A METRIC FOR A CHIRAL POTENTIAL FIELD

UNA MÉTRICA PARA UN CAMPO POTENCIAL QUIRAL

H. Torres-Silva[1]

RESUMEN

En este trabajo se presenta un ejemplo de una métrica específica que geometriza explícitamente un potencial cuadrivector tipo luz (campo quiral). La geometrización muestra que tal vector tiene la misma estructura geométrica que un campo gravitacional Kerr. Se discute una proposición teórica que un cuerpo rotante genera, su gravitación y el calibre de campo tipo magnético que puede ser identificado con un campo quiral geometrizado. Este campo quiral representa un tipo novedoso de campo que no puede ser identificado con alguno de los campos electromagnéticos conocidos. Como aplicación de esta teoría se discute la morfología de los planetas alrededor del sol.

Palabras clave: Potencial vector, campo de fuerza cero, campo quiral, geometrización espacio tiempo, morfología.

ABSTRACT

In this paper we present an example of a specific metric which geometrizes explicitly a light-like four-vector potential (chiral field). The geometrization shows that such a vector has the same geometrical structure as a gravitational Kerr field. We discuss a theoretical proposition that a rotating body generates, besides a special gravitational field, a magnetic-type gauge field which might be identified with a chiral geometrized field. This chiral field represents a novel type of field because we cannot identify it with any of the known electromagnetic fields. As an application of this theory we discuss the morphology of the planets around the sun.

Keywords:Light-like vector potential, force-free field, complete geometrization spacetime, morphology.

INTRODUCTION

In this contribution, we construct a metric which appears appropriate for a geometrization, within the framework of a Riemannian spacetime, of a light-like 4-vector potential field which can be assigned to an electromagnetic-type field. Such field with a 4-vector potential A_α satisfies the relation

$$A_\alpha A^\alpha = 0, \ A_\alpha A^\alpha \big|_{\parallel} = 0, \ A_\alpha A^\alpha \big|_{\perp} = 0 \Rightarrow A_\alpha A^\alpha = 0, \ (1)$$

and A_α is denoted by us as a chiral field. In accordance with our information something emerged for the first time in the work of M Evans in connection with the hypothesis of the existence of a special kind of magnetic field (see, for instance [1]).

The starting point is the well known approach to the geometrization of physical fields involving the construction of spacetime geometries (the so called force-free geometries) within which the geodesic equation proves to be identical to the equation of motion of a particle when interacting with such (nongravitational) fields. This method derives in fact from the generalized Einstein's equivalence principle which asserts that "any trajectory is a geodesic of some geometry" [2]. Furthermore, the laws of motion, in the case of interacting particles, are given by the differential equations of the geodesics for the metric in question at the instantaneous position of each particle [3].

Pursuing this subject, we observe that for the formulation of the geodesic equations also in the presence of nongravitational forces, some efforts have been directed towards applying changes to the metric and other efforts to modifications of the connection [4], in a Riemann or a Riemann-Cartan spacetime. There appeared also papers

---

[1] Instituto de Alta Investigación. Universidad de Tarapacá. Antofagasta Nº 1520. Arica, Chile. E-mail: htorres@uta.cl

which consider the possibility of applying a Finsler or a Randers geometry or a fractal spacetime geometry in order to establish unitary theories of gravitation and electromagnetism in conjunction with a probabilistic interpretation of the geometry of the background spacetime [4].

However, all these alternative interpretations of force-free geometries have not yet reached the same level of elaboration and experimental verification as is the case for Einstein's general theory of relativity the formal structure of which has continuously invited the development of gauge theories. These are reasons that why we maintain in the present work the framework of a Riemannian spacetime which helps us to geometrize a vectorial field. We propose a geometrization of a vectorial field in the sense that the associated physical quantity (e.g., the four-vector potential A_α) enters directly into the metric which may be interpreted, alternatively, as an 'interior' ($T_{\alpha\beta} = 0$, to obtain the microscopic Dirac equation) or 'exterior' ($T_{\alpha\beta} = 0$, to obtain the classical solution of Mercury's orbit) solution of Einstein's equations. However, from an Einsteinian point of view, the field defined by A_α is completely (truly) geometrized (like the gravitational field itself) if it leads to a determination of the geometry of the (curved) vacuum spacetime in which no other (non-geometrized) matter manifests its presence in conjunction with a non-zero energy-momentum tensor. We emphasize that the physical quantities (e.g., density, pressure, electromagnetic field tensor, etc.) which generally appear on the right hand side of Einstein's equations represent non-geometrized quantities, i.e., the source of the (geometrized) gravitational field. Our conjecture is that if $T_{\alpha\beta} = 0$, det $F_{\alpha\beta} \neq 0$, then, A_α is completely (truly) geometrized [5].

In the present paper we adhere to the Einstein's general relativity and thus the energy and momentum of the geometrized chiral field are encapsuled solely in the pseudotensor $t_{\alpha\beta}$ on the same geometrical footing as any gravitational field. We recollect that the general relativity is a very special non-Abelian gauge theory and thus it is possible that a truly spacetime geometrization can be applied also to a non-Abelian analogue of the electromagnetic field. The Yang-Mills field may serve as such a field.

Attempts have also been made to mix directly the standard symmetric Riemannian metric tensor with an antisymmetric (electromagnetic) field tensor, but the new nonsymmetric metric cannot achieve a real geometrization of the electromagnetic field [6].

A possible existence of a light-like 4-vector electromagnetic field would be a proof that the most important metrics of

general relativity, Schwarzschild and Kerr solutions, (which in Eddington coordinates are described also by light-like four vectors) have an electromagnetic analogue. Thus, the Kerr metric, which represents the gravitational field exterior to a spinning source which 'drags' space around with it, has the same geometrical structure as a geometrized chiral field like an Evans-Vigier field. On a microscopic level, the Evans' optical (light) magnet [6] produced by a circulary polarised light beam appears as a natural and physically possible hypothesis. A search for cyclically symmetric equations, similar to spin angular momentum relations but now refering to a magnetic-type field, seems also tempting from a geometrical point of view. Of course, as for gravitation or perhaps for the entire field of physics we do not yet know the physical intrinsic mechanism of such a magneto-rotation induction: 'rotation generates magnetic-type field and magnetic field generates rotation', and yet we attempt to model and describe it here.

A simple experimental proposal for the verification of these hypotheses may be the detection of an Aharonov-Bohm effect as arising, for example, in the usual two-slit electron diffraction experiment in which the solenoid is replaced by a rotating body. Indeed, the gravitational field of a rotating astrophysical lens object plays the role of both a double slit (by its electriclike and curvature inducing effects by gravity) and an 'external' field (with a magneticlike contribution of the gravitation). A proposal for a laboratory experiment for an observation of a gravitational Aharonov-Bohm effect in conjunction with photons is described in [6].

In the final section we present a discussion on the possibility of identifying a chiral field like an a modified Evans-Vigier field within the set of known electromagnetic fields.

SPECIAL METRIC AND BASIC RELATIONS

Let us consider a null-like four-vector with components

$$A_\alpha\left(x^\beta\right) \equiv \left(A_0, A_1, A_2, A_3\right) = \left(A_0, A\right) \qquad (2)$$

We denote by

$$A^2 = \eta^{\alpha\beta} A_\alpha A_\beta = 0 \qquad (3)$$

its Minkowskian module in which

$$\eta_{\alpha\beta} = \lceil +1, -1, -1, -1 \rfloor \qquad (4)$$

is the Minkowski (flat) diagonal metric [8]. We should mention that $A_\alpha(x^\beta)$ is here a standard spacetime

vector which may represent the vector potential of an electromagnetic-type gauge field. For the moment, we cannot foresee if A_α may be associated with a massive or zero-mass field or if we must include the subject of a gauge invariance. Consequently, all the calculations are given in the tangent bundle of spacetime.

We propose to study under which conditions a metric $g_{\alpha\beta}$ having the special form

$$g_{\alpha\beta} = \eta_{\alpha\beta} + KA_\alpha A_\beta \tag{5}$$

where $k \propto T$ is a constant still to be determined, can define a chiral field like an Evans-Vigier field, A chiral field is defined as

$$A \rightarrow (1 + T\nabla\times)A \tag{6}$$

(T is the chiral factor).

The determinant of the metric tensor $g_{\alpha\beta}$ is given by

$$\det\left(g_{\alpha\beta}\right) \equiv g = -\left(1 + KA^2\right) = -1 \tag{7}$$

and thus, the inverse (contravariant) metric is

$$g^{\alpha\beta} = \eta^{\alpha\beta} - KA^\alpha A^\beta \tag{8}$$

The metric (5) is similar to the one which describes a weak gravitational field, i.e.,

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \ g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu} \tag{9}$$

However, for the time being we do not impose yet any condition on the value or the strength of the term $KA_\alpha A_\beta$. There follows that

$$A^\alpha = g^{\alpha\beta} A_\beta = \eta^{\alpha\beta} A_\beta, \ \eta^{\alpha\beta} A_\alpha A_\beta - g^{\alpha\beta} A_\alpha A_\beta = 0 \tag{10}$$

and thus the indices of A^α may be raised and lowered with either the metric $g_{\alpha\beta}$ or the Lorentz metric $\eta_{\alpha\beta}$. It is easy to show that

$$A_\alpha A^\alpha_{;\beta} = A_\alpha A^\alpha_{,\beta} = 0 \tag{11}$$

where the ordinary partial derivatives are denoted by commas (or alternatively by ∂_α and $\partial/\partial\chi^\alpha$), and covariant derivatives by semicolons. The Christoffel symbols are

$$\Gamma^\alpha_{\beta\gamma} = g^{\alpha\sigma}\left[\beta\gamma, \sigma\right]$$
$$= \frac{1}{2}Kg^{\alpha\sigma}\left[\left(A_\sigma A_\beta\right)_{,\gamma} + \left(A_\sigma A_\gamma\right)_{,\beta} - \left(A_\beta A_\gamma\right)_{,\sigma}\right] \tag{12}$$

and $[\beta\gamma, \sigma]$ is the Christoffel symbol of the first kind. Because, $g =$ constants $= -1$, it follows that

$$\Gamma^\alpha_{\beta\alpha} = 0 \tag{13}$$

and thus following [8] the Ricci tensor is given by

$$R_{\beta\gamma} = -\eta^{\alpha\sigma}\left[\beta\gamma, \sigma\right]_{,\alpha} + K\left[A^\alpha A^\sigma\left[\beta\gamma, \sigma\right]_{,\alpha} + \left(A^\alpha_{,\alpha}A^\sigma + A^\alpha A^\sigma_{,\alpha}\right)\left[\beta\gamma, \sigma\right]\right]$$
$$+ \eta^{\alpha\mu}\eta^{\sigma\nu}\left[\beta\sigma, \mu\right]\left[\gamma\alpha, \nu\right]$$
$$- K\left(\eta^{\alpha\mu}A^\sigma A^\nu + \eta^{\sigma\nu}A^\alpha A^\mu\right)\left[\beta\sigma, \mu\right]\left[\gamma\alpha, \nu\right]$$
$$+ K^2 A^\alpha A^\mu A^\sigma A^\nu\left[\beta\sigma, \mu\right]\left[\gamma\alpha, \nu\right]$$
$$\equiv KR_1 + K^2R_2 + K^3R_3 + K^4R_4 \tag{14}$$

FORCE-FREE CHIRAL FIELD

Introducing the parameter s defined by

$$ds^2 = g_{\alpha\beta}dx^\alpha dx^\beta \tag{15}$$

the equations of geodesics,

$$\frac{du^\alpha}{ds} + \Gamma^\alpha_{\beta\gamma}u^\beta u^\gamma = 0, \ u^\mu = \frac{dx^\mu}{ds} \tag{16}$$

become

$$\frac{du^\alpha}{ds} + KA^\alpha\frac{dC}{ds} - KC\eta^{\alpha\sigma}B_{\sigma\gamma}u^\gamma \tag{17}$$

where

$$C = A_\beta u^\beta. \tag{18}$$

At this point it is easy to see that an Evans-Vigier field described by the metric (5) becomes a 'force-free field' with respect to the motion of a charged test particle having the characteristic parameter e/m_0, and subject to the constraint

$$KC = \text{constant} = T\frac{e}{\hbar c} = \frac{e}{m_0 c^2}. \tag{19}$$

Ingeniare. Revista chilena de ingeniería, vol. 16 Nº 1, 2008

93

With this constraint, the geodesic equation (4) reduces formally to the Lorentz equation,

$$\frac{du^{\alpha}}{ds} = T\frac{e}{\hbar c}\eta^{\beta\alpha}F_{\beta\nu}u^{\nu} \qquad (20)$$

We may identify $F_{\beta\nu}$ with an electromagnetic field tensor if the 4-vector potential A_{μ} is related to an electromagnetic potential A_{μ} by a gauge transformation of the second kind

$$A_{\mu} = \mathcal{A}_{\mu} + \frac{\partial\phi}{\partial x^{\mu}} \qquad (21)$$

Since it is possible to demonstrate that constraints such as (6) and (8) are consistent and in fact do not contradict each other along the trajectory of the test-particle (see, for instance, [19]), we can assert that we have achieved a local or a semilocal geometrization (i.e., one along a curve) of the chiral field.

The final conclusion of this section is that any field described by a metric of the form (5) may act on a test particle with a Lorentz-type force (7). In such geometrical terms, a Lorentz-type force was known until now only for a weak gravitational field (see, for instance, [6]).

A COMPLETE GEOMETRIZATION OF A CHIRAL FIELD

Bearing in mind that the metric tensor is given in our account by equations (5) and (7), we need only derive the $R_{\alpha\beta}$, R, and also the Einstein's tensor from the $g_{\alpha\beta}$ and establish in this way the components of the matter tensor $T_{\alpha\beta}$. If this energy-momentum tensor coincides with one which is known for a given (physical, phenomenological) material scheme, we say that (5) represents a solution of Einstein's equations for such a scheme. If we do not posses such a coincidence, we say that we face an exotic matter which might determine the desired properties of the spacetime (e.g., 'traversable wormhole' [21] or 'warp drive' [22]). From this point of view the general theory of relativity is not a closed theory, and sometimes the Einstein's equations seem to form a mathematical identity if a suitable metric is chosen:

$$G_{\alpha\beta} \equiv \kappa\frac{G_{\alpha\beta}}{\kappa} \equiv \kappa T_{\alpha\beta} \qquad (22)$$

In other words, in this case Einstein's equations are used merely for a definition of an energy-momentum tensor which generates a given gravitational field.

In the following we will not use this identity aspect of the Einstein's equations since we intend to geometrize the field A_{α} which may be considered as a gravitational perturbation of a vacuum spacetime. Then the field equations correspond to an 'exterior case' and are given by

$$R_{\beta\gamma} = 0 \qquad (23)$$

where $R_{\beta\gamma}$ is given by equation (14). In a way, the constant K may be called a 'coupling constant' because it characterizes the strength of the perturbation of the vacuum spacetime generated by a chiral field like an Evans-Vigier field. We assume that the form of the metric (5) retains its independence from the value of K. In other words, the metric $g_{\alpha\beta}$ given by (5) remains a solution for any arbitrary value of K. Thus in the expression (14) of $R_{\beta\gamma}$, each coeficient of K and of its powers must be cancelled separately. In this way, following [6], we obtain four equations:

$$R_1 = 0 = -\eta^{\alpha\sigma}\left[\beta\gamma,\sigma\right]_{,\alpha} \qquad (24)$$

$$R_2 = 0 = C_0T\left[A^{\alpha}A^{\sigma}\left[\beta\gamma,\sigma\right]_{,\alpha} + \left(A^{\alpha}_{,\alpha}A^{\sigma} + A^{\alpha}A^{\sigma}_{,\alpha}\right)\left[\beta\gamma,\sigma\right]\right] \\ + \eta^{\alpha\mu}\eta^{\sigma\nu}\left[\beta\sigma,\mu\right]\left[\gamma\alpha,\nu\right] \qquad (25)$$

$$R_3 = 0 = -C_0T\left(\eta^{\alpha\mu}A^{\sigma}A^{\nu} + \eta^{\sigma\nu}A^{\alpha}A^{\mu}\right)\left[\beta\sigma,\mu\right]\left[\gamma\alpha,\nu\right] \qquad (26)$$

$$R_4 = 0 = +C_0^2T^2A^{\alpha}A^{\mu}A^{\sigma}A^{\nu}\left[\beta\sigma,\mu\right]\left[\gamma\alpha,\nu\right] \qquad (27)$$

We note that, in accordance with equation (13), the potential A_{α} generates a new light-like vector a_{α} which, by analogy with the kinematics of a timelike congruence of curves, may be called an 'acceleration-potential vector' and has the following properties:

$$a^{\alpha} = A^{\alpha}_{;\beta}A^{\beta} = A^{\alpha}_{,\beta}A^{\beta} = -b\left(x^{\gamma}\right)A^{\alpha}a^{\alpha} = g^{\alpha\beta}a_{\beta} = \eta^{\alpha\beta}a_{\beta} \qquad (28)$$

$$\eta^{\alpha\beta}a_{\alpha}a_{\beta} = g^{\alpha\beta}a_{\alpha}a_{\beta} = 0 \\ a_{\alpha}a^{\alpha}_{;\beta} = a_{\alpha}a^{\alpha}_{,\beta} = a_{\alpha}A^{\alpha}_{;\beta} = a_{\alpha}A^{\alpha}_{,\beta} = 0 \qquad (29) \\ a^{\alpha}A_{\alpha} = 0$$

94

Ingeniare. Revista chilena de ingeniería, vol. 16 Nº 1, 2008

We notice that equations (12) and (14) are satisfied identically, and that equation (13) is reduced to the definition of the acceleration potential (15). Thus the Einstein field equations (11)-(14) become

$$\Box^2\left(A_\beta A_\gamma\right)\Big|_{\frac{\partial}{\partial t}\to\frac{\partial}{\partial t}(1+T\nabla\times)} = 0 \qquad (30)$$

For the stationary case,

$$\Box^2 \to -\nabla^2(1+T\nabla\times)^2\,, \qquad (31)$$

we have particular vectorial solutions

$$\nabla\times A = \mp\frac{k}{\left(1\mp kT\right)}A\,, \qquad (32)$$

there arise two remarkable solutions of equation (30), namely, the Schwarzschild-type solution, and the Kerr-Schild type metric.

Here kT is related to the angular velocity and, thus, to the angular momentum of the source. We remind the reader that the Kerr metric represents a vacuum field exterior to a spinning source. Hence, a chiral field like an Evans-Vigier field and a typical gravitational field have the same topological properties. It is important to stress that for the Schwarzschild-type solution (31),

$$\nabla\times A^S = 0,\ T\to\infty \text{ (no magnetic-type field)} \quad (33)$$

and for the Kerr-Schild type metric (19)

$$\nabla\times A^{KS} \neq 0,\ T\neq 0 \text{ (magnetic-type field)}. \quad (34)$$

An immediate consequence of these results is that rotating bodies generate, besides a special kind of gravitational field, also some magnetic-type gauge fields defined by light-like vector potentials. For the time being all experimental tests of general relativity (e.g., Advance of the perihelion of Mercury, Bending of light, Gravitational red shift, etc.) are expressed only as functions of the mass of the central gravitating body. In order to evaluate the physical implications of the chiral field we must evaluate all these effects in terms of the light-like vector potential.

PHYSICAL CONTENT OF CHIRAL CONDITION

Four Independent Electromagnetic Invariants

In Classical Electrodynamics there exist only four independent electromagnetic (EM) field invariants [7], namely (in units with c=1),

$$I_0 = A_\alpha A^\alpha\,, \qquad (35)$$

$$I_1 = \frac{1}{2}F_{\alpha\beta}F^{\alpha\beta} = |\mathbf{E}|^2 - |\mathbf{B}|^2\,, \qquad (36)$$

$$I_2 = -\frac{1}{2}F_{\alpha\beta}\mathcal{F}^{\alpha\beta} = 2\mathbf{E}\cdot\mathbf{B}\,, \qquad (37)$$

$$I_3 = -2A_\alpha T^{\alpha\beta}A_\beta\,, \qquad (38)$$

where $F^{\alpha\beta}$ is the EM field tensor, $\mathcal{F}^{\alpha\beta}$ is the dual EM field tensor and $T^{\alpha\beta}$ is the Maxwell stress-energy tensor. Salingaros [7] used these invariants to announce the proposition: plane monochromatic EM (transverse) waves are characterized by vanishing invariants $I_1=I_2=I_3=0$ in the Lorentz gauge. As we mentioned, a chiral field as an Evans-Vigier field are defined by a vanishing invariant I_0 = 0, but contrary to Evans-Vigier field, the conditions for a chiral field are $I_0 = 0$, $I_\parallel= 0$, $I_\perp = 0$ and $I_3 = -2A_\alpha T^{\alpha\beta}A_\beta$.

Rotation and Chiral Field

Following our preceding account, we may now state that a geometrized chiral field like an Evans-Vigier field represents a classical but exotic electromagnetic-type field which possesses similar properties to gravitational fields defined by Schwarzschild and Kerr metrics. The process of geometrizing such field, through association of the vector potential with part of the structure of spacetime, leads to the supposition that, possibly, there exists a fundamental relation between rotation and a magnetic-type field. It should be emphasized that in a sense our results demostrate a generalisation of and the reciprocity to a well known physical phenomenon. Thus, considering a free particle in an external electromagnetic field defined by the tensor $F_{\alpha\beta}$, we observe the generation of a vorticity,

$$\omega_{\alpha\beta} = u_{\alpha;\beta} - u_{\beta;\alpha}\,, \qquad (39)$$

which is related to the field tensor $F_{\alpha\beta}$ via the (London) equation of superconductivity [6]:

$$F_{\alpha\beta} = \frac{\partial A_\beta}{\partial x^\alpha} - \frac{\partial A_\alpha}{\partial x^\beta} = \frac{mc}{e}\omega_{\alpha\beta}\cdot \qquad (40)$$

Equation (40) expresses that the four-vector potential A_α is tangent to the particle trajectories at all points

and thus the particle velocity is proportional to the vector potential as we have seen above. It is important to stress that it is the external vectorial field A_α which determines the motion of a test particle and not vice versa. Moreover, generally, the four-velocity u_α may be defined as the vector-potential of an inertial-gravitational field and may be assigned to each point of the spacetime independently of the fact whether or not a test particle resides at that point [30-31]. Hence, if the vacuum spacetime is perturbed by the presence of the vectorial field A_α we can assert that the source of vorticity is precisely this field.

Our generalisation arises from the fact that not only does a normalized (Dirac) vector potential field [see eq. (41)] generate a vorticity field, but yields also a relation between the angular momentum of a rotating body and a geometrized light-like vector potential. This result is clearly illustrated by equations (32) and (36).

Morphology of the Solar System Set with $\nabla \times B = \kappa B$

It has been shown in detail elsewhere [8] that the Bode numbers and measured velocity ratios of the planets are accurately predicted by the eigenvalues of the Euler--Lagrange equations resulting from the variation of the free energy of the generic plasma that formed the Sun and planets. This theory is reviewed to show that the equation (36) $\nabla \times \mathbf{A} = \kappa \mathbf{A}$ can explain the velocity ratios of planets, the Bode numbers correspond to the roots of the first-order Bessel functions. The extrema of the roots of the zeroth-order Bessel function predict the ratios of the measured planetary velocities almost without error for the outer planets. Both sets of roots correspond to the same eigenvalue solution of the force-free equations. Both the Titius--Bode series and Kepler's harmonic law are predicted by the "relaxed state solution" of the free-energy equation for the generic plasma that formed the Sun and planets. Newton's law of gravitation is not used in the calculations. Here we use the chiral approach where $R_\mu = 0$, and $\mathbf{A} \parallel \mathbf{B}$.

The compressed gas forms a cylindrical volume of plasma which is moving through the background plasma and rotating with a finite angular velocity. As this mass of plasma propagates through the surrounding gas, it loses energy by accelerating the surrounding plasma. The cylinder will lose energy and settle down to a minimum-energy "relaxed" state, a force-free collinear cylindrical structure. It is shown in detail elsewhere -- that the resulting "field equations" for the flow are given by

$$\nabla \times \mathbf{B} = k\mathbf{B} \tag{41}$$

$$v = \pm \left[\frac{\gamma - 1}{\gamma - 2} \right] \frac{\mathbf{B}}{(\mu_0 \rho)^{1/2}} \tag{42}$$

where ρ: fluid density, $\mathbf{B}$: magnetic induction field, v: velocity of the center of mass of a fluid element, and γ: ratio of specific heats of the gas.

A pseudoplane solution to the force-free equation (1) is given by

$$B_r = -k^2 \tilde{a} \frac{J_1(kr)}{kr} \sin \theta$$

$$B_\theta = k^2 \tilde{a} \left\{ \frac{J_1(kr)}{kr} - J_0(kr) \right\} \cos \theta$$

$$B_z = k^2 \tilde{a} J_1(kr) \cos \theta$$

where B_r, B_θ, and B_z are the magnetic induction components in the striated rings of the gas cylinder, and

$$\tilde{a} = k_\theta \theta + k_z z, \quad k^2 = k_1^2 + k_z^2$$

where k_θ, k_1, and k_z are constants supplied by the boundary conditions given by the chiral approach. If we plot of J_1 and J_0 with the functions scaled to the geometry of the solar system, we observe that for $J_1 = 0$,

$$B^2 \sim J_0^2, \quad B_\theta \sim J_0.,$$

This maximizes the magnetic and kinetic energy at the origin. In the cylindrical structure formed by the supernova explosion, the first root corresponds to the structure of the star at the center of the hypothetical solar system, and the second root corresponds to a ring of gas just outside the star. The corresponding flow velocities in the rings is given by eq. 42. The geometry of the configuration is shown in figure 1.

The signs reverse for every other ring (corotational and contrarotational) so that the azimuthal velocities are all prograde. The azimuthal velocity of the gas in each ring has a direct relationship to the velocities of the planets
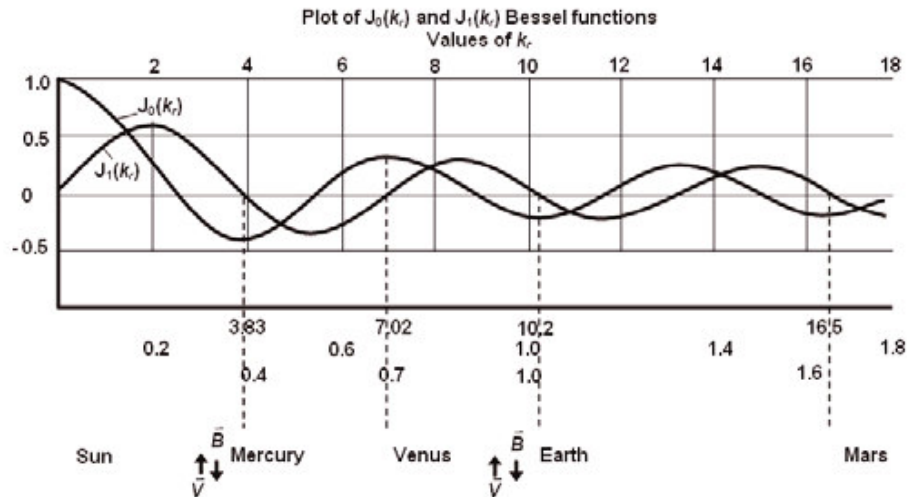
Figure 1. Plots of Bessel functions versus distance (AU) of planets from the sun.

as they exist today. An examination of figure 2 shows that the Bode numbers of the planets out to Jupiter are predicted by the roots of the equations describing the "relaxed state" of the primordial gas. Comparison of the measured velocity ratio with the ratios of the extrema of J_0 (kr) show very close agreement.

For the outer planets, the Bode series fails completely for Neptune and Pluto, but the plasma solutions, the Bessel function roots, give exact predictions. We can observe that if the asymptotic expansions J_1 (kr) and J_0 (kr) where carried out, the theory could be checked all the way out to and including Pluto.

The predicted ratios of the successive peak velocities of the gas in the rings check the measured velocity ratios of the inner planets within a few percent. The velocity ratios for Uranus, Neptune, and Pluto are exact. the relaxed state of the generic plasma predicts both the Bode number series and Kepler's harmonic law $p^2 = a^3$ where p: period of the planet, a: average radius of the planet.

It is suggested that the rings of gas in the planet structure "roll up" azimuthally to form balls of gas that eventually evolve into the planet . The roll up of vortex rings to form balls of gas is a well-known phenomenon which has been observed in laboratory experiments.

A planet is predicted at 1.3 AU. No such planet exists today. It is suggested that the missing planet suffered a catastrophe either in the birthing process or at a later time and that the residue is our moon.
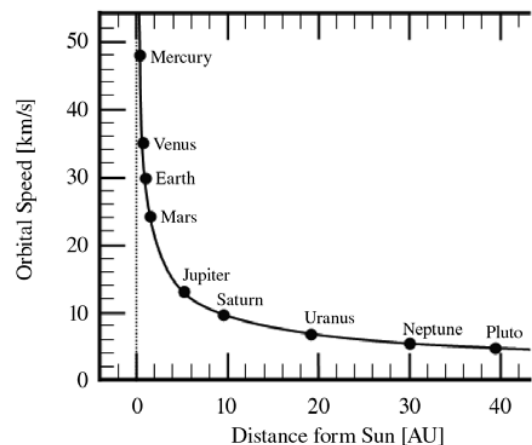

Figure 2. Orbital speed as a function of the distance from the sun.

REFERENCES

[1]   M.W. Evans. "General covariant unified field theory". Abramis Academic. Suffolk. Vol. 1. 2005.

[2]   A.J. Wheeler. "Geometrodynamics". Academic Press, pp. 225-253. 1962.

[3]   A. Einstein. "The Foundation of the General Theory of Relativity". Annalen der Physik

49. 1916. The Principle of Relativity. Dover, pp. 111-164. 1952.

[4]   C. Möller. "Selected Problems in General Relativity". Brandeis University 1960 Summer Institute in Theorical Physics. Lecture Notes. Brandeis University. 1960.

[5]   H. Torres-Silva. "Electrodinámica quiral: eslabón para la unificación del electromagnetismo y la gravitación". Ingeniare. Rev. chil. ing. Vol. 16 Nº 1, pp. 6-23. 2008.

[6]   J. Argyris, C. Ciubotariu and I. Andreadis. "A metric for an Evans Vigier field". Foundation of Physics Letters. Vol. 11, pp. 141. 1998.

[7]   N. Salingaros. "Invariants of the electromagnetic field and electromagnetic waves". Am. J. Phys. Vol. 53, pp. 361. 1985.

[8]   D.R. Wells. "Quantization effects in the plasma universe". IEEE Trans. Plasma Sci. Vol. 17, pp. 270. 1989.