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A Multi-Agent Proposal for the Resolution of BIBD Instances

Una propuesta multi-agente en la resolución de instancias del BIBD

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Abstract

The problem with designing balanced incomplete blocks (BIBD) is enclosed within the combinatorial optimization approach that has been extensively used in experimental design. The present proposal addresses this problem by using local search techniques known as *Hill Climbing*, *Tabu Search*, and an approach based considerable sized the use of *Multi-Agents*, which allows the exploration of diverse areas of search spaces. Furthermore, the use of a vector vision for the consideration associated with vicinity is presented. The experimental results prove the advantage of this technique compared to other proposals that are reported in the current literature.

Key words: Balanced incomplete block design, Vector process, Computer search, Experimental design.

Resumen

El problema del diseño de bloques incompletos equilibrados (BIBD) se enmarca dentro del enfoque de optimización combinatoria que ha sido utilizado ampliamente en el diseño de experimentos. La presente propuesta aborda este problema utilizando técnicas de búsqueda local conocidas como *Ascenso a la Colina* (Hill Climbing), *Búsqueda Tabú* (Tabu Search) y un enfoque basado en el uso de *Multi-Agentes* que permiten la exploración de diversas áreas de espacios de búsqueda de tamaño considerable, además se presenta el uso de una visión vectorial para la consideración asociada a la

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vecindad. Los resultados experimentales evidencian la ventaja de esta técnica frente a otras propuestas mostradas en la literatura actual.

Palabras clave: diseño de bloques incompletos equilibrados, proceso vectorial, búsqueda por computador, diseño de experimentos.

1. Introduction

According to Hinkelman & Kempthorne (1994), experimental designs are found to be classified in hierarchic order, depending on the number of local control or block factors. When the designs have parameters that lack precision, they are said to be Random Blocks Designs, which, depending on some inherent properties, can be identified as: Completely Randomized (CRB), Generalized Randomized (GRB), and Incompletely Randomized (IRB). The last two are characterized because not all the treatments can be applied in each block. Within the field of IRB, it is worth pointing out the design of Incomplete Balanced Blocks, which were introduced by (Yates 1936). On many occasions, experiments might require a reduction in the size of the blocks. The design of complete blocks can reduce the estimated variance of the experimental error; however this reduction might be insufficient because the number of treatments can be very large and the reduction can result impractical.

The design of incomplete blocks can avoid the complete replication of all the experimental treatments. The designs of blocks in general, are arrangements designed with the objective of having systematic control over the variability of the data attributable to external sources and to separate, minimize and even eliminate this variation from the rest of the effects of the factors of interest (Fisher 1926). The design of incomplete blocks has r of t treatments in b blocks of k experimental units with $k < t$. Raghavarao (1988), defines a BIBD as an arrangement of v symbols and b groups in which each one of the $k < v$ symbols must satisfy the following conditions: *i*) Each group has exactly k different symbols, *ii*) each symbol is present in exactly r groups, and *iii*) each pair of symbols are together in exactly λ groups. The parameters of a BIBD must be positive whole integers and can be identified as (v, b, r, k, λ) , where v represents the number of blocks, b the number of varieties in the design, r the number of replications in each variety, k the number of different elements in each block, and λ the number of groups in which each pair of symbols occur. This must satisfy the $\lambda(v - 1) = r(k - 1)$ relation. This kind of technique was initially applied in experimental design (van Lint & Wilson 1992, Mead 1993): however, recently it has been applied in fields such as cryptography (Buratti 1999), codification theory (Lan, Tai, Lin, Memari & Honary 2008), sports league planning (Anderson 1997), and others.

The development of block designs is enclosed within the NP-Hard type of problems (Corneil & Mathon 1978), and it provides an excellent reference point. This is considering that it can scale and it has a wide variety of scenarios with countless instances that can be ranked according to the degree of difficulty to generate a solution. A complete number of algorithms have been applied to the problem, which continues to be unsolvable, even for designs that are relatively small in size

(Gibbons & Ostergard 2007). This article approaches the BIBD problem from a vector spaces stand point and the relationship these vectors have with each other because of their separation angle. In addition a metaheuristic technique is employed based on local search (specifically Hill Climbing) providing a resolution algorithm of great magnitude compared to other specialized algorithms found in the current literature. The BIBD problem has been addressed in recent literature by using a great number of different techniques. Some of the results obtained with these techniques can be found in Colbourn & Dinitz (1996). Much of the previous research has addressed the problem from several points of view. Specifically mathematical programming (Whitaker, Triggs & John 1990, John, Whitaker & Triggs 1993), restriction programming (Flener, Frisch, Hnich, Kzltan, Miguel & Walsh 2001, Puget 2002, Meseguer & Torras 2001). Regarding restriction programming, it is convenient to mention that a point of interest for researchers has been the symmetric property (permutation of rows and columns that can produce the same results) present in the problem. In term of this problem, Puget (2002) proposes a combination of methods to solve the problem through the rupture of symmetries, while Meseguer & Torras (2001) explore two strategies (a heuristic one for the selection of variables and a trimming procedure).

Recent proposals have used heuristic techniques to resolve BIBD. For this reason, the generation of the BIBD type designs was formulated by Bofill, Guimerà & Torras (2003) as a combinatory optimization problem addressed at neural networks. The simulated annealing algorithm that is supplied with a neural network (NN-SA) has proven to offer a better performance than the analogous hybridizations with simulated annealing. These results were improved by Prestwich (2003a, 2003b), using an local search algorithm in conjunction with the addition of new restrictions that were a result of with the symmetric outline of the problem. Rodriguez, Cotta & Fernandez (2009), recently improved the results obtained by the NN-SA and CLS algorithms. They considered the use of two local search techniques (hill climbing, tabu search), as well as a technique based on genetic algorithms. To do this, two forms of vicinity were used, the change of ones for zeros (bit-flip) and the exchange of pairs (0,1) or (1,0) (position-swapping); results reveal the superiority of the local search technique: the Tabu Search with swap vicinity. The same authors presented an approach based on the application of a hybrid nature memetic algorithm (Rodriguez, Cotta & Leiva 2011), in which was extended the number of evaluations of the target function to 2×10^7 and a new crossing operator was incorporated named *Greedy*. A total of 63 of 86 instances was solved with this approach. Finally, Daisuke Yokoya (2009), propose a mathematical model based on the restrictions that a BIBD must meet, the local search technique Tabu Search, and the resolver of linear models CPLEX, in order to provide a solution to the well-known designing balanced incomplete blocks problem. This proposal shows the best results obtained to date; a total of 78 of the 86 instances studied in the proposal were solved.

2. Methods

Using the notation $\langle v, b, r, k, \lambda \rangle$, a $\langle v, b, r, k, \lambda \rangle$ -BIBD problem consists of the grouping of a series of v objects in b groups of $k < v$ objects each in such a way that a specific object in v belongs to r different groups, and any pair of objects in v will be present in exactly $\lambda < b$ groups. A standard form of representation for a solution to the problem could be expressed in terms of the incidence matrix, $M \equiv \{m_{ij}\}_{v \times b}$, as is presented in Hall (1998). This is subject to the following:

$$m_{ij} = \begin{cases} 1 & \text{if treatment } i \text{ appears in block } j \\ 0 & \text{if treatment } j \text{ does not appears in block } j. \end{cases}$$

Furthermore, $\sum_{j=1}^b m_{ij} = r$, $\sum_{i=1}^v m_{ij} = k$, $\sum_{j=1}^b m_{ij}m_{i'j} = \lambda$; $i = 1, 2, \dots, v$; $j = 1, 2, \dots, b$; with $i \neq i'$; $i, i' = 1, 2, \dots, v$. An example of this representation can be observed in Figure 1, which shows the incidence matrix M for a possible solution to the instance $\langle 8, 14, 7, 4, 3 \rangle$ -BIBD.

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 0 |

FIGURE 1: Candidate solution for the instance $\langle 8, 14, 7, 4, 3 \rangle$ -BIBD.

The relation between the parameters $\langle v, b, r, k, \lambda \rangle$ must satisfy $vr = kb$; $r(k-1) = \lambda(v-1)$; $\lambda = \frac{r(k-1)}{(v-1)}$ and $b > v$ (Fisher's (1940) inequality). Also, if $v = b$ and $r = k$, we have a symmetric design.

2.1. Vectorial Approach for $\langle v, b, r, k, \lambda \rangle$ -BIBD

If we analyze the incidence matrix, we can see that the representation of each one of the rows of the matrix is given by a binary components vector in the space R^n . The totality of the vector space represents the possible v treatments of the design to be built. Evidently not all the vectors correspond to feasible treatments because some of them will not meet the necessary conditions that a valid BIBD must have. On the other hand, the condition $\sum_{j=1}^b m_{ij}m_{i'j} = \lambda$ allows a direct relationship to be established between the value of λ and the α angle between two specific vectors of the vector space. According to vector algebra we have that $u, v \in R^b$, with binary components, and with the operations of the scalar product

of vectors, vector module, inequality of Cauchy-Schwarz, and the angle between two vectors, allows us to establish that, for any pair of vectors m_{1j} y m_{2j} , with $j = 1, 2, \dots, b$, the following must be met:

$$m_{1j}m_{2j} = \lambda \quad (1)$$

This represents a necessary and sufficient condition for a specific vector from the vector space to be part of a feasible treatment that will allow the construction of a valid BIBD. Therefore, we deduce that $u.v = \lambda$. In addition, $\sum_{j=1}^b u_j^2 = r$, considering that $u_j \in \{0, 1\}$. Because of this, $|u| = |\nu| = \sqrt{r}$, from which it results that the α angle formed by two vectors that represent a feasible treatment, is given by $\alpha = \arccos\left(\frac{\lambda}{r}\right)$. Additionally to this condition, the parameter k should be considered to determine the number of objects in each group b . This will allow the establishment of another condition that a specific vector present in the vector space should meet in order to be considered a feasible treatment and to establish a valid BIBD. Thus, the total number of elements in the vector space is given by $\binom{b}{r}$, and if the condition $\sum_{i=1}^v m_{ij} = k$ is introduced, only a subspace of the vector space is used.

2.2. Local Search Technique for $\langle v, b, r, k, \lambda \rangle$ -BIBD

This section describes the proposal for the utilization of the metaheuristic techniques Hill Climbing and Tabu Search. It taken into consideration the considered vicinity, the multi-agent proposal, as well as the objective function and the elements of the local search.

2.2.1. Neighborhood

When processing a feasible treatment, for example, considering vector u as a starting point for the construction of a BIBD, whether v is a valid candidate treatment should be verified. For the instance: $\langle 14, 26, 13, 7, 6 \rangle$, with

$$u = \langle 0, 0, 0, 1, 1, 0, 1, 1, 1, 1, 1, 0, 0, 1, 0, 0, 1, 1, 0, 0, 0, 0, 1, 1 \rangle,$$

and considering that $\nu = \langle 0, 1, 1, 1, 0, 1, 0, 0, 1, 1, 0, 1, 0, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 1 \rangle$, then $\alpha = \arccos\left(\frac{\lambda}{r}\right) = \arccos\left(\frac{6}{13}\right) = 1.0911$, with $u.\nu = 6$, $|u| = \sqrt{13}$ and $|u||\nu| = 13$. As such, vector v is a valid candidate treatment. With another vector, for example $\nu' = \langle 1, 1, 1, 0, 1, 1, 0, 0, 1, 1, 1, 0, 0, 0, 0, 0, 0, 1, 0, 1, 1, 0, 0, 1, 0 \rangle$, you can verify that the angle between $\langle u, \nu' \rangle$ or $\langle \nu, \nu' \rangle$ is different because $1.0911 \neq 1.0697$, which indicates that ν' does not belong to the considered vicinity. Another important aspect has to do with the reduction of the vicinity as well as guiding the search; this relates to the condition $\sum_{i=1}^v m_{ij} = k$. Continuing with the previous example, after several iterations, you obtain a candidate BIBD, BI_7 equal to:

$$BI_7 = \begin{pmatrix} 00011011111100100110000011 \\ 11011010100001011001001011 \\ 01101001010011010111000101 \\ 10000100110111111110001000 \\ 00111010100011100010111100 \\ 00000111101001011100110101 \\ 10100111000010101011100011 \end{pmatrix}$$

According to the condition, to following is obtained:

$$32334354532245444453323335$$

If the k parameter is considered, the number of some of the missing in each column would be:

$$4544342324553233324454442$$

Through this condition we can see, that it is possible to try to guide the process primarily considering search those vectors that meet this restriction. The proposal has considered prioritizing the evaluation of those vectors of the vector space with components of the incidence matrix, with values taken from the first λ -columns where there is a higher deficit of ones, besides the angle between the vectors that already conforms the candidate BIBD. As such, a new candidate could be

$$01001111001110110001011000$$

After evaluating the angle conditions and the condition $\sum_{j=1}^b m_{ij}m_{i'j} = \lambda$, the new condition will be considered part of the solution and, because of this, it will be incorporated into the BI candidate.

$$BI_8 = \begin{pmatrix} 00011011111100100110000011 \\ 11011010100001011001001011 \\ 01101001010011010111000101 \\ 10000100110111111110001000 \\ 00111010100011100010111100 \\ 00000111101001011100110101 \\ 10100111000010101011100011 \\ 01001111001110110001011000 \end{pmatrix}$$

However, if the priority be vectors to considered are not feasible treatments, you proceed to take the components according to the lowest number of violations of the restrictions.

There is a marked difference between our approach and the are presented by Rodríguez et al. (2011). The first one uses a vectorial scheme that allows major levels of exploration (i.e diversification) due to the fact that the measure of distance used here to separate the neighbors is different from that used in Rodríguez. This allow zones of search space that are more distant to be explored.

2.2.2. Objective Function

The objective function is expressed throughout, $\sum_{j=1}^b m_{ij}m_{i'j} = \lambda$ and:

$$f^{\langle v, b, r, k, \lambda \rangle}(M) = \sum_{i=1}^v \phi_{ir}(M) + \sum_{j=1}^b \phi'_{jk}(M) + \sum_{i=1}^{v-1} \sum_{j=i+1}^v \phi''_{ij\lambda}(M) \quad (2)$$

$$\text{with } \phi_{ir}(M) = \left| r - \sum_{j=1}^b m_{ij} \right|, \phi'_{jk}(M) = \left| k - \sum_{i=1}^v m_{ij} \right|,$$

$$\phi''_{ij\lambda}(M) = \left| \lambda - \sum_{k=1}^b m_{ik}m_{jk} \right|$$

Note that 2 will only be applied when the totality of the treatments necessary to construct the desired BIBD is available. As long as $\sum_{j=1}^b m_{ij} = r$, it should be applied each time there is a feasible treatment to incorporate. If a BIBD candidate contains v treatments, the global optimum will be reached if there is a configuration M^* so that $f^{\langle v, b, r, k, \lambda \rangle}(M^*) = 0$.

2.2.3. Local Search Techniques

In this proposal, the metaheuristic techniques of Hill Climbing (H_c) and Tabu Search (T_s) have been used in the search for a valid BIBD. Basically, the process consists of taking a candidate treatment ν_i from the vector space $\varphi \in R^b$, so that:

$$\{\nu_i \in \varphi \mid \nu_i \in \binom{b}{r} i = 1, 2, \dots, v\}$$

where $\langle v, b, r \rangle$ represents parameters in the design that is being locked for. If ν_i satisfies the angular conditions and the $\sum_{j=1}^b m_{ij}m_{i'j} = \lambda$, it should be included as a BIBD candidate BI_i . And, if any of the restrictions are not satisfied, the vector is discarded and a new vector is taken from φ . It is clear that the number of neighbors available to be evaluated is a considerable size, mainly for instances with a greater number of blocks. This makes it difficult to evaluate the totality of the vector space. Because of this, for example, for an instance like $\langle 14, 26, 13, 7, 6 \rangle$, there is a total of $6.476475253248 \times 10^{16}$ neighbors to process. For the case of the H_c technique, after the evaluation of b^2 vectors from the vector space without being able to add a new treatment, the search is restarted from another starting point (new vector generated randomly from R^b that will meet the restriction in the incidence matrix where the value of the parameter r will indicate the amount of components with value “one” and the remainders with value “zero”). Clearly BI will contain, once again, only one vector $i = 1$ (BI_1). Additionally, the tabu search algorithm has been considered and is, also defined on the basis of the vector space for its vicinities, with an exploration equal to H_c . Other parameters important to the Tabu algorithm are: a) a vector $\nu_i \in \varphi$ previously evaluated is considered “tabu”, b) the tabu persistence is chosen based on the parameter v from the instance that will be processed, c) the desired criteria consists in finding a new feasible vector that will be a valid treatment, d) after the exploration of b^2 vectors from the vector

space, the search is intensified or diversified. The intensification is undertaken by going back over the best solution that can be found by substituting one of the vectors from the BIBD under construction (BI_{i-1}). This vector will be selected randomly from the previously placed i vectors. Diversification is undertaken by restarting from a new vector of the vector space φ , as long as it meets $\sum_{j=1}^b m_{ij} = r$, with a probability of $1/16$.

Besides the techniques described in Section 2.2.3, we have also introduced two Multi-Agent models based on: (a) the studied metaheuristics (b) our approach and the approach presented by Daisuke Yokoya (2009). Both cases uses a centralized outline with an independent search has been used.

2.2.4. Multi-Agent Proposal: H_c and T_s Agents

This approach only includes the metaheuristics presented at the beginning of this section and a group of 30 individuals initial solutions are applied, which will later be delivered to the cooperating algorithms. The group of algorithms employed corresponds to the two proposals presented in this article H_c and T_s . Other relevant parameters used can consider one of the proposals for each of the two versions: a version that randomly takes a vector from the vector space as an initial candidate a version that takes a possible design as an initial candidate (surely with various vectors from the space that have been proven as valid treatments) from the group of feasible candidates. Additionally, the group of candidate solutions will begin with the application of the algorithm T_s . It will have a random start for approximately 1 second of CPU time, and a total of 30 runs. At one moment of time there will only be two algorithms running simultaneously. After two consecutive runs of a specific algorithm without contribution, it will be replaced by another algorithm from the group. It is said that there is no contribution if the solution obtained by the algorithm is worse than the existing ones, or if the obtained individual is the same as one of the existing ones. The method of assigning an initial solution can be done in several ways: a) take a solution randomly from the group as long as it is not tabu, b) assign according to a quality percentage of the individuals, without considering the tabu condition, and c) assign the best individual of the group without considering the tabu condition. The quality percentage applied has been 25%. A higher percentage was tried but it did not show improvement. The tabu persistence used was based on the value of the parameter v for each instance. The cooperation was executed until a solution was obtained or a maximum of 30 runs was reached. Finally, each run was performed for a maximum of approximately 500 seconds of the CPU.

2.2.5. The $TABU(20)$, H_c and T_s Agents

In this scenario, besides the algorithms H_c and T_s a proposal named $TABU(20)$ presented by Daisuke and Takeo has been included. The mechanism employed is similar to the one presented in section 2.2.4, with slight differences, such as: a) the group of algorithms is composed by three algorithms that correspond to H_c , T_s and $TABU(20)$ in their cooperating version. The non-cooperating version of

T_s has been used to initiate the group of initial candidates, b) a group of 100 initial solutions has been employed, c) for the initiation of the group algorithm, T_s has been used, d) cooperation was executed until a solution was obtained or a maximum of 25 runs was reached, and e) each run was performed for a maximum of approximately 100 seconds of the CPU.

3. Results

The tests were performed on 86 instances taken from Prestwich (2003a), Rodriguez et al. (2009), Bofill et al. (2003) with $vb \leq 1000$ and $k \neq 3$, see table 1. These correspond to the cases that have a higher degree of complexity to find the solution reported in these works. It was taken into account that for those instances where $k = 3$ it is considerably less complicated to find a solution.

TABLE 1: Group of instances considered in the experimentation.

| ID | v | b | r | k | λ | vb | ID | v | b | r | k | λ | vb | ID | v | b | r | k | λ | vb |
|----|-----|-----|-----|-----|-----------|------|----|-----|-----|-----|-----|-----------|------|----|-----|-----|-----|-----|-----------|------|
| 1 | 8 | 14 | 7 | 4 | 3 | 112 | 30 | 9 | 54 | 24 | 4 | 9 | 486 | 59 | 21 | 35 | 15 | 9 | 6 | 735 |
| 2 | 11 | 11 | 5 | 5 | 2 | 121 | 31 | 13 | 39 | 12 | 4 | 3 | 507 | 60 | 10 | 75 | 30 | 4 | 10 | 750 |
| 3 | 10 | 15 | 6 | 4 | 2 | 150 | 32 | 13 | 39 | 15 | 5 | 5 | 507 | 61 | 25 | 30 | 6 | 5 | 1 | 750 |
| 4 | 9 | 18 | 8 | 4 | 3 | 162 | 33 | 16 | 32 | 12 | 6 | 4 | 512 | 62 | 20 | 38 | 19 | 10 | 9 | 760 |
| 5 | 13 | 13 | 4 | 4 | 1 | 169 | 34 | 15 | 35 | 14 | 6 | 5 | 525 | 63 | 16 | 48 | 15 | 5 | 4 | 768 |
| 6 | 10 | 18 | 9 | 5 | 4 | 180 | 35 | 12 | 44 | 22 | 6 | 10 | 528 | 64 | 16 | 48 | 18 | 6 | 6 | 768 |
| 7 | 8 | 28 | 14 | 4 | 6 | 224 | 36 | 23 | 23 | 11 | 11 | 5 | 529 | 65 | 12 | 66 | 22 | 4 | 6 | 792 |
| 8 | 15 | 15 | 7 | 7 | 3 | 225 | 37 | 10 | 54 | 27 | 5 | 12 | 540 | 66 | 12 | 66 | 33 | 6 | 15 | 792 |
| 9 | 11 | 22 | 10 | 5 | 4 | 242 | 38 | 8 | 70 | 35 | 4 | 15 | 560 | 67 | 9 | 90 | 40 | 4 | 15 | 810 |
| 10 | 16 | 16 | 6 | 6 | 2 | 256 | 39 | 17 | 34 | 16 | 8 | 7 | 578 | 68 | 13 | 65 | 20 | 4 | 5 | 845 |
| 11 | 12 | 22 | 11 | 6 | 5 | 264 | 40 | 10 | 60 | 24 | 4 | 8 | 600 | 69 | 11 | 77 | 35 | 5 | 14 | 847 |
| 12 | 10 | 30 | 12 | 4 | 4 | 300 | 41 | 11 | 55 | 20 | 4 | 6 | 605 | 70 | 21 | 42 | 10 | 5 | 2 | 882 |
| 13 | 16 | 20 | 5 | 4 | 1 | 320 | 42 | 11 | 55 | 25 | 5 | 10 | 605 | 71 | 21 | 42 | 12 | 6 | 3 | 882 |
| 14 | 9 | 36 | 16 | 4 | 6 | 324 | 43 | 18 | 34 | 17 | 9 | 8 | 612 | 72 | 21 | 42 | 20 | 10 | 9 | 882 |
| 15 | 8 | 42 | 21 | 4 | 9 | 336 | 44 | 25 | 25 | 9 | 9 | 3 | 625 | 73 | 16 | 56 | 21 | 6 | 7 | 896 |
| 16 | 13 | 26 | 8 | 4 | 2 | 338 | 45 | 15 | 42 | 14 | 5 | 4 | 630 | 74 | 10 | 90 | 36 | 4 | 12 | 900 |
| 17 | 13 | 26 | 12 | 6 | 5 | 338 | 46 | 21 | 30 | 10 | 7 | 3 | 630 | 75 | 15 | 60 | 28 | 7 | 12 | 900 |
| 18 | 10 | 36 | 18 | 5 | 8 | 360 | 47 | 16 | 40 | 10 | 4 | 2 | 640 | 76 | 18 | 51 | 17 | 6 | 5 | 918 |
| 19 | 19 | 19 | 9 | 9 | 4 | 361 | 48 | 16 | 40 | 15 | 6 | 5 | 640 | 77 | 22 | 42 | 21 | 11 | 10 | 924 |
| 20 | 11 | 33 | 15 | 5 | 6 | 363 | 49 | 9 | 72 | 32 | 4 | 12 | 648 | 78 | 15 | 63 | 21 | 5 | 6 | 945 |
| 21 | 14 | 26 | 13 | 7 | 6 | 364 | 50 | 15 | 45 | 21 | 7 | 9 | 675 | 79 | 16 | 60 | 15 | 4 | 3 | 960 |
| 22 | 16 | 24 | 9 | 6 | 3 | 384 | 51 | 13 | 52 | 16 | 4 | 4 | 676 | 80 | 16 | 60 | 30 | 8 | 14 | 960 |
| 23 | 12 | 33 | 11 | 4 | 3 | 396 | 52 | 13 | 52 | 24 | 6 | 10 | 676 | 81 | 31 | 31 | 6 | 6 | 1 | 961 |
| 24 | 21 | 21 | 5 | 5 | 1 | 441 | 53 | 10 | 72 | 36 | 5 | 16 | 720 | 82 | 31 | 31 | 10 | 10 | 3 | 961 |
| 25 | 8 | 56 | 28 | 4 | 12 | 448 | 54 | 19 | 38 | 18 | 9 | 8 | 722 | 83 | 31 | 31 | 15 | 15 | 7 | 961 |
| 26 | 10 | 45 | 18 | 4 | 6 | 450 | 55 | 11 | 66 | 30 | 5 | 12 | 726 | 84 | 11 | 88 | 40 | 5 | 16 | 968 |
| 27 | 15 | 30 | 14 | 7 | 6 | 450 | 56 | 22 | 33 | 12 | 8 | 4 | 726 | 85 | 22 | 44 | 14 | 7 | 4 | 968 |
| 28 | 16 | 30 | 15 | 8 | 7 | 480 | 57 | 15 | 52 | 26 | 7 | 12 | 780 | 86 | 25 | 40 | 16 | 10 | 6 | 1000 |
| 29 | 11 | 44 | 20 | 5 | 8 | 484 | 58 | 27 | 27 | 13 | 13 | 6 | 729 | | | | | | | |

The tests were executed on an Intel computer with a dual core processor at 2.5 GHZ. Three different scenarios have been presented for the tests. These correspond with the local searches, the cooperation between the two proposed local searches, and the cooperation between the local searches and the technique described by Daisuke y Takeo. The maximum time used in each run for the local searches was

approximately 500 seconds: the same time was used in the cooperation proposal that only involves the local searches. In the second cooperation proposal, each run was executed in a maximum of 100 seconds of the CPU. Results present the best times obtained to be resolved each one of the indicated instances. Table 2 presents the different results obtained in the recent research. The first column corresponds to the identification number of each instance, the second column shows the results obtained by Prestwich (2003a), the third column submerges the results presented by Rodríguez et al. (2009), and finally, columns 4 and 5 illustrate the results obtained by Daisuke Yokoya (2009).

TABLE 2: Recent results for the 86 instances. Each column represents the different proposals presented and the time (seconds) used to reach the solution.

| ID | CLS | T_{ss} | BAB | TABU | ID | CLS | T_{ss} | BAB | TABU | ID | CLS | T_{ss} | BAB | TABU |
|----|------|-----------|---------|------|----|-------|-----------|---------|-------|----|-------|-----------|------|--------|
| 1 | 0.00 | <i>Si</i> | 0.00 | 0.00 | 30 | 0.21 | <i>Si</i> | 0.01 | 0.00 | 59 | — | — | — | — |
| 2 | 0.04 | <i>Si</i> | 0.01 | 0.01 | 31 | 1.22 | <i>Si</i> | 0.03 | 0.02 | 60 | 0.92 | <i>Si</i> | 0.02 | 0.02 |
| 3 | 0.04 | <i>Si</i> | 2068.03 | 0.02 | 32 | 11.20 | <i>Si</i> | 0.05 | 0.01 | 61 | 14.30 | <i>Si</i> | 0.04 | 0.03 |
| 4 | 0.05 | <i>Si</i> | 0.01 | 0.00 | 33 | — | — | 1985.33 | 3.57 | 62 | — | — | — | — |
| 5 | 0.01 | <i>Si</i> | 0.00 | 0.00 | 34 | — | — | 3.29 | 0.05 | 63 | 43.20 | <i>Si</i> | 0.11 | 0.15 |
| 6 | 0.12 | <i>Si</i> | 0.01 | 0.01 | 35 | 27.90 | <i>Si</i> | 0.04 | 0.03 | 64 | — | — | 0.38 | 0.37 |
| 7 | 0.06 | <i>Si</i> | 0.00 | 0.00 | 36 | — | <i>Si</i> | — | 0.07 | 65 | 1.38 | <i>Si</i> | 0.05 | 0.02 |
| 8 | 0.61 | <i>Si</i> | 0.01 | 0.01 | 37 | 0.98 | <i>Si</i> | 0.04 | 0.02 | 66 | 12.80 | <i>Si</i> | 0.04 | 0.03 |
| 9 | 1.20 | <i>Si</i> | 0.01 | 0.01 | 38 | 0.13 | <i>Si</i> | 0.01 | 0.00 | 67 | 0.69 | <i>Si</i> | 0.02 | 0.02 |
| 10 | 0.54 | <i>Si</i> | — | 0.02 | 39 | — | — | — | 7.37 | 68 | 1.40 | <i>Si</i> | 0.04 | 0.01 |
| 11 | 1.23 | <i>Si</i> | 0.02 | 0.02 | 40 | 0.75 | <i>Si</i> | 0.01 | 0.01 | 69 | 5.11 | <i>Si</i> | 0.03 | 0.02 |
| 12 | 0.12 | <i>Si</i> | 0.05 | 0.01 | 41 | 0.80 | <i>Si</i> | 0.03 | 0.01 | 70 | — | — | — | 798.86 |
| 13 | 0.16 | <i>Si</i> | 0.01 | 0.01 | 42 | 4.45 | <i>Si</i> | 0.02 | 0.06 | 71 | — | — | — | — |
| 14 | 0.21 | <i>Si</i> | 0.01 | 0.00 | 43 | — | — | — | 30.91 | 72 | — | — | — | — |
| 15 | 0.25 | <i>Si</i> | 0.01 | 0.00 | 44 | — | — | — | 0.24 | 73 | — | — | 0.12 | 0.11 |
| 16 | 0.50 | <i>Si</i> | 0.04 | 0.06 | 45 | 42.90 | <i>Si</i> | 0.14 | 0.05 | 74 | 0.99 | <i>Si</i> | 0.04 | 0.01 |
| 17 | 7.93 | <i>Si</i> | 17.54 | 0.08 | 46 | — | — | — | 61.42 | 75 | — | <i>Si</i> | 0.09 | 0.05 |
| 18 | 1.08 | <i>Si</i> | 0.03 | 0.03 | 47 | 4.22 | <i>Si</i> | 0.05 | 0.08 | 76 | — | — | 0.78 | 1.34 |
| 19 | 9.73 | <i>Si</i> | 0.21 | 0.05 | 48 | — | — | 2.72 | 0.84 | 77 | — | — | — | — |
| 20 | 1.79 | <i>Si</i> | 0.04 | 0.01 | 49 | 0.54 | <i>Si</i> | 0.01 | 0.01 | 78 | 30.70 | <i>Si</i> | 0.09 | 0.20 |
| 21 | — | — | 0.38 | 0.88 | 50 | — | — | 0.06 | 0.06 | 79 | 6.31 | <i>Si</i> | 0.05 | 0.02 |
| 22 | 9.80 | <i>Si</i> | 0.03 | 0.05 | 51 | 3.76 | <i>Si</i> | 0.03 | 0.01 | 80 | — | — | 0.34 | 1.12 |
| 23 | 0.33 | <i>Si</i> | — | 0.22 | 52 | 49.70 | <i>Si</i> | 0.04 | 0.02 | 81 | 2.04 | <i>Si</i> | 0.07 | 0.04 |
| 24 | 0.22 | <i>Si</i> | 0.31 | 0.02 | 53 | 1.33 | <i>Si</i> | 0.02 | 0.01 | 82 | — | — | — | 1.70 |
| 25 | 0.12 | <i>Si</i> | 0.01 | 0.01 | 54 | — | — | — | — | 83 | — | — | — | 0.10 |
| 26 | 0.08 | <i>Si</i> | 0.02 | 0.01 | 55 | 1.52 | <i>Si</i> | 0.04 | 0.01 | 84 | 4.07 | <i>Si</i> | 0.03 | 0.04 |
| 27 | — | — | 10.13 | 0.95 | 56 | — | — | — | — | 85 | — | — | — | — |
| 28 | — | — | 237.50 | 0.44 | 57 | — | — | 0.11 | 0.07 | 86 | — | — | — | — |
| 29 | 1.13 | <i>Si</i> | 0.04 | 0.01 | 58 | — | — | — | 0.54 | | | | | |

Table 3 compiles the experimental results obtained from this investigation is proposal. Columns 2 – 3 present the results obtained when applying the proposed local search methods Hill Climbing (H_c) and Tabu Search (T_s). We can observe that algorithm T_s is able to resolve 72 of the 86 instances compared whit H_c , which resolves 70 of the 86. The response times in both proposals are relatively similar. In column 4 we observe the results when applying a cooperation model in which algorithms H_c and T_s are applied. In this case 74 of the 86 instances are resolved. Finally column 5 summarizes the best times obtained by the cooperative proposal, including the algorithms. There are presented here as will as in the model presented in Daisuke Yokoya (2009). Table 4 presents a summary of the

number of instances resolved by the different revised algorithms that are presented in this research.

TABLE 3: Experimental results for the 86 instances. The best results are from the proposals H_c , T_s , cooperation $H_c + T_s$ and cooperation $TABU(10) + H_c + T_s$ ($TABU++$). Each column represents the different proposals presented and the time (seconds) used to reach the solution.

| ID | H_c | T_s | $H_c + T_s$ | $TABU++$ | ID | H_c | T_s | $H_c + T_s$ | $TABU++$ | | | | | |
|----|-------|-------|-------------|----------|----|-------|--------|-------------|----------|----|-------|--------|--------|-------|
| 1 | 0.03 | 0.03 | 0.02 | 0.00 | 30 | 0.01 | 0.01 | 0.01 | 0.00 | 59 | — | — | — | — |
| 2 | 0.02 | 0.03 | 0.02 | 0.00 | 31 | 0.01 | 0.03 | 0.03 | 0.00 | 60 | 0.05 | 0.03 | 0.03 | 0.01 |
| 3 | 0.03 | 0.01 | 0.01 | 0.00 | 32 | 0.05 | 0.03 | 0.03 | 0.01 | 61 | 3.56 | 36.50 | 0.02 | 0.00 |
| 4 | 0.01 | 0.01 | 0.01 | 0.01 | 33 | 13.72 | 24.78 | 49.72 | 0.94 | 62 | — | — | — | — |
| 5 | 0.03 | 0.03 | 0.03 | 0.00 | 34 | 5.70 | 18.38 | 0.81 | 0.80 | 63 | 4.11 | 5.59 | 0.17 | 0.16 |
| 6 | 0.06 | 0.05 | 0.06 | 0.03 | 35 | 2.69 | 2.17 | 0.41 | 0.39 | 64 | 19.92 | 27.48 | 0.73 | 0.72 |
| 7 | 0.02 | 0.01 | 0.03 | 0.00 | 36 | 0.72 | 0.61 | 0.56 | 0.55 | 65 | 0.05 | 0.05 | 0.03 | 0.02 |
| 8 | 0.03 | 0.01 | 0.01 | 0.01 | 37 | 0.13 | 0.09 | 0.09 | 0.06 | 66 | 28.36 | 37.09 | 35.36 | 0.66 |
| 9 | 0.08 | 0.25 | 0.23 | 0.22 | 38 | 0.05 | 0.03 | 0.05 | 0.03 | 67 | 0.36 | 0.36 | 0.34 | 0.31 |
| 10 | 0.03 | 0.03 | 0.01 | 0.00 | 39 | — | 326.77 | 312.66 | 14.92 | 68 | 0.03 | 0.03 | 0.03 | 0.01 |
| 11 | 0.03 | 0.03 | 0.03 | 0.01 | 40 | 0.05 | 0.03 | 0.03 | 0.01 | 69 | 1.42 | 26.36 | 1.02 | 0.98 |
| 12 | 0.03 | 0.01 | 0.01 | 0.00 | 41 | 0.03 | 0.01 | 0.03 | 0.00 | 70 | — | — | — | 1.19 |
| 13 | 0.13 | 0.23 | 0.23 | 0.22 | 42 | 0.08 | 0.06 | 0.06 | 0.05 | 71 | — | — | — | — |
| 14 | 0.03 | 0.03 | 0.03 | 0.00 | 43 | — | — | — | 1.64 | 72 | — | — | — | — |
| 15 | 0.01 | 0.03 | 0.01 | 0.00 | 44 | 11.92 | 38.16 | 0.39 | 0.38 | 73 | 9.17 | 7.72 | 1.01 | 0.92 |
| 16 | 0.28 | 0.76 | 0.72 | 0.70 | 45 | 7.58 | 2.94 | 0.11 | 0.09 | 74 | 0.19 | 0.14 | 0.13 | 0.11 |
| 17 | 0.03 | 0.03 | 0.03 | 0.01 | 46 | — | — | 89.30 | 4.19 | 75 | 37.23 | 29.00 | 27.70 | 0.28 |
| 18 | 0.05 | 0.05 | 0.05 | 0.03 | 47 | 9.02 | 2.13 | 0.01 | 0.00 | 76 | 29.67 | 11.83 | 0.91 | 0.88 |
| 19 | 0.14 | 0.76 | 0.73 | 0.72 | 48 | 7.66 | 35.25 | 0.28 | 0.28 | 77 | — | — | — | — |
| 20 | 0.01 | 0.03 | 0.01 | 0.00 | 49 | 16.53 | 13.92 | 0.03 | 0.01 | 78 | 0.44 | 0.33 | 0.33 | 0.30 |
| 21 | 13.11 | 2.72 | 0.72 | 0.72 | 50 | 37.73 | 44.92 | 42.59 | 1.14 | 79 | 0.03 | 0.03 | 0.05 | 0.01 |
| 22 | 0.17 | 0.19 | 0.19 | 0.17 | 51 | 0.03 | 0.01 | 0.03 | 0.00 | 80 | — | — | — | 0.30 |
| 23 | 0.01 | 0.01 | 0.01 | 0.00 | 52 | 1.39 | 1.26 | 0.83 | 0.81 | 81 | 0.03 | 0.03 | 0.01 | 0.01 |
| 24 | 0.11 | 30.26 | 0.01 | 0.00 | 53 | 1.59 | 1.36 | 0.67 | 0.66 | 82 | — | — | 469.81 | 26.38 |
| 25 | 4.33 | 3.89 | 0.02 | 0.01 | 54 | — | — | — | 58.14 | 83 | — | 229.88 | 187.25 | 5.67 |
| 26 | 0.01 | 0.03 | 0.03 | 0.00 | 55 | 1.14 | 0.98 | 0.92 | 0.91 | 84 | 7.39 | 6.95 | 0.66 | 0.64 |
| 27 | 6.69 | 3.08 | 0.22 | 0.20 | 56 | — | — | — | — | 85 | — | — | — | — |
| 28 | 19.16 | 10.44 | 9.94 | 0.44 | 57 | 13.50 | 1.83 | 61.41 | 43.17 | 86 | — | — | — | — |
| 29 | 0.05 | 0.03 | 0.03 | 0.01 | 58 | 26.92 | 36.97 | 98.72 | 0.70 | | | | | |

TABLE 4: Summary of results.

| Method | Resolved | Unresolved |
|-------------------|----------|------------|
| CLS | 55 | 31 |
| Tabu Search Swap | 57 | 29 |
| BAB BIBD(LP,BWD) | 64 | 22 |
| BAB BIBD(LP, FWD) | 66 | 20 |
| TABU BIBD(10) | 77 | 9 |
| H_c | 70 | 16 |
| T_s | 72 | 14 |
| $H_c + T_s$ | 74 | 12 |
| $TABU++$ | 78 | 8 |

4. Conclusions

We have presented an algorithm with a constructive approach which allows a great number of instances to be resolved for the problem of the design of balanced incomplete blocks using a technique of local search based on a multi-start. It can

be confirmed that by using cooperative mechanisms and incorporating multiple agents you can achieve a better performance from the techniques. However, there are still instances for which it has not been possible to reach optimal solutions, but it should be mentioned that the feasible solutions found are very close to the optimal globals. However, an advantage of our approach lies in the major capacity to explore of new zones (major diversification). This is unlike other approaches that use major intensification, for example the vectorial approach. Normally one of the disadvantages of the process of intensification is the wide possibility of finding many local optima: something that can be solved by using the diversification method.

In practical situations, adoption of a complete block design is not appropriate and in several cases, not at all feasible. This fact prompted the development of various kinds of incomplete block designs, which, in turn, have been used extensively for experiments in different areas. Moreover, these designs opened up many challenging problems in combinatorial optimization. In addition to the classical IBD (incomplete block designs), the BIB (balanced incomplete block) and PBIB (partially balanced incomplete block) designs are still found to be useful in several applications.

We believe that the benefits of this proposal seem to be very promising. For this reason we have presented the work hypothesis that an adequate specialization of the search algorithms could open the door to resolve many of these instances. Therefore, in terms of future work, we consider hybridizing the presented proposals, employing genetic algorithms and methods of local search. From another perspective, we can attempt to employ a different model the presentation the candidate solutions, as well as, study other local search techniques: Iterated Local Search (ILS), Variable Neighborhood Search (VNS).

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