first step in defining the weight coefficients is to define the square comparison matrix. Two elements of the hierarchy (models) are compared using the Saaty's classic scale and by defining the degree of certainty of a given claim (according to expression 11). The degree of inconsistency of the given matrix is 0.08.

$$A = \begin{matrix} & C_1 & C_2 & C_3 & C_4 & C_5 & C_6 \\ C_1 & 1;1.0 & 4;0.2 & 6;0.2 & 5;0.4 & 2;0.5 & 9;0.2 \\ C_2 & 4^{-1};0.2 & 1;1.0 & 3;0.6 & 5;0.9 & 3^{-1};0.4 & 6;0.5 \\ C_3 & 6^{-1}:0.2 & 3^{-1}0.6 & 1;1.0 & 2;0.4 & 4^{-1};0.6 & 7;0.6 \\ C_4 & 5^{-1};0.4 & 5^{-1};0.9 & 2^{-1};0,4 & 1;1.0 & 6^{-1};0.4 & 4;1.0 \\ C_5 & 2^{-1};0.5 & 3;0.4 & 4;0.6 & 6;0.4 & 1;1.0 & 7;0.4 \\ C_6 & 9^{-1};0.2 & 6^{-1};0.5 & 7^{-1};0.6 & 4^{-1};1.0 & 7^{-1};0.4 & 1;1.0 \end{matrix}$$

The values of the matrix A are converted into fuzzy numbers by applying the fuzzyficated Saaty's scale (Table 2), so a new matrix A' is obtained.

$$A' = \begin{matrix} & C_1 & C_2 & C_3 & C_4 & C_5 & C_6 \\ C_1 & 1 & \tilde{4} & \tilde{6} & \tilde{5} & \tilde{2} & \tilde{9} \\ C_2 & \tilde{4} & 1 & \tilde{3} & \tilde{5} & \tilde{3}^{-1} & \tilde{6} \\ C_3 & \tilde{6}^{-1} & \tilde{3}^{-1} & 1 & \tilde{2} & \tilde{4}^{-1} & \tilde{7} \\ C_4 & \tilde{5}^{-1} & \tilde{5}^{-1} & \tilde{2}^{-1} & 1 & \tilde{6}^{-1} & \tilde{4} \\ C_5 & \tilde{2}^{-1} & \tilde{3} & \tilde{4} & \tilde{6} & 1 & \tilde{7} \\ K_6 & \tilde{9}^{-1} & \tilde{6}^{-1} & \tilde{7}^{-1} & \tilde{4}^{-1} & \tilde{7}^{-1} & 1 \end{matrix}$$

The weight vector $w$ of every criterion of the matrix A' is the sum of the linguistic expressions that describe the criteria in the same row of the matrix A', which is divided by the sum of all linguistic expressions that describe the criteria of the matrix A'.

After the calculation is performed, the weight vectors of the criteria are obtained:

$$w_i = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \end{bmatrix} = \begin{bmatrix} 0.282; 0.393; 0.382 \\ 0.212; 0.159; 0.162 \\ 0.112; 0.093; 0.105 \\ 0.089; 0.060; 0.058 \\ 0.272; 0.269; 0.254 \\ 0.033; 0.026; 0.039 \end{bmatrix}$$

## *Ranking alternatives*

The model is applied using the illustrative values of seven alternatives shown in the initial decision matrix and taken from (Kurtov, et al., 2014, p.709).

$$\begin{array}{cc} & \begin{array}{cccccc} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 \\ w_1 & w_2 & w_3 & w_4 & w_5 & w_6 \\ \min & \max & \max & \max & \max & \min \end{array} \\ P = \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \\ A_7 \end{array} & \begin{bmatrix} 1100 & 15 & \tilde{1} & \tilde{4} & \tilde{3} & 3 \\ 2000 & 23 & \tilde{4} & \tilde{3} & \tilde{4} & 6 \\ 3800 & 27 & \tilde{3} & \tilde{5} & \tilde{4} & 2 \\ 500 & 5 & \tilde{2} & \tilde{4} & \tilde{2} & 1 \\ 1850 & 32 & \tilde{5} & \tilde{1} & \tilde{3} & 4 \\ 3200 & 10 & \tilde{5} & \tilde{1} & \tilde{3} & 9 \\ 900 & 20 & \tilde{3} & \tilde{3} & \tilde{5} & 5 \end{bmatrix} \end{array}$$

The six-step TOPSIS method application results in the values of the distance of alternatives from the ideal alternative, based on which they are ranked:

$$\tilde{Q}_i^* = \begin{array}{c} \tilde{Q}_1^* \\ \tilde{Q}_2^* \\ \tilde{Q}_3^* \\ \tilde{Q}_4^* \\ \tilde{Q}_5^* \\ \tilde{Q}_6^* \\ \tilde{Q}_7^* \end{array} \begin{bmatrix} 0.578; 0.664; 0.658 \\ 0.597; 0.573; 0.574 \\ 0.410; 0.293; 0.300 \\ 0.538; 0.650; 0.647 \\ 0.611; 0.591; 0.598 \\ 0.276; 0.242; 0.252 \\ 0.748; 0.812; 0.802 \end{bmatrix}$$

Finally, defuzzification is performed by applying expression 12, and the final values of the distance of alternatives from the ideal alternative are obtained:

$$Q_i^* = \begin{matrix} Q_1^* \\ Q_2^* \\ Q_3^* \\ Q_4^* \\ Q_5^* \\ Q_6^* \\ Q_7^* \end{matrix} \begin{bmatrix} 0.633 \\ 0.581 \\ 0.334 \\ 0.612 \\ 0.600 \\ 0.257 \\ 0.787 \end{bmatrix}$$

Based on the obtained distances from the ideal alternative, it can be concluded that alternative seven ($A_7$) is the most appropriate alternative, i.e., alternative six ($A_6$) is the least favorable one.

## Discussion and Conclusions

A practical example of the new scale demonstrated a possibility of using the hybrid fuzzy AHP-TOPSIS model, i.e., its performance in ranking the offered alternatives. In order to determine the differences between the application of the classic Saaty's scale and the scale demonstrated in this paper, a comparative overview of the output results is presented (Table 3).

*Table 3* – Comparative summary of the solutions
Таблица 3 – Сравнительный обзор решений
*Tabela 3* – Upredni pregled dobijenih rešenja

| Alternatives | Classic Saaty's scale | | Fuzzyficated Saaty's scale | |
|---|---|---|---|---|
| | $Q_i^*$ | Rank | $Q_i^*$ | Rank |
| $A_1$ | 0.664 | 2 | 0.633 | 2 |
| $A_2$ | 0.573 | 5 | 0.581 | 5 |
| $A_3$ | 0.293 | 6 | 0.334 | 6 |
| $A_4$ | 0.650 | 3 | 0.612 | 3 |
| $A_5$ | 0.591 | 4 | 0.600 | 4 |
| $A_6$ | 0.242 | 7 | 0.257 | 7 |
| $A_7$ | 0.812 | 1 | 0.787 | 1 |

Analyzing the output results, it can be observed that the rank of alternatives has not changed. However, the values obtained by applying the classic Saaty's scale and the fuzzyficated Saaty's scale are different. The differences between the alternatives $A_1$, $A_4$ and $A_5$ are significantly reduced, and the difference between the alternatives $A_1$ and $A_5$ is reduced more than twice. It is also evident that the difference between the alternatives $A_3$ and $A_6$ increased.

Although the differences shown are significant, there is no change in the rank of alternatives. However, it would be important to determine whether changes in the elements of the initial decision matrix may result in a change of the rank of alternatives when using the classic and fuzzyficated Saaty's scale. These changes will be presented with two examples.

*Example 1*: Changing the value of the alternative $A_1$ by the criterion $C_2$ in the initial decision-making matrix from 15m to 11m would lead to a different way of ranking alternatives (Table 4).

*Table 4* – Comparative summary of the solutions with the change of the initial elements
*Таблица 4* – Сравнительный обзор решений с изменением исходных элементов
*Tabela 4* – Uporedni pregled dobijenih rešenja sa promenom početnih elemenata

| Alternatives | Classic Saaty's scale | | Fuzzyficated Saaty's scale | |
|---|---|---|---|---|
| | $Q_i^*$ | Rank | $Q_i^*$ | |
| $A_1$ | 0.646 | 3 | 0.611 | 2 |
| $A_2$ | 0.574 | 5 | 0.582 | 5 |
| $A_3$ | 0.295 | 6 | 0.336 | 6 |
| $A_4$ | 0.648 | 2 | 0.610 | 3 |
| $A_5$ | 0.592 | 4 | 0.601 | 4 |
| $A_6$ | 0.242 | 7 | 0.257 | 7 |
| $A_7$ | 0.811 | 1 | 0.786 | 1 |

In the mentioned example, it is noted that, when the classic Saaty's scale is used, the alternative $A_1$ is ranked as the third. However, when the fuzzyficated scale is used, it is ranked as the second, while the alternative $A_4$ switched the position with the $A_1$.

*Example 2*: Changing the value of the alternative $A_5$ by the criterion $C_1$ in the initial decision-making matrix from 1850 m to 1500 m would lead to a different way of ranking alternatives (Table 5). In the mentioned example, it is noted that when using the classic Saaty's scale the alternative $A_1$ is ranked as the second, while when using the fuzzyficated scale the alternative $A_5$ is ranked as the second, and vice versa.

*Table 5* – Comparative summary of the solutions with the change of the initial elements
*Таблица 5* – Сравнительный обзор решений с изменением исходных элементов
*Tabela 5* – Uporedni pregled dobijenih rešenja sa promenom početnih elemenata

| Alternatives | Classic Saaty's scale | | Fuzzyficated Saaty's scale | |
|---|---|---|---|---|
| | $Q_i^*$ | Rank | $Q_i^*$ | Rank |
| $A_1$ | 0.667 | 2 | 0.636 | 3 |
| $A_2$ | 0.573 | 5 | 0.580 | 5 |
| $A_3$ | 0.290 | 6 | 0.331 | 6 |
| $A_4$ | 0.654 | 4 | 0.616 | 4 |
| $A_5$ | 0.661 | 3 | 0.661 | 2 |
| $A_6$ | 0.241 | 7 | 0.256 | 7 |
| $A_7$ | 0.813 | 1 | 0.789 | 1 |

Both examples show the importance of the degree of certainty of decision makers when ranking alternatives. Of course, the degree of certainty should not be the deciding factor in ranking, but an additional element that comes to the fore at the limit values. This can be analyzed through the already given examples.

If we consider the calculation with the values from the initial decision matrix, it can be noted that the rank of alternatives is identical when using both classic and fuzzyficated scale. In examples 1 and 2, the rank of alternatives has changed. If we continue and further reduce the value of the alternative $A_1$ by the criterion $C_2$ in the initial decision-making matrix from the original 15 m, and next 11m, to 10, new solutions would be obtained (Table 6).

*Table 6* – Comparative summary of the solutions with the change of the initial elements
*Таблица 6* – Сравнительный обзор решений с изменением исходных элементов
*Tabela 6* – Uporedni pregled dobijenih rešenja sa promenom početnih elemenata

| Alternatives | Classic Saaty's scale | | Fuzzyficated Saaty's scale | |
|---|---|---|---|---|
| | $Q_i^*$ | Rank | $Q_i^*$ | Rank |
| $A_1$ | 0.641 | 3 | 0.605 | 3 |
| $A_2$ | 0.574 | 5 | 0.582 | 5 |
| $A_3$ | 0.295 | 6 | 0.337 | 6 |
| $A_4$ | 0.648 | 2 | 0.609 | 2 |
| $A_5$ | 0.592 | 4 | 0.602 | 4 |
| $A_6$ | 0.242 | 7 | 0.257 | 7 |
| $A_7$ | 0.811 | 1 | 0.786 | 1 |

Table 6 shows that, when using different scales, the rank of alternatives is identical again, except that the alternatives $A_1$ and $A_4$ switched their positions (which is considered to be expected because the characteristics of the alternative $A_1$ improved). The situation is similar in example 2. If the values of the alternative $A_5$ by the criterion $C_1$ in the initial decision-making matrix are reduced from 1850 m, and 1500 m, to 1400 m, the identical situation occurs as in the previous case (Table 7). The ranks of alternatives are identical applying either scale, except that there is the expected replacement in the rank of alternatives due to improving the alternative value by one criterion.

*Table 7* – Comparative summary of the solutions with the change of the initial elements
*Таблица 7* – Сравнительный обзор решений с изменением исходных элементов
*Tabela 7* – Uporedni pregled dobijenih rešenja sa promenom početnih elemenata

| Alternatives | Classic Saaty's scale | | Fuzzyficated Saaty's scale | |
|---|---|---|---|---|
| | $Q_i^*$ | Rank | $Q_i^*$ | Rank |
| $A_1$ | 0.668 | 3 | 0.637 | 3 |
| $A_2$ | 0.572 | 5 | 0.580 | 5 |
| $A_3$ | 0.289 | 6 | 0.330 | 6 |
| $A_4$ | 0.655 | 4 | 0.617 | 4 |
| $A_5$ | 0.680 | 2 | 0.677 | 2 |
| $A_6$ | 0.241 | 7 | 0.255 | 7 |
| $A_7$ | 0.814 | 1 | 0.789 | 1 |

In all cases it can be observed that $Q_i^*$ changes, i.e., that the differences between the alternatives increase/decrease depending on the degree of certainty of decision makers.

From all of the above mentioned, it can be concluded that the new fuzzyficated Saaty's scale improves decision making taking into account the degree of certainty of decision makers in the shown pairwise comparison. Considering the degree of certainty of decision makers, a change occurs in ranking alternatives at the limit values, thereby maintaining the decisive role of the comparison itself, which is the essence of the Saaty's scale.

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# ФАЗЗИФИКАЦИЯ СКАЛЫ СААТИ И ОБЗОР ГИБРИДНОЙ МОДЕЛИ FUZZY AHP – TOPSIS: ПРИМЕРЫ ВЫБОРА ОГНЕВОЙ ПОЗИЦИИ БРИГАДНОЙ АРТИЛЛЕРИЙСКОЙ ГРУППЫ В ОБОРОНИТЕЛЬНЫХ ОПЕРАЦИЯХ

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*Резюме:*

*В настоящей работе представлен новый метод фаззификации шкалы Саати. В данной фаззификации доверительный интервал нечетких чисел, описывающих степени парных сравнений не определен заранее − до начала процесса сравнения. Он вычисляется в течение и по окончании сравнения, на основании степени уверенности лица, принимающего решение (эксперта).*

*Таким образом, доверительный интервал может отличаться в сравнениях, несмотря на одинаковую степень сравнения. Доверительные интервалы также могут отличаться при групповом принятии решений, в зависимости от выбора решения эксперта. Данные положения поддерживаются новой фаззифицированной шкалой Саати.*

*Применение новой шкалы представлено в гибридной модели fuzzy AHP – TOPSIS, при выборе огневой позиции бригадной артиллерийской группы в оборонительной операции. В завершении произведена оценка влияния степени уверенности на конечное решение, и доказано, что данный элемент влияет на выходные результаты.*

*Влияние отражается в изменениях значений весовых векторов, то есть, выходных результатов (расстояние от идеальной альтернативы), а также на изменение ранга альтернатив, но только при граничных значениях.*

Ключевые слова: *фаззи логика, огневая позиция, гибридная модель, TOPSIS, AHP.*

# FAZIFIKACIJA SAATY-JEVE SKALE I PRIKAZ HIBRIDNOG MODELA FUZZY AHP – TOPSIS: PRIMER IZBORA VATRENOG POLOŽAJA BRIGADNE ARTILJERIJSKE GRUPE U ODBRAMBENOJ OPERACIJI

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OBLAST: matematika, operaciona istraživanja
VRSTA ČLANKA: originalni naučni članak
JEZIK ČLANKA: engleski

*Sažetak:*

*Osnovu rada čini prikaz novog načina fazifikacije Saaty-jeve skale. U ovoj fazifikaciji interval poverenja fuzzy brojeva, kojima su opisani stepeni poređenja u parovima, nije određen pre samog procesa poređenja. On se definiše (proračunava) u toku i nakon poređenja, a na osnovu stepena uverenosti donosilaca odluka – eksperata. Tako se interval poverenja može razlikovati od jednog do drugog poređenja, bez obzira na to što se radi o istom stepenu poređenja. Takođe, intervali poverenja se razlikuju od jednog do drugog donosioca odluke – eksperta, kada je reč o grupnom odlučivanju. Ovakvo obrazloženje podržava nova fazifikovana Saaty-jeva skala. Primena nove skale prikazana je u hibridnom modelu fuzzy AHP – TOPSIS prilikom izbora vatrenog položaja brigadne artiljerijske grupe u odbrambenoj operaciji. Na kraju je analiziran uticaj stepena uverenosti na konačnu odluku, gde je dokazano da ovaj element utiče na izlazne rezultate. Uticaj se ogleda u promeni veličine težinskih vektora, odnosno izlaznih rezultata (udaljenosti od idealne alternative), kao i na promeni ranga alternativa, ali samo na graničnim vrednostima.*

Ključne reči: *fuzzy logika, vatreni položaj, hibridni model, TOPSIS, AHP.*

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