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The effect of carrier diffusion and recombination in semiconductors on the photoacoustic signal

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A quantitative derivation is presented for the heat transport in bipolar semiconductors, taking into account generation and heating of carriers on the surface due to an incident modulated laser beam on the surface and finite carrier diffusion and recombination in the solid. The temperature distribution as function of the position and time in the semiconductor is calculated using appropriate boundary conditions according to the photoacoustic experimental conditions. In addition, special emphasis is pointed out in the heat power density generated in the sample due to the recombination of the electron-hole pair and the effect of the inhomogeneous temperature distribution on the thermal generation rate of carriers in the photoacoustic signal.

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1. Introduction

Thermal-wave physics is becoming a valuable tool in the study of material thermal parameters as well as in the semiconductor industry for characterizing processes in the manufacturing of microelectronic devices[1-2]. These waves are created whenever there is a periodic heat generation. The most common mechanism for producing thermal waves is the absorption of an intensity modulated light beam by a sample. Of the several mechanisms available for detecting these waves, the gas microphone photoacoustic detection is one of the most widely used. In this case the sample is usually irradiated by a modulated light beam, which is then absorbed by the material and converted into heat. The heat diffuses to the sample surfaces and then into the surrounding gas of the photoacoustic cell. Finally, the thermal expansion of the gas generates the photoacoustic signal. The physical interpretation of thermal waves produced in the photothermal experiments and the difference with wave phenomenon have been discussed in detail in Refs. [3] and [4]. In the case of semiconductors, the photoacoustic signal provides us, besides the thermal parameters, with additional information regarding the carrier-transport properties. Qualitatively, this may understood as follows. The absorption of radiation with photon energy greater the band-gap excitation produces an excess carrier distribution in the semiconductor with an energy above (below) the conduction (valence) band. In a time scale of a few picoseconds these photoinjected carriers distribute the excess of energy between them via Coulomb interaction and finally this extra energy is given to the lattice by relaxing to the bottom (top) of the conduction (valence) band via the carrier-phonon interaction. As the excess carrier diffuses through the sample, the electron-hole pairs eventually recombine producing a second source of heat which also diffuses into the semiconductor.

One of the simplest and at the same time quite effective model describing the process of heat transport consists in assign to each quasiparticle system an individual temperature. Then the thermal problem in the total system can be reduced to the determination of the space-time evolution of these temperatures taking into consideration the energy exchange between subsystems. For example, the two-temperature approach has been used to analyze the thermal wave propagation in semiconductors[4] taking into account the electron-phonon energy interaction. In this paper we restrict ourselves to study heat transport in semiconductors in one-temperature approximation, which is valid when the thickness of the sample is greater than the cooling length[5] of the temperature distribution of the quasiparticle systems. In Refs. [6] a theory of the photoacoustic effect in the fundamental absorption region of ambipolar semiconductor has been developed allowing for the formation of free electrons and holes, their diffusion and recombination. In this work the authors used a plane periodic heat source $Qe^{i\Omega t}$ at the surface of a semi-infinite semiconductor as boundary condition to study the thermal-wave propagation. However, in order to observe the photothermal effect, the sample must be illuminated with a light which is intensity-modulated by a mechanical chopper ($I = I_0 + \Delta I \; e^{i\Omega t}$ with $\Delta I \leq I_0$) to generate thermal waves inside the solid. In this situation the heat generated at the surface of the sample has the form $Q = Q_0 + \Delta Q e^{i\Omega t}$ where

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$\Delta Q \leqslant Q_0$. More recently,using photoacoustic amplitude and phase measurements in heat transmission configuration, carrier transport properties were investigated [7-8]. Expressions for the distributions of the periodic parts of the thermal sources (thermalization, nonradiative surface and bulk recombination) were analytically evaluated and the photoacoustic signal at the front and rear surface of the sample were calculated including the absorption process on the front side.

In this work, we further extend the aforementioned models for the photoacoustic signal by taking into account the one so far neglected important feature of the behavior of carriers under time-varying excitation, namely the effect of the gradient temperature on the recombination rate of carriers in the sample. This term takes into account the change of the rate of thermal generation of electrons and holes.

2. Theoretical model

Let us consider an incident monochromatic light beam on the surface of the semiconductor at $x = 0$ with intensity $I = I_0 + \Delta I \ e^{i\Omega t}$, modulated at an angular frequency $\Omega = 2\pi f$, where I_0 is the incident monochromatic light flux. It is assumed that the energy of the excitation beam $h\nu$ is greater than the band gap energy of the semiconductor ε_g. Under modulated monochromatic light excitation with an angular frequency Ω, the flux of photons penetrating to depth x in the semiconductor is given by $e^{-\beta x}\left(I_0 + \Delta I \ e^{i\Omega t}\right)$ where β is the optical absorption coefficient of the solid sample. For sake of simplicity we shall also assume that the light is only absorbed in a relatively thin layer of the sample smaller than the thickness of the semiconductor and the carriers diffusion length. In this case a source of heat and carrier generation at the surface of the sample may be considered.

In the limit of small effective cooling length as compared with the sample dimensions and strong energy interaction between the quasiparticle system [5] (electrons, holes and phonons), the system of quasiparticles can be described in the one temperature approximation $T(x,t)$ and the coupled heat-diffusion equations reduce to the usual one dimensional equation.

$$\frac{\partial^2 T(x,t)}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T(x,t)}{\partial t} + \frac{Q_s(x,t)}{\kappa} \tag{1}$$

In Eq. (1) $\alpha = \kappa/\rho c$ is the thermal diffusivity, ρ the density, c the heat capacity and κ the thermal conductivity of the material. Here, $Q_s(x,t)$ is the thermal power density produced at a point x of the sample due to the electron-hole recombination. In terms of this photoinjected excess carrier concentration, the thermal power density generated in the sample can be written as $Q_s = \varepsilon_g R$ with R being the electron-hole pair recombination rate. Of course, in general, an electron-hole pair can recombine radiatively or nonradiatively. In fact, in many semiconductors the nonradiative transition is the dominant process. In the present work, because the attenuation length of the light ($\approx \beta^{-1}$) is too small as compared with the characteristic lengths to concern in transport theory and size of the sample, the emitted photons in the radiative process are absorbed at the same point in the sample producing heat. For gap-less and small gap semiconductors, instead of ε_g a more complicated function of temperature appears. Therefore, the equations governing the photocarriers generation and diffusion at the modulation frequency Ω in the semiconductor are given in the form

$$\frac{\partial n}{\partial t} = \frac{\partial j_n}{\partial x} - eR_n, \quad \frac{\partial p}{\partial t} = -\frac{\partial j_p}{\partial x} - eR_p \tag{2}$$

$$j_n = \sigma_n E + eD_n\frac{\partial n}{\partial x} + \sigma_n\alpha_n\frac{\partial T}{\partial x}, \tag{3}$$

$$j_P = \sigma_P E - eD_p\frac{\partial p}{\partial x} + \sigma_p\alpha_p\frac{\partial T}{\partial x}, \tag{4}$$

$$\frac{\partial E}{\partial x} = \frac{4\pi e}{\in}(p - n + N_d), \tag{5}$$

where n and p are the local concentrations, j_n and j_p the carrier densities induced by a gradient of temperature and carrier concentration, σ_n and σ_p the electrical conductivities, α_n and α_p the thermoelectric coefficients, and D_n and D_P the diffusion coefficient of electrons and holes, respectively. Here, E is the internal electric field set up due to different diffusion coefficients of electrons and holes and N_d is the concentration of charge impurity.

For small concentration of nonequilibrium carriers $\delta n = n - n_0 \ll n_0$ and $\delta p = p - p_0 \ll p_0$ (where n_0, p_0 are the electron and hole concentrations in equilibrium, respectively), the recombination rates are usually assumed to be of the following form[9] $R_n = \delta n/\tau_n$ and $R_p = \delta p/\tau_p$, where τ_n and τ_p are the life time of electrons and holes respectively. However, as can be seen such approximations cannot be correct, the physical origin of these inconsistency, in steady state conditions, comes because they do not satisfy the Maxwell equation $\nabla(j_n + j_p) = 0$ unless that $R_n \equiv R_p$[10].

In order to evaluate the ambipolar kinetic equations let us assume the following approximations: the total current density vanishes i.e. $j = j_n + j_p = 0$ (open circuit), the electron and hole recombination rates are equal i.e. $R_n = R_p = R$ and charge quasineutrality i.e. $\delta n = \delta p$ (i.e. the excess free electron density is equal to the excess free hole density), this latter approach is valid if the thickness of the semiconductor is too large as compared with the Debye length[11]. Under charge quasineutrality approximation, Eq.(5) can be neglected. Here, the equations governing the photocarrier generation and diffusion at the modulation frequency Ω in the semiconductor may be written in the form:

$$j_p = -j_n = -eD\frac{\partial \delta p}{\partial \mathrm{x}} - A\frac{\partial T}{\partial \mathrm{x}} \tag{6}$$

and

$$-\frac{1}{e}\frac{\partial \delta p}{\partial t} + D\frac{\partial^2 \delta p}{\partial \mathrm{x}^2} + \frac{A}{e}\frac{\partial^2 T}{\partial \mathrm{x}^2} - R(\delta p) = 0 \tag{7}$$

where

$D \equiv (\sigma_p D_n + \sigma_p D_n)/(\sigma_n + \sigma_p)$ and $A \equiv \sigma_n \sigma_p(\alpha_n - \alpha_p)/(\sigma_n + \sigma_p)$.

The third term in Eq. (7) takes into account the effect of the thermoelectric field on the ambipolar diffusion of carriers in the sample, this important effect has been usually neglected in previous theories of the photoacoustic effect in semiconductors.

Now, the point to be considered in the case of semiconductors is the importance of taking into account besides carrier diffusion and recombination, the non-stationary temperature distribution in the description of carrier concentration under time-varying excitation. To incorporate these features into the photoacoustic signal, the carrier concentration has to be determinated (see below) and in addition, it is necessary to consider the influence of heating on the thermal carrier generation which is proportional to n_i^2 (n_i is the intrinsic carrier concentration) and in this case the interband recombination rates are given by $\delta p/\tau - \gamma\delta T$, where $\delta T(\mathrm{x},t) = T(\mathrm{x},t) - T_0$, T_0 is the ambient temperature and $\gamma \equiv [2n_i/(n_0 + p_0)\tau](dn_i/dt)$[10].

The solutions $T(\mathrm{x},t)$ and $\delta p(\mathrm{x},t)$ in Eqs. (1)-(7) should be supplemented by boundary conditions at x = 0. In the photoacoustic experiments, the most common mechanism to produce thermal waves and nonequilibrium carriers is the absorption of an intensity modulated light beam. It is clear that when the intensity of the radiation is fixed, the light energy partially is converted into heat and partially into surface carrier generation. Under these considerations the boundary conditions for the temperature distribution and the excess of carrier concentration are given by

$$-\kappa\frac{\partial T(\mathrm{x},t)}{\partial \mathrm{x}}\bigg|_{\mathrm{x}=0} = Q_0 + \Delta Q_0 e^{i\omega t},$$

$$-D\frac{\partial \delta p(\mathrm{x},t)}{\partial \mathrm{x}}\bigg|_{\mathrm{x}=0} = I_0 + \Delta I_0 e^{i\omega t} \tag{8}$$

where Q_0 is the average overtime of the total heat flux $Q(\mathrm{x},t)$ at the surface of the sample and it is proportional to the intensity of the incident radiation I_0.

In general, the efficiency at which the absorbed light by the semiconductor is converted into heat and an electron-hole pair is an unknown parameter and it can be surface temperature dependent, in this work it will be considered constant. The numerical value of the efficiency factor can be estimated as follows: The intensity of the incident radiation is distributed between carrier generation and sample heating in the form

$$Q_0 = (\hbar\omega - \varepsilon_g)I_o, \qquad \Delta Q_0 = (\hbar\omega - \varepsilon_g)\Delta I_o. \tag{9}$$

On the other hand, in the high surface recombination limit (in this case the surface recombination velocity is infinite), the photoinjected carriers recombine very fast near the surface of the semiconductor given all the excess of energy to phonons. In this limit, the efficiency to produce heat in the sample is high.

For a semiconductor with thickness much greater than the carrier diffusion length i.e. $d \gg L_D = \sqrt{(D\tau)}$, the following boundary condition for the carriers at x = d can be used

$$\delta p\big|_{\mathrm{x}=d} = 0 \tag{10}$$

and

$$-\kappa\frac{\partial T}{\partial \mathrm{x}}\bigg|_{\mathrm{x}=d} = \eta\left(T(d) - T_0\right) \tag{11}$$

for the heat flux. Here η represents the surface thermal conductivity between the sample and the gas at ambient temperature T_0. Note that when η goes to infinite (perfect thermal surface), since the heat flux is finite, the temperature distribution must be continuous at the surface $x = d$ ($T(\mathrm{x},t)|_{x=d} = T_0$). For finite surface thermal conductivity, $T(\mathrm{x},t)|_{x=d} \neq T_0$.

3. Nonequilibrium carrier concentration and temperature distribution in the semiconductor

In the photohermal experiments, the most common mechanism to produce thermal waves is the absorption by the sample of an incident modulated light beam $I_0 + \Delta I\, e^{i\Omega t}$. The static heat flux at the surface of the sample is proportional to I_0 and it is the responsible one to give rise to the static temperature distribution in the material, while the dynamic contribution part $\Delta I\, e^{i\Omega t}$ is associated with the modulated heat source with sinusoidal time dependence at the surface x = 0. Similar considerations can be used for the carrier concentration. Therefore, it is naturally to seek solutions for the temperature distribution and the carrier concentration in the form

$$T(\mathrm{x},t) = T_s(\mathrm{x}) + T_d(\mathrm{x}) e^{i\omega t}\,, \tag{12}$$

$$\delta p(\mathrm{x},t) = \delta p_s(\mathrm{x}) + \delta p_d(\mathrm{x}) e^{i\omega t}\,, \tag{13}$$

where Ω is the chopper frequency and $T_s(\mathrm{x})$ and $T_d(\mathrm{x})$ are the static and dynamic contribution to the temperaturedistribution and $\delta p_s(\mathrm{x})$, $\delta p_d(\mathrm{x})$ the static and dynamic contribution to the charge concentration in the sample, respectively.

a) Static part contribution: $T_s(\mathrm{x})$ and $\delta p_s(\mathrm{x})$

In this case the laser beam excitation is not modulated by the chopper and as a consequence the heat flux through the sample is constant. For simplicity in the calculation is convenient to introduce the following dimensionless parameters: $x = \mathrm{x}/L_D$, $p_s = \delta p_s/p_0$, $\theta_s = (T_s - T_0)/T_\alpha$, $a = T_\alpha/T_\gamma$, $T_\alpha = D\varepsilon_g p_0/\kappa$, $T_\gamma = p_0/\gamma$. With these definitions the static contribution to the temperature and carrier concentration are given by

$$\frac{d^2 p_s}{dx^2} - p_s + a\theta_s = 0\,, \tag{14}$$

$$\frac{d^2 \theta_s}{dx^2} + p_s - a\theta_s = 0\,, \tag{15}$$

For a semiconductor, it is usually assumed that $A\varepsilon_g/e\kappa \ll 1$. This approximation neglects the effect of the thermoelectric field (second term in Eq. (7)). Note that, if the intrinsic carrier concentration is temperature independent i.e. $\gamma \longrightarrow 0$ then, $T_\gamma \longrightarrow \infty$ and $a \longrightarrow 0$.

We are now in a position to consider the influence of the thermal field on the temperature distribution and carrier concentration in the semiconductor due to the absorption of the incident modulated radiation by the sample. Using the following boundary conditions

$$\left.\frac{dp_s}{dx}\right|_{x=0} = \tilde{I}_0, \qquad p_s\big|_{x=d} = 0; \tag{16}$$

$$\left.\frac{d\theta_s}{dx}\right|_{x=0} = -\tilde{Q}_0, \qquad \left.\frac{d\theta_s}{dx}\right|_{x=d} = -\tilde{\eta}\,\theta_s\big|_{x=d}\,, \tag{17}$$

where

$$\tilde{I}_0 \equiv \frac{I_0 L_D}{D p_0}, \qquad \tilde{Q}_0 \equiv \frac{Q_0 L_D}{\kappa T_\alpha}, \qquad \tilde{\eta} \equiv \frac{\eta L_D}{\kappa}.$$

The solution of the thermal and the carrier diffusion equations leads the following expression for the static part of the carrier concentration and temperature distribution, respectively

$$p_s = C_{s1} + C_{s2}x + C_{s3}e^{\lambda_0 x} + C_{s4}e^{-\lambda_0 x}, \tag{18}$$

$$\theta_s = \frac{C_{s1}}{a} + \frac{C_{s2}}{a}x - C_{s3}e^{\lambda_0 x} - C_{s4}e^{-\lambda_0 x}, \tag{19}$$

with $\lambda_0 = \sqrt{(a+1)}$ and the constants C_{sq} are given by

$$C_{s1} = \frac{a}{\lambda_0{}^3[a\sinh(\lambda_0 d) + \tilde{\eta}\lambda_0\cosh(\lambda_0 d)]} \times \Big\{[a\lambda_0 d\sinh(\lambda_0 d) + (1+\tilde{\eta}\lambda_0{}^2 d)\cosh(\lambda_0 d)](\tilde{I}_0 + \tilde{Q}_0) - \tilde{I}_0 + a\tilde{Q}_0\Big\},$$

$$C_{s2} = -\frac{\tilde{I}_0 + \tilde{Q}_0}{1+a}a,$$

$$C_{s3} = \frac{[a(1-e^{\lambda_0 d}) - \tilde{\eta}\lambda_0]\tilde{I}_0 - a[e^{\lambda_0 d} - 1 - \lambda_0(\tilde{\eta} - \lambda_0)]\tilde{Q}_0}{2\lambda^3 e^{\lambda_0 d}[a\sinh(\lambda_0 d) + \tilde{\eta}\lambda_0\cosh(\lambda_0 d)]}\,,$$

$$C_{s4} = \frac{1}{2\lambda_0{}^3[a\sinh(\lambda_0 d) + \tilde{\eta}\lambda_0\cosh(\lambda_0 d)]} \times \Big\{a(e^{\lambda_0 d} - 1) + \tilde{\eta}\lambda e^{\lambda_0 d}]\tilde{I}_0 + a[e^{\lambda_0 d} - 1 - \lambda e^{\lambda_0 d}(\tilde{\eta} + \lambda_0)]\tilde{Q}_0\Big\}$$

In the limit of small intrinsic carrier concentration $n_i \ll \kappa T^2/D\varepsilon_g{}^2$, the change of the thermal generation rate is small i.e. $a \ll 1$ and the above equations reduce to the ones obtained in previous theories on transport in semiconductors without non-stationary heating:

$$C_{s1} = \frac{(1+\tilde{\eta}d)(\tilde{I}_o + \tilde{Q}_o)\cosh d - \tilde{I}_o}{\tilde{\eta}\cosh d}a, \quad C_{s2} = -(\tilde{I}_o + \tilde{Q}_o)a,$$

$$C_{s3} = -\frac{\tilde{I}_o}{I + e^{2d}}\,, \quad C_{s4} = \frac{e^{2d}\tilde{I}_o}{I + e^{2d}}\,.$$

b) Dynamic part contribution: $T_d(\mathrm{x})$ and $\delta p_d(\mathrm{x})$

From Eqs. (1) and (2) the dynamical contribution to the temperature distribution and carrier concentration in the semiconductor are described by the next expressions

$$\frac{d^2 p_d}{dx^2} - p_d - i\frac{\Omega}{\Omega_\tau}p_d + a\theta_d + i\frac{\Omega}{\tilde{\Omega}}a\theta_d = 0, \tag{20}$$

$$\frac{d^2\theta_d}{dx^2} + p_d - a\theta_d - i\frac{D}{\alpha}\frac{\Omega}{\Omega_\tau}\theta_d = 0, \tag{21}$$

where $\theta_d = T_d/T_\alpha$, $\Omega_\tau = 1/\tau$, $\tilde{\Omega} = e\alpha\gamma/A$.

As can be seen, Eq. (20) is a diffusion equation and therefore the frequency $\tilde{\Omega}$ has not meaning like in electrodynamic theory.

The solutions of Eqs. (20) and (21) are complemented with the following boundary conditions

$$\left.\frac{dp_d}{dx}\right|_{x=0} = -\Delta\tilde{I}_o\,, \qquad p_d\big|_{x=d} = 0\,, \tag{22}$$

$$\left.\frac{d\theta_d}{dx}\right|_{x=0} = -\Delta\tilde{Q}_o\,, \qquad \left.\frac{d\theta_d}{dx}\right|_{x=d} = -\tilde{\eta}\,\theta_d\big|_{x=d}\,, \tag{23}$$

where

$$\Delta\tilde{I}_o \equiv \frac{\Delta I_o L_D}{D p_o}, \qquad \Delta\tilde{Q}_o \equiv \frac{\Delta Q_o L_D}{\kappa T_\alpha}\,.$$

Using Eqs. (20) and (21) together with the boundary conditions Eqs. (22) and (23) the temperature distribution and carrier concentration in the material can be written as

$$p_d = C_{d1}e^{\lambda_1 x} + C_{d2}e^{-\lambda_1 x} + C_{d3}e^{\lambda_2 x} + C_{d4}e^{-\lambda_2 x}\,, \tag{24}$$

$$\theta_d = F_1 C_{d1}e^{\lambda_1 x} + F_1 C_{d2}e^{-\lambda_1 x} + F_2 C_{d3}e^{\lambda_2 x} + F_2 C_{d4}e^{-\lambda_2 x}\,, \tag{25}$$

with

$$F_1 \equiv \left[a + i\frac{D}{\alpha}\frac{\Omega}{\Omega_\tau} - \lambda_1^2\right]^{-1}, \quad F_2 \equiv \left[a + i\frac{D}{\alpha}\frac{\Omega}{\Omega_\tau} - \lambda_2^2\right]^{-1}, \tag{26}$$

$$\lambda_{1,2}^2 = \left\{\frac{1}{2} 1 + a + i\left(1 + \frac{D}{\alpha}\right)\frac{\Omega}{\Omega_\tau}\right.$$

$$\left. \pm\sqrt{\left[1 - a + i\left(1 - \frac{D}{\alpha}\right)\frac{\Omega}{\Omega_\tau}\right]^2 + 4a\left(1 + i\frac{\Omega}{\tilde{\Omega}}\right)}\right\}, \tag{27}$$

and the constants

$$C_{d1} = \{[\tilde{\eta}(F_1 - F_2) - F_1\lambda_1)\cosh(\lambda_2 d) - F_2\lambda_2 sinh(\lambda_2 d)]$$
$$\times(F_2\Delta\tilde{I}_0 - \Delta\tilde{Q}_0) + F_2\lambda_1\exp(\lambda_1 d)(F_1\Delta\tilde{I}_0 - \Delta\tilde{Q}_0)\}$$
$$/(\lambda_1 B\exp(\lambda_1 B\exp(\lambda_1 d)),$$

$$C_{d2} = \{[(\tilde{\eta}(F_1 - F_2) + F_1\lambda_1)\cosh(\lambda_2 d) - F_2\lambda_2 sinh(\lambda_2 d)]$$
$$\times\exp(\lambda_1 d)(\Delta\tilde{Q}_0 - F_2\Delta\tilde{I}_0) - F_2\lambda_1(\Delta\tilde{Q}_0 - F_1\Delta\tilde{I}_0)\}/(\lambda_1 B)$$

$$C_{d3} = \{[(\tilde{\eta}(F_1 - F_2) + F_2\lambda_2)\cosh(\lambda_1 d) + F_1\lambda_1 sinh(\lambda_1 d)]$$
$$\times(\Delta\tilde{Q}_0 - F_1\Delta\tilde{I}_0) - \exp(\lambda_2 d)F_1\lambda_2(\Delta\tilde{Q}_0 - F_2\Delta\tilde{I}_0)\}$$
$$/(\lambda_2 B\exp(\lambda_2 d)),$$

$$C_{d4} = \{[(\tilde{\eta}(F_1 - F_2) - F_2\lambda_2)\cosh(\lambda_1 d) + F_1\lambda_1 sinh(\lambda_1 d)]$$
$$\times\exp(\lambda_2 d)(F_1\Delta\tilde{I}_0 - \Delta\tilde{Q}_0) + F_1\lambda_2(F_2\Delta\tilde{I}_0 - \Delta\tilde{Q}_0)\}/(\lambda_2 B),$$

$$B \equiv 2(F_1 - F_2)\{F_1\cosh(\lambda_2 d)[\tilde{\eta}\cosh(\lambda_1 d) + \lambda_1 sinh(\lambda_1 d)]$$
$$- F_2\cosh(\lambda_1 d)[\tilde{\eta}\cosh(\lambda_2 d) + \lambda_2 sinh(\lambda_2 d)]\}. \tag{28}$$

In the limit when the change of the thermal generation is neglected ($a\longrightarrow 0$), Eqs. (28) reduces to

$$C_{d1} = -\frac{\Delta\tilde{I}_0}{[1 + \exp(2\lambda_1 d)]\lambda_1}, \qquad C_{d2} = \frac{\exp(2\lambda_1 d)\Delta\tilde{I}_0}{[1 + \exp(2\lambda_1 d)]\lambda_1},$$

$$C_{d3} = C_{d4} = 0,$$

$$F_2 C_{d3} = \frac{(\lambda_2 - \tilde{\eta})\cosh(\lambda_1 d)(\Delta\tilde{Q}_0 - F_1\Delta\tilde{I}_0) + F_1\lambda_2\exp(\lambda_2 d)\Delta\tilde{I}_0}{2\cosh(\lambda_1 d)[\tilde{\eta}\cosh(\lambda_2 d) + \lambda_2 sinh(\lambda_2 d)]\lambda_2\exp(\lambda_2 d)},$$

$$F_2 C_{d4} = \frac{(\lambda_2 + \tilde{\eta})\cosh(\lambda_1 d)\exp(\lambda_2 d)(\Delta\tilde{Q}_0 - F_1\Delta\tilde{I}_0) + F_1\lambda_2\Delta\tilde{I}_0}{2\cosh(\lambda_1 d)[\tilde{\eta}\cosh(\lambda_2 d) + \lambda_2 sinh(\lambda_2 d)]\lambda_2},$$

$$\lambda_1^2 = 1 + i\frac{\Omega}{\Omega_\tau}, \qquad \lambda_2^2 = i\frac{D}{\alpha}\frac{\Omega}{\Omega_\tau}.$$

In Fig. 1 we present the frequency dependence of the amplitude and phase signal obtained from Eqs. (24) and (25) for different values of the change of thermal generation rate. It is clearly depicted in this plot the influence of the parameter a in the low-frequency region. As can be seen in Fig. 1a the photoacoustic signal which is proportional to the amplitude, increases with the decreasing of the change of the thermal generation parameter in the low frequency range and they converge to the same value for large chopper frequency. On the other hand, the phase associated with the photoacoustic signal exhibits a maximum when the change of the thermal generation rate vanishes and it disappears with increasing a for low chopper frequency.

From the expressions (24)-(28), it follows that carrier concentration as well the temperature distribution depend substantially on the relationship between the size of the sample and the thermal attenuation length $L = \sqrt{(2\alpha/\Omega)}$. For example if the thickness of the semiconductor d is smaller than L (quasistatic approximation), the temperature distribution is almost constant across the sample and in this situation the time to heat the sample is considerably dependent on the sample thickness whereas the signal phase increases monotonically with the chopper frequency (see Fig. 2). On the other hand, in the limit of high chopper frequency $L \ll d$ the temperature fluctuation decays exponentially from the surface of the semiconductor and in this case the energy relaxation time is frequency dependent

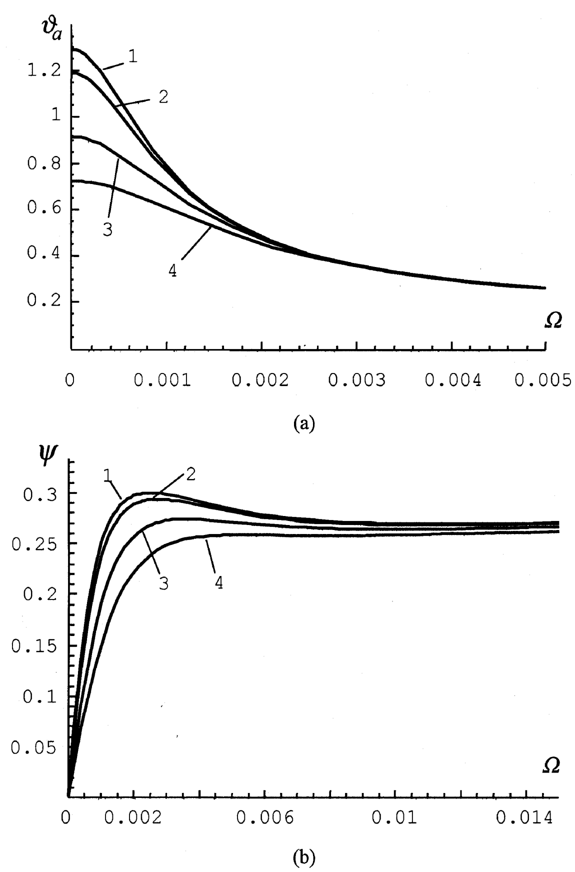


Fig. 1. Influence of the thermal generation rate change on the amplitude a) and phase (b) of the PAE response: **1** - $a = 0$; **2** - $a = 0.1$; **3** - $a = 0.5$; **4** - $a = 1$ ($d = 1$, $\tilde{\eta} = 1$, $\Delta\tilde{Q}_o = \Delta\tilde{I}_o$, $D/\alpha = 10^3$, $\vartheta_a \equiv \theta_a/(\Delta\tilde{Q}_o + \Delta\tilde{I}_o)$, $\theta_a \equiv \sqrt{\mathrm{Re}[\theta_d(0)]^2 + \mathrm{Im}[\theta_d(0)]^2}$, $\psi \equiv \arccos(\mathrm{Re}[\theta_d(0)]/\theta_a)/\pi$). The modulation frequency is normalized by carrier's lifetime τ.

i.e. $\tau_s \sim L \sim \Omega^{-1/2}$ while the signal phase increases until it reaches a maximum for some frequency and then remains constant as function of the chopper frequency.

In Fig. 3 the normalized amplitude and phase signal are shown as a function of the normalized chopper frequency in the limit of poor, intermediate and perfect surface thermal conductivity for vanish thermal generation rate and intermediate carrier diffusion length. As can be seen, the amplitude has an exponential dependence with the frequency whereas the phase has a pronounced maximum for a small parameter of the surface conductivity and it tends to disappear in the limit of perfect surface thermal conductivity in the low frequency limit.

Finally, in Fig. 4 we compare the amplitude and phase signal for various fixed values of the ratio between the radiation energy necessary to create an electron-hole pair ΔI and the light converted into heat in the semiconductor ΔQ.

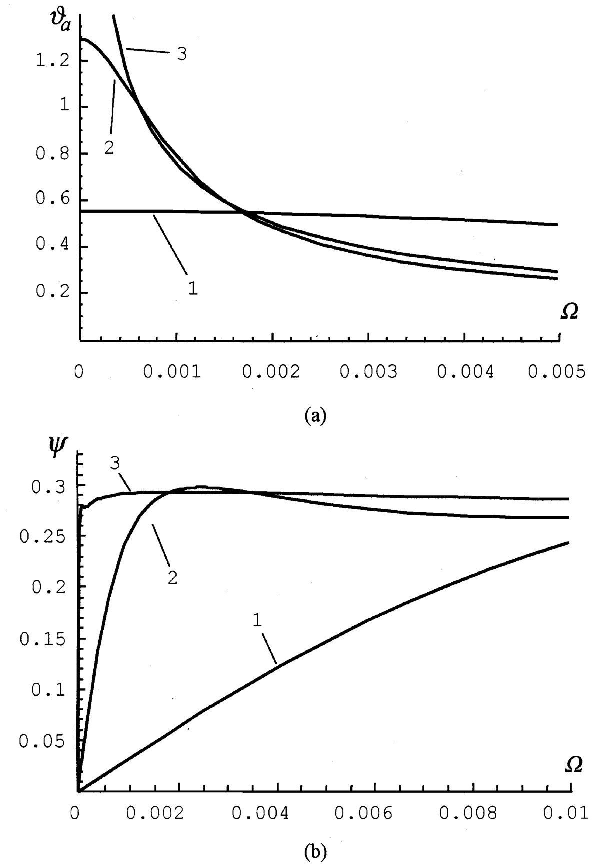
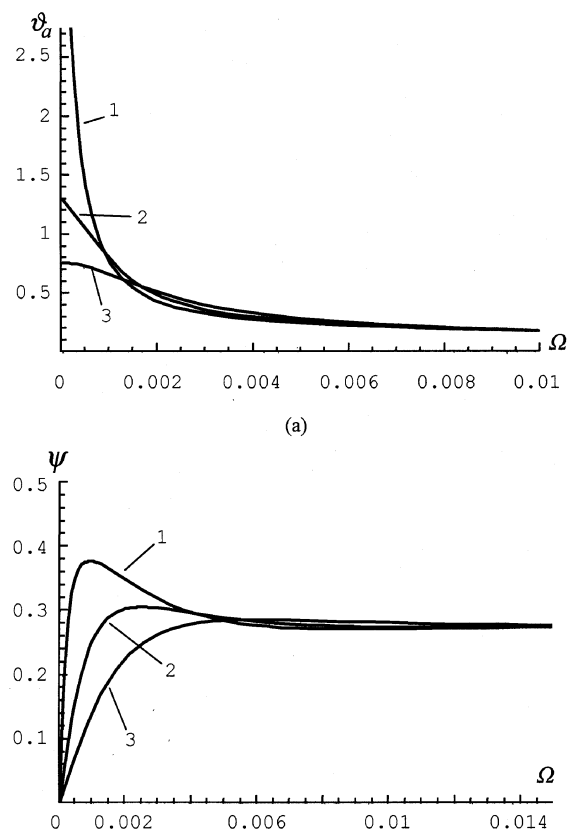


Fig. 2. Frequency characteristics of the PAE in samples of different sizes: **1** - $d = 0.1$; **2** - $d = 1$; **3** - $d = 10$ ($a = 1$, $\tilde{\eta} = 1$; $\Delta\tilde{Q}_o = \Delta\tilde{I}_o$, $D/\alpha = 10^3$).

In this situation the amplitude of the thermal wave for $\Delta I = 0$ is greater than the other ones when electron-hole transition is finite, while the phase has similar behavior as in Fig. 1b.

4. The resonance effect

We now turn to the discussion of one of the amazing results obtained so far and compare them with the corresponding results with previous theories on the photoacoustic effect reported in the literature. In the theoretical models of the photoacoustic effect in semiconductors the carrier diffusion and recombination mechanism have been considered in the transport properties. However, as has been showed in this work, the thermoelectric field (thermal waves) mechanism has a strong effect on the electron-hole pair recombination rate through the parameter γ. Taking into account this new mechanism in our theoretical model, the solutions for the carrier concentration and temperature distribution are given by Eqs. (24) and (25), respectively. According to these results, if the change of the thermal recombination rate is



(a)

(b)

Fig 3. Influence of the surface thermal conductivity on the frequency characteristic of the PAE: **1** - $\tilde{\gamma} = 0.2$; **2** - $\tilde{\gamma} = 1$; **3** - $\tilde{\gamma} = 5$ ($a = 0$, $d = 1$; $\Delta\tilde{Q}_o = \Delta\tilde{I}_o$, $D/\alpha = 10^3$).

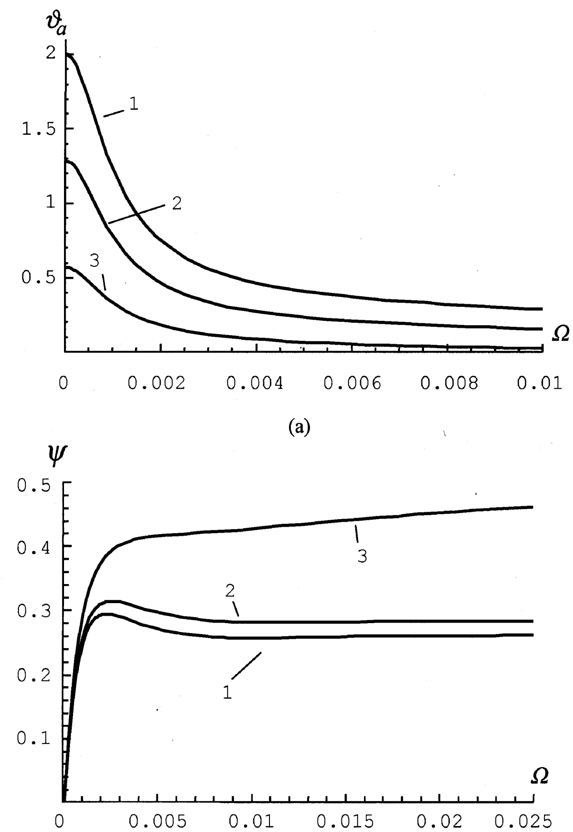
equal to unit i.e. $a = 1$ ($\gamma = \kappa/D\varepsilon_g$) and the incident modulated light frequency is given by $\Omega_R = 2\alpha/|\alpha - D|\tau$, a resonance effect should be observed, the temperature amplitude becoming very large. We should expect the very large amplitudes in the temperature fluctuations at resonance to give rise to a strong peak in photoacoustic signal at the chopper resonance frequency. However this phenomenon does not occur in typical photothermal experiments. In order to see this more clearly, let us solve the coupled carrier and heat diffusion equations at the resonance frequency Ω_R. In this case the carrier concentration and the temperature distribution become

$$p_r = (C_{r1} + C_{r2}x)e^{\lambda x} + (C_{r3} + C_{r4}x)e^{-\lambda x}, \tag{29}$$

$$\theta_r = -i(C_{r1} - 2i\lambda C_{r2} + C_{r2}x)e^{\lambda x} - i(C_{r3} + 2i\lambda C_{r4} + C_{r4}x)e^{-\lambda x}, \tag{30}$$

$$\lambda^2 = 1 + \frac{i}{2}\left(1 + \frac{D}{\alpha}\right)\frac{\Omega_R}{\Omega_\tau} = 1 + i\frac{D+\alpha}{D-\alpha}, \tag{31}$$

where the integration constants can be written as



(a)

(b)

Fig. 4. Frequency characteristics of the PAE for different proportions between fractions of the light, causing carrier generation and direct heating of the specimen: **1** - $\Delta\tilde{I}_o = 0$; **2** - $\Delta\tilde{Q}_o = \Delta\tilde{I}_o$; **3** - $\Delta\tilde{Q}_o = 0$ ($a = 0$, $d = 1$; $\tilde{\gamma} = 1$, $D/\alpha = 10^3$).

$$C_{r1} = -\{[(\lambda d + 1 - 2i\tilde{\gamma}\lambda)(i\Delta\tilde{Q}_0 - \Delta\tilde{I}_0) + 2i\lambda^2\Delta\tilde{I}_0] \times \lambda d\exp(\lambda d) + [(i + 2\lambda^2)sinh(\lambda d) + 2\tilde{\gamma}\lambda\cosh(\lambda d)] \times [(i + 2\lambda^2)\Delta\tilde{I}_0 + \Delta\tilde{Q}_0]\}/[B\lambda\exp(\lambda d)],$$

$$C_{r2} = \{[sinh(\lambda d) - 2i\tilde{\gamma}\lambda\cosh(\lambda d) + \lambda d\exp(\lambda d)] \times (i\Delta\tilde{Q}_0 - \Delta\tilde{I}_0) + 2i\lambda^2[i\Delta\tilde{Q}_0\cosh(\lambda d) + \Delta\tilde{I}_0 sinh(\lambda d)]\} /[B\exp(\lambda d)],$$

$$C_{r3} = \{[(i + 2\lambda^2)sinh(\lambda d) + 2\tilde{\gamma}\lambda\cosh(\lambda d)]\{(i + 2\lambda^2)\Delta\tilde{I}_0 + \Delta\tilde{Q}_0]\exp(\lambda d) + \lambda d(1 - \lambda d + 2i\tilde{\gamma}\lambda)(i\Delta\tilde{Q}_0 - \Delta\tilde{I}_0) + 2i\lambda^3 d\Delta\tilde{I}_0\}/(B\lambda),$$

$$C_{r4} = (\{[2i\lambda^2 - 1)\Delta\tilde{I}_0 + i\Delta\tilde{Q}_0]sinh(\lambda d) + 2[\tilde{\gamma}\lambda(i\Delta\tilde{I}_0 + \Delta\tilde{Q}_0) + \lambda^2\Delta\tilde{Q}_0]\cosh(\lambda d)\}\exp(\lambda d) + i\lambda d(i\Delta\tilde{I}_0 + \Delta\tilde{Q}_0))/B$$

$$B \equiv 2\lambda^2[2i\lambda d + 4\tilde{\gamma}\lambda\cosh^2(\lambda d) + (i + 2\lambda^2)sinh(2\lambda d)].$$

As can be seen, this resonance singularity under consideration does not appear in the temperature amplitude and as a consequence there is not a peak in the photoacoustic experiments. This is because Eqs. (1) and (7) represent a diffusion process instead of a wave phenomenon. However, we consider this effect sufficiently interesting and worth to being mentioned.

5. Conclusions

In this paper we have presented a theoretical analysis of the photoacoustic effect in semiconductors taking into account the influence of both carrier diffusion and recombination effects. In addition, we have also included in our calculations for the first time, the effect of the thermoelectric field on the thermal generation process. We have derived exact solutions for the carrier concentration an temperature distribution in the sample.

It is shown that the influence of the thermoelectric field on the thermal generation rates, the temperature distribution in the semiconductor a suitable resonance effect appears, however, this is a spurious effect in these solutions.

The above findings tell us that the photoacoustic effect can be used not only for obtaining information about the thermal parameters of the material but can also be used as an alternative tool for measuring carrier recombination time, carrier diffusion coefficient and for example, the thermal dependence of the intrinsic concentration. Furthermore, the photoacoustic spectroscopy may give us information about the electron-phonon energy interaction in the two temperature approximation i.e each quasiparticle system is described by its own temperature T_e and T_p, respectively.

Acknowledgments

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