Feedback regulation of a DC motor via interconnection and damping assignment

Atilio Morillo [(1)] *Miguel Ríos Bolívar* [(2)] *Vivian Acosta* [(2)]
[(1)] *Centro de Investigación en Matemática Aplicada*
Universidad del Zulia. Maracaibo, Venezuela
[(2)] *Facultad de Ingeniería*
Universidad de los Andes. Mérida, Venezuela
Email: amorillo7@cantv.net, riosm@ula.ve, acostav@ula.ve

Abstract

In this work, we consider the application of the Interconnection And Damping Assignment (IDA) control methodology, recently proposed in the literature to the asymptotic position regulation problema of a brushed DC motor driving a mechanical load. To achieve this objective the electromechanical system is firstly transformed into the general port controlled Hamiltonian form, then the IDA desing procedure is applied to synthesize the stabilizing control law. The Hamiltonian structure of the closed loop system is preserved and the asymptotic stability of the mechanical position is verified by digital simulations.

Keywords: Nonlinear control systems, motor control, stabilizing controllers.

Regulación por retroalimentación de un motor DC vía interconexión y asignación de amortiguamiento

Resumen

En este trabajo se considera la metodología de Interconexión y Asignación de Amortiguamiento (IDA), recientemente propuesta en la literatura, en la aplicación para el problema de la regulación asintótica de la posición de un motor DC de escobillas que manipula una carga mecánica. Para lograr este objetivo el sistema electromecánico se transforma en primer lugar en la forma Hamiltoniana controlada por puertos, y a continuación se aplica el procedimiento de diseño IDA para sintetizar la ley de control estabilizante. La estructura Hamiltoniana es preservada en el sistema en lazo cerrado y la estabilidad asintótica de la posición mecánica es verificada mediante simulaciones digitales.

Palabras clave: Sistemas de control no lineales, control de motores, controladores estabilizantes.

1. INTRODUCTION

Energy shaping control methods have attracted a lot of interest recently. A common outstanding feature of these methods, in the face of the stabilization problem, is studying physical systems as the interconnection of simpler sub-systems or components either storing or disipating energy [1]. This viewpoint allow us to obtain physical models in two alternative structures (Lagrangian or Hamiltonian) [1-3]. Energy shaping methods pursue to preserve the physycal structure (Lagrangian or hamiltonian in the in the closed-loop. This characteristic has the advantage that the closed-loop energy function can be used as Lyapunov (or storage) funtion for stability analysis purposes.

Whilst the controller design method that shape the potential energy have been known for years [1], the challenging problem of shaping the total energy of electromechanical systems has been addressed more recently [4-6]. To this end it is required that the overall energy function has a minimum at the desired equilibrium point. This is guaranteed in mechanical systems by firstly modifying the inertia matrix (in the kinetic energy) and then by shaping the potential energy.

The Interconnection and Damping Assignment (IDA) passivity based control approach is used to achieve this goal in [6], for mechanical systems in the general Port-Controlled Hamiltonian (PCH) form [7]. It is importan to emphasize that in the work by [8], the complete equivalence of the PCH/IDA methodology to that of the controlled Lagrangian methodology is proven. In particular, the Lagrangian form of the gyroscopic terms corresponding to the Poisson structure modification is identified.

On the other hand, the total energy problem for generalized electromechanical systems has been addressed by [9] to solve the asymptotic position regulation problem of fully actuated system with linear magnetic materials consisting of inductances, permanent magnets and one mechanical coordinate. This objective is achieved by modififying the interconnection structure of the system and adding gyroscopic terms to the energy function so that the minimum of the total closed-loop system is only determined by the mechanical potential energy.

In this paper we address the problem of achieving asymptotic position regulation, by following the PCH/IDA methodology reported in [9], of a brushed DC motor driving a mechanical load.

2. IDA CONTROL OF ELECTROMECHANICAL SYSTEMS IN THE PCH FORM

For the seek of clarity, we revisit the modelling procedure explained in [9], which is base on the energy conversión methodology detailed in [2]. To represent the genralized electromechanical system. It consists of n_e windings with linear magnetic materials and it is also assumed that all parameters are constant and known. Then, by applying the Gauss`s law and the Ampere`s law, the following affine relationship arises

$$\lambda = L(\theta)\ i + \mu(\theta) \tag{1}$$

between the flux linkage $\lambda \in \Re^n$ and the current vector $i \in \Re^n$, with $\theta \in \Re$ the mechanical angular position, and $L(\theta) = L^T(\theta) > 0$ the n_e x n_e multi-port inductance matrix. The vector $\mu(\theta)$ represents the flux linkages.

By assuming fully actuated electrical coordinates, the voltage equilibrium equation yields

$$\dot{\lambda} + R\ \ i = u \tag{2}$$

where $u \in \Re^n$ is the vector of voltage applied to the windings, $R = R^T > 0$ is the matrix of electrical resistance of the windings.

Coupling between the electrical and mechanical subsystem is established through the torque of electrical origin [2] for further details).

$$\begin{aligned}\tau(i,\theta) &= \frac{1}{2} i^T \nabla_\theta L(\theta)\ i + i^T \nabla_\theta \mu(\theta) \\ &= \frac{1}{2} i^T \frac{\partial L(\theta)}{\partial \theta} i + i^T \frac{\partial \mu(\theta)}{\partial \theta}\end{aligned} \tag{3}$$

In the sequel, we will use the notation

$$\nabla_w H(z,w) := \partial H(z,w) \big/ \partial w$$

To complete the model, the latter equation is replaced in the mechanical dynamics

$$J\ddot{\theta} = -r_m \dot{\theta} + \tau(i,\theta) - \nabla_\theta V(\theta) \tag{4}$$

where $J>0$ is the rotational inertia of the mechanical subsystem $r_m \geq 0$ is the viscous friction coefficient, and $V(\theta)$ is the potential energy function.

To apply the IDA control design approach, it is needed to express the model (1)-(4) in PCH form

$$\dot{x} = \left[\Im(x) - \Re(x)\right]\ \nabla_x H(x) + g(x)u \tag{5}$$

Where $x \in \Re^n$ is the state space, $\mathfrak{F}(x) = -\mathfrak{F}^r(x)$, g(x) are the internal and external interconnection structures, respectively, $\Re(x) = \Re^r(x) \geq 0$ and is the damping structure [3].

To achieve this objective, the total energy function is introduced [7]

$$H(x)=\frac{1}{2}\left[\lambda-\mu(\theta)\right]^{T}L(\theta)^{-1}\left[\lambda-\mu(\theta)\right]+V(\theta)+\frac{1}{2}\frac{p^{2}}{J} \qquad (6)$$

where $p=J\dot{\theta}$ is the mechanical momentum; and the state vector $x=\left[\lambda^{T},\theta,p\right]^{T}$ is defined.

Thus (1)-(4) can be rewritten in the compact form

$$\begin{bmatrix}\dot{\lambda}\\ \dot{\theta}\\ \dot{p}\end{bmatrix}=\begin{bmatrix}-R & 0 & 0\\ 0 & 0 & 1\\ 0 & -1 & -r_{m}\end{bmatrix}\begin{bmatrix}\nabla_{\lambda}H(x)\\ \nabla_{\theta}H(x)\\ \nabla_{p}H(x)\end{bmatrix}+\begin{bmatrix}I\\ 0\\ 0\end{bmatrix}u \qquad (7)$$

and comparing with the PCH model (5) we identify

$$\Im=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 1\\ 0 & -1 & 0\end{bmatrix},\ \Re=\begin{bmatrix}R & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & r_{m}\end{bmatrix},\ g=\begin{bmatrix}I\\ 0\\ 0\end{bmatrix}$$

The control problem is the asymptotic regulation of θ to a constant position $\theta_{*}\in\Theta\subset\Re$. The equilibria of the electromechanical system (7) are of the form

$$x_{*}=\left[\lambda_{*},\theta_{*},0\right]^{T} \qquad (8)$$

where $\lambda_{*}=L(\theta_{*})\ i_{*}+\mu(\theta_{*})$ and i_{*} is the solution of (3),(4) for $\theta=\theta_{*}$, that is

$$\frac{1}{2}i_{*}^{T}\nabla_{\theta}L(\theta_{*})\ i_{*}+i_{*}^{T}\nabla_{\theta}\mu(\theta_{*})=0 \qquad (9)$$

The corresponding control is $u_{*}=R\ i$. Thus, all equilibria correspond to nonzero current, hence to nonzero electrical energy.

2.1 IDA control synthesis

In order to assign the equilibria of the closed-loop system via selection of the potencial energy only, [9] have chosen an energy function of the form

$$H_{d}(\theta)=\frac{1}{2}\left[\lambda-\mu_{d}(\theta,p)\right]^{T}L(\theta)^{-1}\left[\lambda-\mu_{d}(\theta,p)\right]+V_{d}(\theta)+\frac{1}{2}\frac{p^{2}}{J} \qquad (10)$$

where $\mu_d(\theta,p)$ is fixed such that $\lambda_* = \mu_d(\theta_*,0)$. Thus, the equilibria will coincide with the extrema of $V_d(\theta)$, and it is only needed to select a function with a unique isolated minimum at θ_*.

Then, to assign the proposed energy function preserving the PCH structure, the original interconnection and damping structures are modified to take the form

$$\begin{bmatrix}\dot{\lambda}\\ \dot{\theta}\\ \dot{p}\end{bmatrix}=\begin{bmatrix}-R & \alpha(x) & \beta(x)\\ -\alpha^{T}(x) & 0 & 1\\ -\beta^{T}(x) & -1 & -r_{a}(p)\end{bmatrix}\begin{bmatrix}\nabla_{\lambda}H_{d}(x)\\ \nabla_{\theta}H_{d}(x)\\ \nabla_{p}H_{d}(x)\end{bmatrix} \qquad (11)$$

where *α(x)*, *β(x)*, $r_a(p)$ are the free parameters to be used for assigning the desired energy function.

By following the proof of the Proposition 1 in [9], it can be shown that matching the first n_e rows of the desired dynamics (11) with the corresponding rows of the original dynamics (7), we can obtain the control law

$$\begin{aligned}u=R\ i_{d}-\alpha\left\{\frac{1}{2}\left[i^{T}\nabla_{\theta}L(\theta)\ i-i_{d}^{T}\nabla_{\theta}L(\theta)\ i_{d}\right]\right\}\\ +\alpha\left\{(i-i_{d})^{T}\left[L(\theta)\nabla_{\theta}i_{d}+\nabla_{\theta}\mu(\theta)\right]-\nabla_{\theta}V_{d}(\theta)\right\}+\beta\left[\frac{p}{J}-(i-i_{d})^{T}L(\theta)\nabla_{p}i_{d}\right]\end{aligned} \qquad (12)$$

defining

$$i_{d}:=L(\theta)^{-1}\left[\mu_{d}(\theta,p)-\mu(\theta)\right] \qquad (13)$$

and proposing the *α(x)*, *β(x)* functions

$$\alpha=-L(\theta)\nabla_{p}i_{d} \qquad (14)$$

$$\beta=L(\theta)\left[\nabla_{\theta}i_{d}+r_{a}(p)\nabla_{p}i_{d}\right] \qquad (15)$$

Respectively. The suitable choice of the parameter *β* yields

$$\frac{1}{2}i_{d}^{T}\nabla_{\theta}i_{d}+i_{d}^{T}\nabla_{\theta}\mu(\theta)+V_{d}(\theta)-\nabla_{\theta}V_{\theta}-\left[r_{m}-r_{a}(p)\frac{p}{J}\right]=0 \qquad (16)$$

which is a quadratic algebraic equation in i_d. That is, by leaving the inductance matrix unchanged in (10), it is posible to transform the matching conditions into simple algebraic equations, instead of partial differential equations, as in the case of mechanical systems. Furthermore, it is proved in [9] that the electromechanical system (7) in closed-loop with the control law (12), and $V_d(\theta)$ a positive definite function, has an asymptotically stable equilibrium point at (8). Under these conditions the original dynamics (7) in closed loop with (12) matches the desired dynamics (11) in the set

$$D=\left\{(\lambda,\theta,p)\in\Re^{n_e+2} \mid \nabla_\theta V(\theta)-\nabla_\theta V_d(\theta)+\left[r_m-r(a)\right]\frac{p}{J}\in T\right\} \quad (17)$$

where $T := \left[\tau_m, \tau_M\right] \subset \Re$ is the interval of admisible torques.

2.2 IDA control of a brushed DC motor

We consider in this section the application of the IDA control methodology above for the asymptotic regulation of the angular position of a brushed DC motor driving a mechanical load. The system model is taken from [10] by considering that most electromechanical systems can be separated into three different parts:

- a dynamic mechanical subsystem, including in this case a position dependent load and the motor rotor;
- a dynamic electrical subsystem which includes all relevant electrical effects;
- a static relationship representing the conversión of electrical energy into Mechanical energy.

The mechanical and electrical subsystem dynamics are respectively represented by the equations

$$\begin{aligned} M\ddot{q} + B\dot{q} + N\sin(q) &= i \\ L\frac{di}{dt} &= v - Ri - K_B\dot{q} \end{aligned} \quad (18)$$

where

- M constant lumped inertia
- N constant lumped load term
- B friction coefficient
- q(t) angular load position
- i(t) electrical current
- L rotor inductance
- R rotor resistance
- K_B back-emf coefficient
- v(t) input control voltaje

Considering the generalized electromechanical system (1)-(4) and specializing it to this case, we can identify $n_e = 1$, $L(\theta) = L \in \Re$, $V(\theta) = N_0\,(1-\cos(\theta))$ and $\mu(\theta) = K_B(\theta) = \tau_L(\theta)$; where $N_0 := N\tau_L$, τ_L is the constant torque coefficient, and we can also obtain the relationships

$$\lambda = Li + \tau_L\theta \qquad N_0 := N\tau_L \quad (19)$$

$$\dot{\lambda} = L\frac{di}{dt} + \tau_L\dot{\theta} \quad (20)$$

$$L\frac{di}{dt} + \tau_L\dot{\theta} + Ri = u \quad (21)$$

$$\tau(i,\theta) = \tau_L i \quad (22)$$

$$J\ddot{\theta} + r_m\dot{\theta} + N_0\sin(\theta) = \tau_L i \quad (23)$$

Note that the equation (23) coincides with the mechanical subsystem equation in (18) for $J = M\tau_L$, $r_m = B\tau_L$, $N_0 = N\tau_L$; whilst (21) coincides with the electrical subsystem equation in (18) for $\tau_L = K_B$.

To transform the brushed DC motor system model into the PCH form, we propose the total energy function

$$H(x) = \frac{1}{2L}\left(\lambda-\tau_L\theta\right)^2 + N_0\left(1-\cos(\theta)\right) + \frac{p^2}{2J} \quad (24)$$

Then, by considering the new state vector $x=[\lambda,\ \theta,\ p]^T$ proposed in [9], the brushed DC motor can be written

$$\begin{aligned} \dot{\lambda} &= -Ri + u \\ \dot{\theta} &= \frac{p}{J} \\ \dot{p} &= J\ddot{\theta} \end{aligned} \quad (25)$$

which can be rewritten in the compact form

$$\begin{bmatrix} \dot{\lambda} \\ \dot{\theta} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} -R & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & -r_m \end{bmatrix} \begin{bmatrix} \nabla_\lambda H(x) \\ \nabla_\theta H(x) \\ \nabla_p H(x) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u \qquad (26)$$

Thus, we can identify the matrices of the system (5)

$$\Im = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}, \quad g = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \Re = \begin{bmatrix} R & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & r_m \end{bmatrix}$$

Note that, the equation $\dot{p}$ in (25) is equivalent to $J\ddot{\theta} + r_m\dot{\theta} + N_0 \sin(\theta) = \tau_L i$.

By following the design procedure explained in the previous section, we choose the functions $V_d(\theta)$ and $r_a(p)$ as suggested in Rodriguez and Ortega (2003).

$$V_d(\theta) = K_p \frac{(\theta - \theta_*)^2}{\sqrt{\left(1 + (\theta - \theta_*)^2\right)}}, \quad K_p > 0 \qquad (27)$$

$$r_a(p) = r_{a_1} + r_{a_2} \frac{p^2}{1 + p^2}; \quad r_{a_1}, r_{a_2} > 0$$

Concerning the brushed DC motor equations, we can identify

$$L(\theta) = L \quad \Rightarrow \quad \nabla_\theta L(\theta) = 0$$

$$\mu(\theta) = \tau_L \theta \quad \Rightarrow \quad \nabla_\theta \mu(\theta) = L \qquad (28)$$

$$\nabla_\theta V_d(\theta) = K_p \tilde{\theta} \frac{(2 + \tilde{\theta})}{\sqrt{1 + \tilde{\theta}^2}}$$

$$V(\theta) = N_0(1 - \cos(\theta)) \quad \Rightarrow \quad \nabla_\theta V(\theta) = N_0 \sin(\theta)$$

Then by defining $\tilde{\theta} := \theta - \theta_*$ and replacing these functions into (16), we can obtain after some further manipulations

$$i_d = \frac{1}{\tau_L}\left[N_o \sin(\theta) + \left\{r_m - r_{a_1} - r_{a_2}\frac{p^2}{1+p^2}\right\}\frac{p}{J}\right] - \frac{K_p\tilde{\theta}}{\tau_L}\frac{2+\tilde{\theta}^2}{\sqrt{\left(1+\tilde{\theta}^2\right)^3}} \qquad (29)$$

The equation (29) can be replaced into (14) and (15) to yield

$$\alpha(x) = -\frac{L}{\tau_L J}\left(r_m - r_{a_1} - r_{a_2}\left[\frac{p^2\left(3+p^2\right)}{\left(1+p^2\right)^2}\right]\right) \qquad (30)$$

$$\beta(x) = \frac{1}{\tau_L}\left[N_0 \cos(\theta) + K_p\left[\frac{-2+\tilde{\theta}^2}{\sqrt{\left(1+\tilde{\theta}^2\right)^5}}\right]\right] + \frac{L}{\tau_L J}\left[r_{a_1} + r_{a_2}\left(\frac{p^2}{1+p^2}\right)\right] \times \left[r_m - r_{a_1} - r_{a_2}\left(\frac{p^2\left(3+p^2\right)}{\left(1+p^2\right)^2}\right)\right]$$

Finally, by substituting the corresponding equations for α, β and i_d into (12), we obtain the stabilizing feedback control law

$$u = Ri_d - \alpha\left(i - i_d\right)\left[\frac{L}{\tau_L}\left(N_0 \cos(\theta) + \tau_L + K_p\left[\frac{-2+\tilde{\theta}^2}{\sqrt{\left(1+\tilde{\theta}^2\right)^5}}\right]\right) - K_p\tilde{\theta}\left[\frac{2+\tilde{\theta}^2}{\sqrt{\left(1+\tilde{\theta}^2\right)^3}}\right]\right]$$
$$+\beta\left[\frac{p}{J} - (i - i_d)L\frac{1}{\tau_L J}\left(r_m - r_{a_1} - r_{a_2}\left(\frac{p^2\left(3+p^2\right)}{\left(1+p^2\right)^2}\right)\right)\right] \qquad (31)$$

This systematic design procedure allows to deal with an important class of electromechanical systems. An adaptive partial state feedback control design method has been proposed by [11] for asymptotic position regulation of electromechanical systems, when only measurement of the electrical coordinates and of the mechanical position are available.

Digital simulations were carried out to evaluate the performance of the closed loop system under control of the feedback law (31). The system parameters used in simulations were:

$$M = 0.0052542 Kg \times \frac{m^2}{rad} \qquad R = 5\Omega$$

$$N = 2.2839 Kg \times \frac{m}{seg^2} \qquad L = 25 \times 10^{-3} H$$

$$B = 0.018 N \times \frac{seg}{rad} \qquad K_p = 8$$

These parameters are for an equilibrium position $\bar{X}_1 = 1.5707\ rad$, corresponding to an equilibrium voltage value $\bar{U} = 11.4195\,volts$ and a current value $\bar{X}_3 = 2.2839\,amp$. Figure 1 shows the controlled state variable response and the input control voltage. This figure verifies the asymptotic behavior of the controlled state variables of the brushed DC motor.

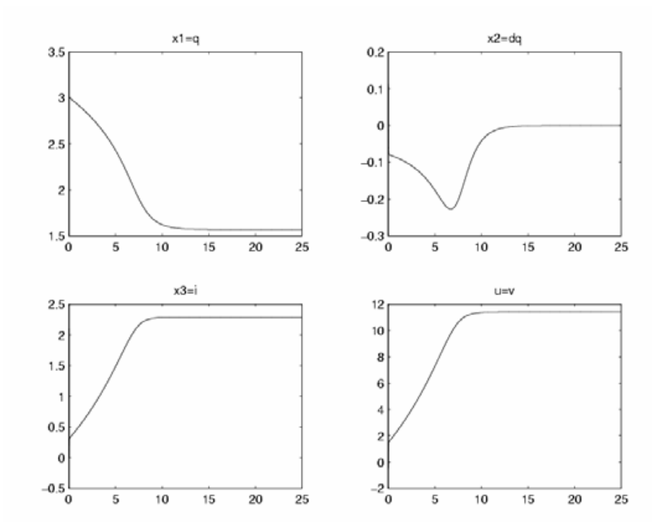

Figure 1. Controlled state response of the brushed DC motor.

3. CONCLUSIONES

We have applied the systematic PCH/IDA control methodology to a brushed DC motor driving a mechanical load for achieving asymptotic stability of the mechanical position. It was shown that modifying the interconnection and damping structures, and with a suitable choice of the free parameters, the matching conditions become simple algebraic equations. Digital simulations demonstrated the asymptotic stability of the controlled response of the brushed DC motor. The output feedback control of this system under parametric uncertainty is an interesting problema to be addressed.

5. REFERENCES

[1] Ortega, R., A. Loria, P. J. Nicklasson and H. Sira-Ramirez (1998). Passivity-based Control of Euler Lagrange Systems. Springer-Verlag. Berlin.

[2] Meisel, J. (1969). Principles of Electromechanical Energy Conversion. Mc Graw- Hill. New York.

[3] Van Der Schaft, A. J. (2000). L_2-Gain and Passivity Techniques in Nonlinear Control. Springer-verlag. Berlin.

[4] Bloch, A. M., N. Leonard and J. E.Marsden (2000). "Controlled Lagrangian and Stabilization of Mechanical systems. 1. The First matching Theorem". IEEE Trans. on Automatic Control 45, 2259, 2259-2270.

[5] Hamberg, J. (2000). "Controlled Lagrangian Symmetries and Condition for Strong Matching". In: Proc. IFAC Workshop on Lagrangian and Hamiltonian Methods for Nonlinear Control (N. Leonard, R. Ortega, Ed), 62-67.

[6] Ortega, R.,M. Spong, F. Gómez-Stern and G. Blankestein (2002a). "Stabilization of Underactuated Mechanical Systems via Interconnection and Damping Assignment". IEEE Trans. Automatic Control. 47 (8), 1218-1233.

[7] Ortega, R. A. J. van der Schaft. B. Masche and G. Escobar (2002b). "Stabilization of Port_Controlled Hamiltonian Systems: Energy Balance and Passivation". Automatica, 38(4), 585-596.

[8] Chang, D., A. M. Bloch, N. Leonard, J. E. Marsden and C. Woolsey (2002). "The Equivalence of Controlled Lagrangian and Controlled Hamiltonian Systems". Control and the calculus of Variations 8, 393.422.

[9] Rodriguez H., and R. Ortega (2003). "Stabilization of Electromechanical Systems via Interconnection and Damping Assignment". Int. J. Robust Nonlinear Control. 13, 1095-1111.

[10] Dawson, D. M., J. Hu. and T: C. Burg (1998). Nonlinear Control of electric Machinery. Marcel dekker, Inc. New York.

[11] Rodriguez, H., A. Astolfi, and R. Ortega (2003). "Adaptive Partial Feedback Stabilization of a Class of electromechanical Systems via Inmersion and Invariance". Proc. 2003, american Control Conference.