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A NOTE ON THE MEIR-KEELER THEOREM IN THE CONTEXT OF *b*-METRIC SPACES

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Abstract:

In this note we consider the famous Meir-Keeler's theorem in the context of b-metric spaces. Our result generalizes, improves, compliments, unifies and enriches several known ones in the existing literature. Also, our proof of Meir-Keeler's theorem in the context of standard metric spaces is much shorter and nicer than the ones in (Ćirić, 2003) and (Meir & Keeler, 1969, pp.326-329).

Keywords: b-metric space, b-complete, b-Cauchy, Meir-Keeler conditions, Picard sequence.

Definitions, notations and preliminaries

Let (X,d) be a standard metric space and $f:X\to X$ be a self-mapping. In the context of these spaces, the following (Meir-Keeler) conditions are well known: For each $\varepsilon>0$ there exists $\delta=\delta(\varepsilon)>0$ such that for all $x,y\in X$ holds

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$\varepsilon \le d(x,y) < \varepsilon + \delta$ implies $d(fx, fy) < \varepsilon$ (1)

or

$\varepsilon < d(x,y) < \varepsilon + \delta$ implies $d(fx, fy) \le \varepsilon$ (2)

or f is contractive and

$\varepsilon \le d(x,y) < \varepsilon + \delta$ implies $d(fx, fy) \le \varepsilon$. (3)

One says that the mapping f defined on the standard metric space (X,d) is contractive if $d(fx, fy) < d(x,y)$ holds, whenever $x \ne y$.

For more details, see (Ćirić, 2003, pp.30-33, pp.56-58).

In 1969, Meir-Keeler proved the following:

Theorem 1 (Meir & Keeler, 1969, pp.326-329, Theorem) Let (X,d) be a complete metric space and let f be a self-mapping on X satisfying (1). Then f has a unique fixed point, say $u \in X$, and for each $x \in X, \lim_{n\to\infty} f^n x = u$.

Inspired by the above Meir-Keeler theorem, Ćirić proved the following, slightly more general result:

Theorem 2 (Ćirić, 2003, Theorem 2.5) Let (X,d) be a complete metric space and let f be a self-mapping on X satisfying (2). Then f has a unique fixed point, say $u \in X$, and for each $x \in X, \lim_{n\to\infty} f^n x = u$.

The example which follows shows that Ćirić's result is a proper generalization of the famous Meir-Keeler theorem:

Example 3 Let $X = [0,1] \cup \{3n-1\}_{n\in N} \cup \left\{3n + \frac{1}{3n}\right\}_{n\in N}$ be a subset of real numbers with the Euclidean metric and let f be a self-mapping on X defined by

$$fx = 0, \;\; if \;\; 0 \le x \le 1 \;\; and \;\; x \in \{3n-1\}_{n\in N},$$

$$fx = 1, \;\; if \;\; x \in \left\{3n + \frac{1}{3n}\right\}_{n\in N}.$$

Then one can verify that f satisfies (2) while it does not satisfy Meir-Keeler condition (1). For all details, see (Ćirić, 2003, p.33).

Remark 1 Both previous theorems are true if the self-mapping $f : X \to X$ satisfies condition (3).

Bakhtin (Bakhtin, 1989, pp.26-37) and Czerwik (Czerwik, 1993, pp.5-11) introduced *b*-metric spaces (as a generalization of metric spaces) and proved the contraction principle in this context. In the last period, many authors have obtained fixed point results for single-valued or set-valued functions, in the context of *b*-metric spaces. Now we give the definition of a *b*-metric space:

Definition 1.1 (Bakhtin, 1989, pp.26-37), (Czerwik, 1993, pp.5-11) Let X be a nonempty set and let $s \geq 1$ be a given real number. The function $d : X \times X \to [0, \infty)$ is said to be a *b*-metric if, and only if, for all $x, y, z \in X$ the following conditions hold:

b1) $d(x, y) = 0$ if, and only if, $x = y$;

b2) $d(x, y) = d(y, x)$;

b3) $d(x, z) \leq s[d(x, y) + d(y, z)]$.

A triplet $(X, d, s \geq 1)$ is called a *b*-metric space with the coefficient s.

It should be noted that the class of *b*-metric spaces is effectively larger than that of standard metric spaces, since a b-metric is a metric when $s = 1$. The following example shows that, in general, a *b*-metric does not necessarily need to be a metric (Chandok et al, 2017, pp.331-345), (Došenović et al, 2017, pp.851-865), (Dubey et al, 2014), (Dung & Hang, 2018, pp.298-304), (Faraji & Nourouzi, 2017, pp.77-86), (Jovanović et al, 2010), (Jovanović, 2016), (Kir & Kiziltunc, 2016, pp.13-16), (Kirk & Shahzad, 2014).

Example 4 Let (X, ρ) be a standard metric space, and $d(x, y) = (\rho(x, y))^p$, $p > 1$ is a real number. Then d is a *b*-metric with $s = 2^{p-1}$, but d is not a standard metric on X.

Otherwise, for more concepts such as *b*-convergence, *b*-completeness, *b*-Cauchy and *b*-closed set in *b*-metric spaces, we refer the reader to (Došenović et al, 2017, pp.851-865), (Dubey et al, 2014), (Dung & Hang, 2018, pp.298-304), (Faraji & Nourouzi, 2017, pp.77-86), (Jovanović et al, 2010), (Jovanović, 2016), (Kir & Kiziltunc, 2016, pp.13-16), (Kirk & Shahzad, 2014), (Koleva & Zlatanov, 2016, pp.31-34), (Chifu & Petruşel, 2017, pp.2499-2507), (Kumar et al, 2014, pp.19-22), (Miculescu & Mihail, 2017, pp.1-11), (Paunović et al, 2017, pp.4162-4174), (Singh et al, 2008, pp.401-416), (Sintunavarat, 2016, pp.397-416), (Suzuki, 2017), (Zare & Arab, 2016, pp.56-67).

The following two lemmas are very significant in the theory of a fixed point in the context of *b*-metric spaces.

Lemma 1.2 (Jovanović et al, 2010, p.15, Lemma 3.1) Let $\{a_n\}_{n\in N\cup\{0\}}$ be a sequence in a *b*-metric space $(X,d,s\geq 1)$ such that

$$d(a_n,a_{n+1})\leq kd(a_{n-1},a_n)$$

for some $k\in\left[0,\frac{1}{s}\right)$, and each $n=1,2,...$ Then $\{a_n\}$ is a *b*-Cauchy sequence in a *b*-metric space $(X,d,s\geq 1)$.

Lemma 1.3 (Miculescu & Mihail, 2017, pp.1-11, Lemma 2.2) Let $\{a_n\}_{n\in N\cup\{0\}}$ be a sequence in a *b*-metric space $(X,d,s\geq 1)$ such that

$$d(a_n,a_{n+1})\leq kd(a_{n-1},a_n)$$

for some $k\in[0,1)$, and each $n=1,2,...$ Then $\{a_n\}$ is a *b*-Cauchy sequence in a *b*-metric space $(X,d,s\geq 1)$.

Remark 2 In (Došenović et al, 2017, pp.851-865), it is proven that the previous lemmas are equivalent.

Since in general a *b*-metric is not necessarily continuous, many papers related with *b*-metric spaces used the following lemmas to prove the main results.

Lemma 1.4 (Aghajani et al, 2014, pp.941-960, Lemma 2.1) Let $(X,d,s\geq 1)$ be a *b*-metric space. Suppose that $\{a_n\}$ and $\{b_n\}$ are *b*-convergent to *a* and *b*, respectively. Then

$$\frac{1}{s^2}d(a,b)\leq \liminf_{n\to\infty} d(a_n,b_n)\leq \limsup_{n\to\infty} d(a_n,b_n)\leq s^2 d(a,b).$$

In particular, if $a=b$, then we have $\lim_{n\to\infty} d(a_n, b_n) = 0$. Moreover, for each $c \in X$, we have

$$\frac{1}{s} d(a,c) \le \liminf_{n\to\infty} d(a_n, c) \le \limsup_{n\to\infty} d(a_n, c) \le sd(a,c).$$

Lemma 1.5 (Paunović et al, 2017, pp.4162-4174, Lemma 2.3) Let $(X, d, s \ge 1)$ be a *b*-metric space and $\{a_n\}$ a sequence in X such that

$\lim_{n\to\infty} d(a_n, a_{n+1}) = 0$.

If $\{a_n\}$ is not *b*-Cauchy, then there exist $\varepsilon > 0$ and two sequences $\{m(k)\}$ and $\{n(k)\}$ of positive integers such that the following items hold:

$$\varepsilon \le \liminf_{k\to\infty} d(a_{m(k)}, a_{n(k)}) \le \limsup_{k\to\infty} d(a_{m(k)}, a_{n(k)}) \le \varepsilon s,$$

$$\frac{\varepsilon}{s} \le \liminf_{k\to\infty} d(a_{m(k)}, a_{n(k)+1}) \le \limsup_{k\to\infty} d(a_{m(k)}, a_{n(k)+1}) \le \varepsilon s^2,$$

$$\frac{\varepsilon}{s} \le \liminf_{k\to\infty} d(a_{m(k)+1}, a_{n(k)}) \le \limsup_{k\to\infty} d(a_{m(k)+1}, a_{n(k)}) \le \varepsilon s^2,$$

$$\frac{\varepsilon}{s^2} \le \liminf_{k\to\infty} d(a_{m(k)+1}, a_{n(k)+1}) \le \limsup_{k\to\infty} d(a_{m(k)+1}, a_{n(k)+1}) \le \varepsilon s^3.$$

In particular, if $s=1$ and $\{a_n\}$ is not a *b*-Cauchy sequence, then there exists $\varepsilon > 0$ as well as two sequences $\{m(k)\}$ and $\{n(k)\}$ of positive integers such that the sequences

$$d(a_{m(k)}, a_{n(k)}), d(a_{m(k)}, a_{n(k)+1}), d(a_{m(k)+1}, a_{n(k)}) \text{ and } d(a_{m(k)+1}, a_{n(k)+1}) \quad (4)$$

tend to ε^+ as $k \to \infty$.

Main result

Now, according to the last Lemma (the condition $s=1$), we formulate and prove the following result:

Theorem 5 Let (X, d) be a complete metric space and let f be a contractive self-mapping on X satisfying the next condition:

Given $\varepsilon > 0$, there exists $\delta > 0$ such that for all $x, y \in X$

$$\varepsilon \le d(x,y) < \varepsilon + \delta \text{ implies } d(fx, fy) \le \varepsilon. \quad (5)$$

Then f has a unique fixed point, say $u \in X$, and for each $x \in X$, $\lim_{n\to\infty} f^n x = u$.

Proof. Let x_0 in X be arbitrary. Consider the sequence of iterates $\{f^n x_0\}_{n=0}^{+\infty}$. If $d(f^n x_0, f^{n+1}x_0) = d(f^n x_0, ff^n x_0) = 0$ for some $n \in N$, then $a_n = f^n x_0$ is a fixed point of *f*. Assume now that $d(f^n x_0, f^{n+1}x_0) > 0$ for all $n \in N$. Since f is contractive, the sequence $\{d(f^n x_0, f^{n+1}x_0)\}_{n=0}^{+\infty}$ is strictly decreasing. Therefore, there exists the limit of this sequence, say ε, and $d(f^n x_0, f^{n+1}x_0) > \varepsilon$ for all $n \in N$. Assume that $\varepsilon > 0$. In this case, by hypothesis, there exists a suitable $\delta = \delta(\varepsilon) > 0$ such that (5) holds. From the definition of ε, it follows that there is $n \in N$ such that

$$\varepsilon \le d(f^n x_0, f^{n+1}x_0) < \varepsilon + \delta. \tag{6}$$

According to (5), we get that

$$d(ff^n x_0, ff^{n+1}x_0) = d(f^{n+1}x_0, f^{n+2}x_0) \le \varepsilon,$$

a contradiction. Therefore $\lim_{n\to\infty} d(f^n x_0, f^{n+1}x_0) = 0$.

Now we show that $\{f^n x_0\}_{n=0}^{+\infty}$ is a Cauchy sequence. If this is not the case, by applying Lemma 1.5 to the sequence $\{f^n x_0\}_{n=0}^{+\infty}$, we get that there exist $\varepsilon > 0$ and two sequences of positive integers $\{m(k)\}$ and $\{n(k)\}$ such that $n(k) > m(k) > k$, and sequences (4) tend to ε^+ as $k \to \infty$. Using the condition (5) with $x = a_{m(k)}, y = a_{n(k)}$ and the $\delta = \delta(\varepsilon) > 0$, ones obtains that there exists a positive integer l such that for each $k \ge l$, we have

$$\varepsilon \le d(a_{m(k)}, a_{n(k)}) = d(fa_{m(k)-1}, fa_{n(k)-1}) < \varepsilon + \delta \ implies \ d(fa_{m(k)}, fa_{n(k)}) \le \varepsilon.$$

This contradicts the fact that

$$d(fa_{m(k)}, fa_{n(k)}) = d(a_{m(k)+1}, a_{n(k)+1}) \to \varepsilon^+ \ as \ k \to \infty.$$

Hence, $\{f^n x_0\}_{n=0}^{+\infty}$ is a Cauchy sequence.

The proof is further as in (Ćirić, 2003) and (Meir & Keeler, 1969, pp.326-329).

To our knowledge, it is not known whether Meir-Keeler's and Ćiric's theorems hold in the context of a b-metric space. Also, there is no known example that confirms that conditions (1) or (2) or (3) holds in the context of *b*-metric spaces but that *f* does not have a fixed point.

However, with a stronger condition than (1), we have the positive result. Hence, our main result is the following:

Theorem 6 Let $(X, d, s>1)$ be a *b*-complete *b*-metric space and let f self-mapping on X satisfy the following condition:

Given $\varepsilon > 0$, there exists $\delta > 0$ such that

$$\varepsilon \le d(x, y) < \varepsilon + \delta \textit{ implies } s^a d(fx, fy) < \varepsilon, \qquad (7)$$

where $a > 0$ is given.

Then f has a unique fixed point, say $u \in X$, and for each $x \in X$, $\lim_{n\to\infty} f^n x = u$.

Proof. It is clear that for all $x, y \in X$ we obtain

$$d(fx, fy) \le kd(x, y), \qquad (8)$$

where $k = \frac{1}{s^a} \in [0,1)$.

Let $a_0 \in X$ be an arbitrary point. Define the sequence $\{a_n\}$ by $a_{n+1} = fa_n$ for all $n \ge 0$. If $a_n = a_{n+1}$ for some *n*, then a_n is a fixed point (unique) of *f* and the results follows.

So, suppose that $a_n \ne a_{n+1}$ for all $n \ge 0$. From the condition (8), we obtain

$$d(a_n, a_{n+1}) \le kd(a_{n-1}, a_n). \qquad (9)$$

Further, according to (Miculescu & Mihail, 2017, pp.1-11, Lemma 2.2.) we obtain that $\{a_n\}$ is a *b*-Cauchy sequence in a *b*-metric space (X, d). By the *b*-completeness of (X, d), there exists $u \in x$ such that

$$\lim_{n\to\infty} a_n = u. \qquad (10)$$

Finaly, (8) and (10) imply that $fu = u$, i.e. u is a unique fixed point of *f* in X.

For the following facts and definitions, we refer to (Aghajani et al, 2014, pp.941-960), (Jovanović, 2016) and (Kirk & Shahzad, 2014) and the references therein.

Definition 2.1 Let f and g be self-mappings of a nonempty set X such that $f(X) \subset g(X)$. Let $x_0 \in X$ be an arbitrary point. Then $fx_0 \in g(X)$, so we can assume that $fx_0 = gx_1 = y_0$ (say) for some $x_1 \in X$. Again, $fx_1 \in g(X)$, so we can choose $x_2 \in X$ such that $fx_1 = gx_2 = y_1$ (say). Similarly, we can construct two sequences $\{x_n\}$ and $\{y_n\}$ such that $y_n = fx_n = gx_{n+1}$ for all $n \geq 0$. Here the sequence $\{y_n\}$ is called a corresponding Jungck sequence for the point $x_0 \in X$.

Definition 2.2 Let f and g be the self-mappings of a nonempty set X. If $z = fx = gx$ for some x in X, then x is called a coincidence point of f and g, and z is called a point of coincidence of f and g. The mappings f and g are called weakly compatible if they commute at their coincidence points.

Lemma 2.3 Let f and g be the weakly compatible self-maps of a nonempty set X. If f and g have a unique point of coincidence $z = fx = gx$, then z is the unique common fixed point of f and g.

Now, we announce the following result which generalizes Theorem 5 in several directions:

Theorem 7 Let $(X, d, s > 1)$ be a *b*-complete *b*-metric space and let $f, g : X \to X$ be two self-maps such that $f(X) \subset g(X)$, one of these two subsets of X being b-complete. Suppose the following conditions hold:

$$\text{for each } \varepsilon > 0 \text{ there exists } \delta > 0 \text{ such that}$$

$$\varepsilon \leq d(gx, gy) < \varepsilon + \delta \text{ implies } s^a d(fx, fy) < \varepsilon$$

$$\text{and } fx = fy \text{ whenever } gx = gy,$$

where $a > 0$ is given.

Then f and g have a unique point of coincidence, say $z \in X$. Moreover, for each $x_0 \in X$, the corresponding Jungck sequence $\{y_n\}$ can be chosen such that $\lim_{n\to\infty} y_n = z$. In addition, if f and g are weakly compatible, then they have a unique common fixed point.

Finally, we have an open question:

Prove or disprove the following:

- Let $(X, d, s > 1)$ be a *b*-complete b-metric space and $f, g : X \to X$ be two given mappings such that $f(X) \subset g(X)$, one of these two subsets of X being *b*-complete. Assume that the following conditions hold:

for each $\varepsilon > 0$, there exists $\delta = \delta(\varepsilon) > 0$ such that $\varepsilon \le d(gx, gy) < \varepsilon + \delta$ implies $d(fx, fy) < \varepsilon$ and $fx = fy$, whenever $gx = gy$.

Then f and g have a unique point of coincidence, say $z \in X$. Moreover, if f and g are weakly compatible, then they have a unique common fixed point.

References

Aghajani, A., Abbas, M., & Roshan, J. 2014. Common fixed point of generalized weak contractive mappings in partially ordered b-metric spaces. *Mathematica Slovaca,* 64(4), pp.941-960. Available at: https://doi.org/10.2478/s12175-014-0250-6.

Bakhtin, I.A. 1989. The contraction principle in quasimetric spaces. *Funct. Anal,* 30, pp.26-37.

Chandok, S., Jovanovic, M., & Radenovic, S. 2017. Ordered b-metric spaces and Geraghty type contractive mappings. *Vojnotehnički glasnik/Military Technical Courier,* 65(2), pp.331-345. Available at: https://doi.org/10.5937/vojtehg65-13266.

Chifu, C., & Petruşel, G. 2017. Fixed point results for multivalued hardy-rogers contractions in b-metric spaces. *Filomat,* 31(8), pp.2499-2507. Available at: https://doi.org/10.2298/fil1708499c.

Czerwik, S. 1993. Contraction mappings in b-metric spaces. *Acta Math. Inform., Univ. Ostrav,* 1(1), pp.5–11. Available at: https://dml.cz/handle/10338.dmlcz/120469. Accessed: 10.10.2018.

Ćirić, Lj. 2003. *Fixed Point Theory: Contraction Mapping Principle*.Belgrade: Faculty of Mechanical Engineering.

Došenović, T., Pavlović, M., & Radenović, S. 2017. Contractive conditions in b-metric spaces. *Vojnotehnički glasnik/Military Technical Courier,* 65(4), pp.851-865. Available at: https://doi.org/10.5937/vojtehg65-14817.

Dubey, A.K., Shukla, R., & Dubey, R.P. 2014. Some fixed point results in b-metric spaces. *Asian J. Math. Appl.,* article ID ama0147.

Dung, N.V., & Hang, V.T.L. 2018. On The Completion Of b-Metric Spaces. *Bulletin of the Australian Mathematical Society,* 98(2), pp.298-304. Available at: https://doi.org/10.1017/s0004972718000394.

Faraji, H., & Nourouzi, K. 2017. A generalization of Kannan and Chatterjea fixed point theorem on complete b-metric spaces. *Sahand Communications in Mathematical Analysis (SCMA),* 6(1), pp.77-86. Available at: https://doi.org/10.22130/SCMA.2017.23831.

Jovanović, M., Kadelburg, Z., & Radenović, S. 2010. Common Fixed Point Results in Metric-Type Spaces. *Fixed Point Theory and Applications,* 2010, Article ID:978121. Available at: https://doi.org/10.1155/2010/978121.

Jovanović, M. 2016. *Contribution to the theory of abstract metric spaces*. Belgrade. Available at: http://nardus.mpn.gov.rs/handle/123456789/7975. Accessed: 10.10.2018.

Kir, M., & Kiziltunc, H. 2016. On Some Well Known Fixed Point Theorems in b-Metric Spaces. *Turkish Journal of Analysis and Number Theory,* 1(1), pp.13-16. Available at: https://doi.org/10.12691/tjant-1-1-4.

Kirk, W., & Shahzad, N. 2014. *Fixed Point Theory in Distance Spaces*. Switzerland: Springer International Publishing.

Koleva, R., & Zlatanov, B. 2016. On fixed points for Chatterjea's maps in b-metric spaces. *Turkish Journal of Analysis and Number Theory,* 4(2), pp.31–34. Available at: http://www.sciepub.com/TJANT/abstract/6009. Accessed: 10.10.2018.

Kumar, M.P., Sachdeva, S., & K. Banerjee, S. 2014. Some Fixed Point Theorems in b-metric Space. *Turkish Journal of Analysis and Number Theory,* 2(1), pp.19-22. Available at: https://doi.org/10.12691/tjant-2-1-5.

Meir, A., & Keeler, E. 1969. A theorem on contraction mappings. *Journal of Mathematical Analysis and Applications,* 28(2), pp.326-329. Available at: https://doi.org/10.1016/0022-247x(69)90031-6.

Miculescu, R., & Mihail, A. 2017. New fixed point theorems for set-valued contractions in b-metric spaces. *Journal of Fixed Point Theory and Applications,* 19(3), pp.2153-2163. Available at: https://doi.org/10.1007/s11784-016-0400-2.

Paunović, Lj., Kaushik, P., & Kumar, S. 2017. Some applications with new admissibility contractions in b-metric spaces. *The Journal of Nonlinear Sciences and Applications,* 10(08), pp.4162-4174. Available at: https://doi.org/10.22436/jnsa.010.08.12.

Singh, S.L., Czerwik, S., Krol, K., & Singh, A. 2008. Coincidences and fixed points of hybrid contractions. *Tamsui Oxf. J. Math. Sci.,* 24, pp.401-416.

Sintunavarat, W. 2016. Nonlinear integral equations with new admissibility types in b-metric spaces. *J. Fixed Point Theory Appl.,* 18(2), pp.397–416. Available at: https://doi.org/10.1007/s11784-015-0276-6.

Suzuki, T. 2017. Basic inequality on a b-metric space and its applications. *Journal of Inequalities and Applications,* 2017:256. Available at: https://doi.org/10.1186/s13660-017-1528-3.

Zare, K., & Arab, R. 2016. Common fixed point results for infinite families in partially ordered b-metric spaces and applications. *Electronic Journal of Mathematical Analysis and Applications,* 4(2), pp.56–67. Available at: http://math-frac.org/Journals/EJMAA/Vol4(2)_July_2016/Vol4(2)_Papers/06_EJMAA_Vol4(2)_July_2016_pp_56-67.pdf. Accessed: 15.10.2018.

ЗАМЕТКА О ТЕОРЕМЕ МЕИРА-КИЛЕРА В КОНТЕКСТЕ *b*-МЕТРИЧЕСКИХ ПРОСТРАНСТВ

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Резюме:

В данной работе рассматривается знаменитая теорема Меира-Килера в контексте b-метрических пространств. Наш результат обобщает, улучшает, дополняет и объединяет ранее полученные результаты, которые были опубликованы в научной литературе. Наше доказательство намного короче и лучше, чем доказательства, представленные в иных работах (Ћирић, 2003) и (Meir & Keeler, 1969, pp.326-329).

Ключевые слова: b-метрическое пространство, b-полная система функций, b-Коши, условия *Меира-Килера, последовательности Пикарда.*

БЕЛЕШКА О MEIR-KEELER-ОВОЈ ТЕОРЕМИ У КОНТЕКСТУ *b*-МЕТРИЧКИХ ПРОСТОРА

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ОБЛАСТ: математика (математичка тематска класификација: примарна 47H10, секундарна 54H25)
ВРСТА ЧЛАНКА: оригинални научни чланак
ЈЕЗИК ЧЛАНКА: енглески

Сажетак:

У овом раду разматрана је позната Meir-Keeler-ова теорема у контексту b-метричких простора. Наш резултат генерализује, побољшава, даје допринос, уједињује и обогаћује познате резултате у научној литератури. Такође, наш доказ Meir-Keeler-ове теореме у контексту стандардних метричких простора је много краћи и прикладнији него у радовима Ћирића, (2003) и Meir & Keeler-а (1969, pp.326-329).

Кључне речи: b-метрички простор, b-комплетан, b-Cauchy-јев, Meir-Keeler-ови услови, Picard-ов низ.

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